PROBABILITY AND STATISTICS IN ACCESS EXAMS TO SPANISH UNIVERSITIES

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Exams to get access to Spanish universities are performed region-wide every year. These exams are based on the curriculum established for the Spanish Baccalaureate. We perform a crossed analysis between curricular guidelines and items appearing in the exams, in four Spanish regions, so as to detect prevalent units, similarities and differences. Thus, we check what competencies and curricular units appear in the official curriculum related to Statistics and Probability, and how they are assessed in the exams. We show how these exams influence teaching methodologies; over-representing mechanical exercises instead of interpretive questions, especially applications of Statistics and Probability in students' everyday life. This fact considerably lessens the importance of a teaching/learning methodology based on competencies.

INTRODUCTION

This communication focuses on how Statistics & Probability is considered in Spain, in Applied Mathematics to Social Sciences at University access exams (hereinafter, PAU, from Spanish *Prueba de Acceso a la Universidad*) since the last reform in Spanish Baccalaureate (2010 was the first time this curricular reform was evaluated in Spain at PAU) from 2006 (JE, 2006).

The syllabus of Applied Mathematics to Social Sciences subject is the basis for designing the exams. These exams are compulsory for every student after Secondary Education (*Bachillerato*, in Spanish Educational system) to enter University. The syllabus of Applied Mathematics to Social Sciences is fixed by National government (MEC, 2007), but Autonomous Communities can also add items or extend them in greater depth (CECPA, 2008; CEA, 2008; CECM, 2008; DE, 2009). Applied Mathematics to Social Sciences is a different subject from Mathematics; the later are basically oriented to Science and Technology students (and does not contain Statistics and Probability), whilst the former are designed for future Social Sciences students (Primary Teacher, Education, Business, Economy, Accounting, etc.).

The general structure of the PAU is the same in the whole country. PAU consists of two phases: a general phase and a specific one. General phase is compulsory and students can choose to do Applied Mathematics to Social Sciences as one of the four compulsory exams. Specific phase is optional and students have to choose what subjects they want to do, but different from general phase, so they can choose Applied Mathematics to Social Sciences, if not examined in general phase. In addition, exams consist of two options and students must choose and solve completely one of them.

We have analyzed PAU exams from four Autonomous Communities (Andalucía, Asturias, Madrid and País Vasco), focusing on a descriptive analysis of appearing curricular units, so as to detect prevalence units, similarities and differences and studying how the prevalence of some curricular units can influence in teaching-learning process of Statistics & Probability in the last course of Baccalaureate in Spain, because it is known that the assessments of educational system determine the way of teachers orientate their work in the classroom, trying to adapting their methodology. Within Spanish contexts we can mention studies by Boal et al. (2008), Contreras de la Fuente et al. (2010), Ferrer (2006), Muñoz de Miguel et al. (1988) and Nortes and Nortes (2010). Other studies have analyzed similar exams in other countries, e.g., High School Exit Exams in the USA (Bishop, 2000; Ou, 2010; Warren & Edwards, 2005).

METHODOLOGY

Data has been collected from released exams on universities websites (Junta de Andalucía, Universidad de Oviedo, Universidad Complutense de Madrid, and Universidad del País Vasco). There are different numbers of exams in each community, due to the differences of PAU-

organization in each of them: some communities organize general phase and specific phase separately and others jointly, so it implies that the number of released exams is different in each of them. Other communities also publish their reserve exams, as in the case of Andalucía. Exams have two options; in order to obtain clear results, we have considered each option as an individual exam, so the number of exams analyzed is: 36 exams in Andalucía, 24 exams in Asturias, 22 exams in Madrid and 12 exams in País Vasco.

In order to quantify contents and competences appearing in exams, we have designed curricular units. To construct them, we have considered the contents and the evaluation criteria both expressed in the curricula (CEC, 2008; CECM, 2008; CEA, 2008; DE, 2009), grouping them in basic units, which help us to produce significant results in order to appreciate prevalent units. Obtained curricular units are shown in Table 1. It is necessary to underline that Spanish curriculum presents separately contents and evaluation criteria (properly, learning outcomes), therefore this gathering what we have performed, allows us to overview the syllabus from a jointly point of view.

Table 1: Curricular units for Statistics

S1: Probability	Probability with easy events. Composed events. Laplace's rule.
S2: Independence of events	Definition. Rules. Probability of dependent or independent events.
S3: Conditional probability	Definition. Contingency tables. Probability of conditional events.
S4: Bayes	Law of total probability. Bayes' theorem. Tree diagrams. Calculating posterior probability.
S5: Decision making	Making decisions in uncertainty problems in Social Sciences
S6: Practical applications of limit theorems	Central limit theorem. Normal approximation of binomial distribution. Law of large numbers. Identification of normality. Determining the type of distribution.
S7: Sampling	Determining the type of sampling and the sample size. Representativeness.
S8: Confidence intervals	Probability distributions of sample mean and proportion. Confidence interval for mean and proportion.
S9: Hypothesis testing	For mean, proportion, and difference of means or proportions. Determining significant differences between means or proportions of two populations.
S10: Inferring conclusions	Ability to infer conclusions from data obtained in Inferential Statistics. Formulating critical reasoning.
S11: Distributions	Binomial and Normal distribution. Calculus of probabilities about them.

ANALYSIS RESULTS

We first calculate frequency of curricular units in the Autonomous Communities considered but, in order to avoid the effect of the difference in the number of exams, the percentage of appearance is showed in Figure 1.

Statistics & Probability

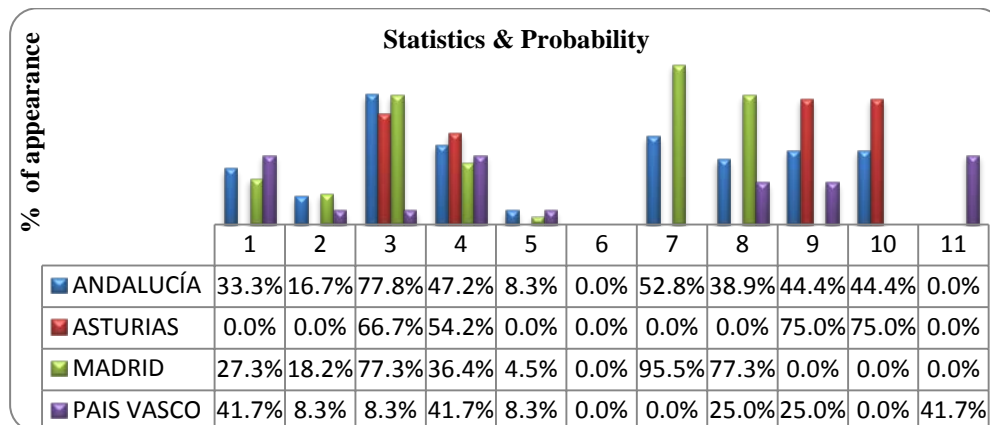|	1	2	3	4	5	6	7	8	9	10	11
ANDALUCÍA	33.3%	16.7%	77.8%	47.2%	8.3%	0.0%	52.8%	38.9%	44.4%	44.4%	0.0%
ASTURIAS	0.0%	0.0%	66.7%	54.2%	0.0%	0.0%	0.0%	0.0%	75.0%	75.0%	0.0%
MADRID	27.3%	18.2%	77.3%	36.4%	4.5%	0.0%	95.5%	77.3%	0.0%	0.0%	0.0%
PAIS VASCO	41.7%	8.3%	8.3%	41.7%	8.3%	0.0%	0.0%	25.0%	25.0%	0.0%	41.7%

Figure 1. Percentage of appearance of Curricular Units for Statistics & Probability

We also have to take into account that Statistics & Probability is not the only topic in the curriculum of Applied Mathematics to Social Sciences. It also gathers Algebra and Calculus. Therefore, Statistics & Probability do not have always the same weight in the exams. In Andalucía, Madrid and País Vasco the set of questions of Statistics & Probability represents 50% of the whole exam, whereas in Asturias it varies from 25% to 50% depending on the option.

As we can see in Figure 1, the distribution of curricular units in exams is very heterogeneous, S3 is the most posed in average, S6 never appears, and other units have few occurrences or even null frequency in some Communities. A possible explanation to null frequency of S6 is that it refers to limit theorems, which is a difficult item to be asked in an isolated question, but it is a necessary tool when using confidence intervals or hypothesis testing for proportions.

If we pay attention to the kind of activities proposed in the exams, we note that all exercises in Asturias and País Vasco are related to real-life problems that have to be translated into statistical or probabilistic language; this is a very important skill in the curriculum: *translating real situations to mathematical language*. Common topics in daily life such as alcohol consumption in young people, granting of loans or health and safety at work, shape the problems. On the other hand, in Madrid and Andalucía there is a mixture of real-situations problems with others where probability of different events must be calculated without any contextualization.

Analyzing the results of each Community individually we can observe that: in Asturias the structure of Probability exercises is always the same: use of Bayes' formula and total probability; meanwhile Statistics exercises usually include hypothesis testing, for mean and proportion.

In Madrid, Probability exercises vary more than in Asturias: Bayes' formula, the law of total probably, tree diagrams, contingency tables or Laplace´s rule. Exams include very few problems where students have to make a decision; despite it is required in the official curriculum (MEC, 2007). On the other hand, in Statistics exercises, questions about confidence intervals for the mean are posed (but never for proportion) and also questions about minimum sample size to attain a given maximum error, but we never find hypotheses testing exercises.

Andalucía shares with Madrid the same scheme of Probability questions. Regarding the Statistics items, there are three major types of exercises: confidence intervals (in 14 of 36 exams), hypothesis testing (16 of 36) and sampling or distribution of sample mean (14 of 36).

Finally, in País Vasco exams have several exercises for calculating probabilities based on normal distribution and problems to be solved by using Bayes' formula or total probability. There are also probability exercises where student have to use combinatorial analysis or counting techniques (which does not properly belong to the curriculum). In a lower percentage, we have found very basic problems about statistical inference, 3 on hypotheses testing (solving the test) and 3 on confidence intervals (construct the interval).

CONCLUSIONS

In Spain, although every Autonomous Community organizes PAU individually, the scores are used to provide a general access to all public universities, so this can cause several problems if there are notable differences in the exams. In the case of Statistics & Probability, differences in the most frequent questions can cause notable mathematical lacks in students (due to conditioning on teachers' activity) and different grades in the competences and objectives achievement.

Most exams repeat the scheme from one to another, making PAU repetitive and stable in this kind of exercises. This fact has positive points because students work these curricular units in depth but, on the other hand, it determines the teaching-learning process which emphasize in working the same type of exercises during all the course, and developing only algorithmic skills, in order to achieve good grades in PAU. Therefore, essential and original points in the official curriculum for Applied Mathematics such as the interpretation of issues related to social sciences from a mathematical point of view, critical assessment of the obtained results or the importance of inferential statistic to study commercial, economical and political situations are poorly represented in PAU.

The Royal Decree (MEC, 2007) establishes the objectives of Applied Mathematics to Social Science. Among these objectives are *"to apply mathematics in different situations to analyze, interpret and evaluate social phenomena in order to understand and appropriately express aspects of social and economic reality."* In addition, it also indicates the need of "*interpret,*

understand and appreciate the reality, establishing relationships between mathematics and the social, cultural or economic terms." In particular, this is especially important in the case of the Probability and Statistics. However, in general, the questions about real life problems are underrated in PAU. These results are related to the obtained in different national frameworks (in Jellison Holme *et al.*, 2010, an interesting meta-analysis of HSEE can be found), pointing out the effect of exit of exams on the real curriculum, and the risk of transforming teaching in test preparation.

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