STOCHASTIC AND DETERMINISTIC VIEWS OF STATISTICS: A PEDAGOGICAL PERSPECTIVE

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Learning and teaching statistics is a juxtaposition of two lines of reasoning—deterministic and stochastic. Research has shown that the deterministic line of reasoning is in a superior position to stochastic reasoning. Hence, the stochastic line of reasoning must be strengthened to create a better balance between the two lines in order to work toward better statistics learning and teaching. By placing greater emphasis on variability in the data, we will contribute to better stochastic reasoning. This paper adopts the variation theory of learning approach as a pedagogical tool to emphasize variability in statistics and to help students develop their stochastic reasoning. This, we claim, might lead to the sustainable learning of statistics.

BACKGROUND

This paper is within the theoretical framework of an ongoing research project entitled Sustainable Learning of Statistics (SLS). It is a four-year project (2021–2024) funded by the Swedish Research Council and owned by the University of Gothenburg. The purpose of this project is to investigate how sustainable learning in the domain of statistics can be enhanced, particularly in statistics taught in postgraduate research methods courses. We use the variation theory of learning (Marton, 2014) to pursue two aims: (a) to provide a theoretical and practical case for how sustainable learning in the domain of statistics can be enhanced, and (b) to test the conjectures of the variation theory of learning and contribute to its theoretical development.

We see that sustainable learning is characterized by (a) learning that enables students to deal with novel situations in powerful ways and (b) learning that persists and continues to grow.

This paper highlights a core issue in this project: enabling learners to focus on variability in statistics to a greater extent. This will most likely facilitate more sustainable learning of statistics. As the following sections will show, our argument is based on two sources: (a) learning and teaching statistics as a juxtaposition of deterministic and stochastic perspectives and (b) findings in the research approach of variation theory.

DETERMINISTIC AND STOCHASTIC REASONING

We see the learning and teaching of statistics in terms of the juxtaposition of two lines of reasoning: (a) deterministic reasoning, which is oriented toward exact numbers and causal explanations, and (b) stochastic reasoning, which is oriented toward distributed alternatives and probabilistic reasoning.

When tracking the development of statistics, it can be seen that statistics originated in a deterministic context, where it was about “counting” populations. In *The History of Statistics*, Fienberg (1991) showed that, as early as 3050 B.C., the ancient Chinese and Egyptians compiled data concerning wealth, agriculture, animals, the military, and society. The term “statistics,” which comes from the Latin “status,” means “states,” and what it reflects of governmental data and information might confirm the deterministic view of the emergence of this science. Statistics has been growing and developing toward abandoning determinism and causality to facing reality. This reality demands dealing with uncertainty and complexity. Statistics has grown from counting things into explaining, estimating, and predicting phenomena: studying the population through studying a sample and studying the future through studying the past and the present. This requires a nondeterministic mindset to deal with the existence of randomness and complexities.

Over the last 30 years, the statistics education community has been calling for statistical reasoning to be taught as a balance between stochastic and deterministic reasoning. For example, Pfannkuch and Brown (1996) stated that the task of educators is to enable students to be comfortable thinking probabilistically and deterministically.

Research in psychology and education has provided evidence that people generally demonstrate less developed ways of reasoning amid uncertainty (Shaughnessy, 1977). More recent studies show

In S. A. Peters, L. Zapata-Cardona, F. Bonafini, & A. Fan (Eds.), *Bridging the Gap: Empowering & Educating Today's Learners in Statistics. Proceedings of the 11th International Conference on Teaching Statistics (ICOTS11 2022), Rosario, Argentina.* International Association for Statistical Education. iase-web.org

some evidence of students' tendency to use deterministic reasoning (e.g., Nilsson, 2013; Sánchez et al., 2018). In our current SLS project, deterministic reasoning can be seen in graduate-level students. For example, one of the questions that was discussed in a pre-teaching interview was about a certain manufacturer that produces 50% brown candies. The question was about which bag was more likely to have more than 70% brown candies, a large family-size bag, or a small fun-size bag. One student's answer was that both bags had the same chance. He could not see that there was more variability in the proportion of browns among the smaller samples. He insisted that 70% was a fixed proportion; accordingly, there was no difference.

Based on the above, a question should be raised: How can we affect students' deterministic ways of thinking about statistical problems so that they will learn to adopt a certain form of statistical reasoning that reflects stochastic reasoning?

WHAT MAKES LEARNING POSSIBLE?

Variation theory (Marton, 2014; Marton & Booth, 1997) is a learning theory that explains how learners come to understand or see phenomena in certain ways. The core tenet of variation theory is that people learn through experiencing differences and similarities (in this order). Hence, it also implies that learning can be brought about by affording learners the experience of such differences and similarities. Thus, it represents a theoretical framework that can direct teachers' attention toward what must be done to provide learners with necessary learning opportunities.

According to variation theory, learning takes place when the learner's way of understanding changes from one way to another, more complex way. Variation theory clarifies and explains how such changes can be made possible. To see something in a specific way, the learner must discern certain aspects or attributes of the object of learning, and the only opportunity to discern them is when they vary (Marton, 2014). Critical aspects are aspects that must be discerned by a learner to develop a certain understanding of an object of learning. For example, if the object of learning is to understand the concept of "mean" in statistics, the learner has to discern aspects such as the following: the mean is representative of all data values; the sum of deviations from the mean equals zero; the mean is influenced by the extreme values; thus, it is not always a good idea to use it to represent data; and to find the mean, add up all the values and then divide by the number of values. If a learner can calculate the mean but does not see it as a representative indicator of the data, the former aspect is not a critical aspect, but the latter is a critical aspect that a student has to discern. If one learner discerns certain aspects of something and another learner discerns different aspects, we say that the two learners see the same thing in different ways. Thus, a way of seeing something can be defined in terms of the aspects that are discerned at a certain point in time.

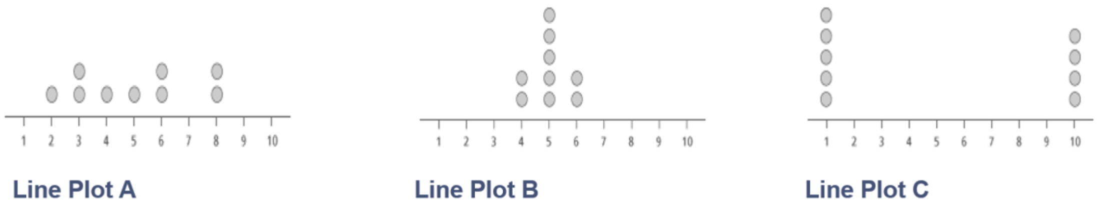
The most effective way to help learners learn is to focus on providing opportunities for them to experience variations in critical aspects of the object of learning. Here, we want to clarify that deliberately using specific patterns of variation and invariance does not guarantee that learning will happen. Instead, we argue that patterns of variation make learning possible by seeing an object of learning in a specific way. Hence, it is a necessary but not sufficient condition (Marton & Tsui, 2004). To understand what is made possible to learn and what is not possible to learn in a learning situation, it is necessary to pay close attention to what aspects are varied and what aspects are invariant in the situation (Marton, 2014). We term this variation and invariance a pattern of variation. A substantial number of studies have been carried out indicating that variation (in learning) is a necessary condition of learning, and it has proven to be a powerful pedagogical tool with the potential to make learning sustainable (Marton, 2014).

In teaching statistics, opportunities for learners to interact with patterns of variation and invariance could be provided through noticing, acknowledging, measuring, describing, modeling, explaining, and predicting the variability in data (e.g., Reading & Shaughnessy, 2004). For example, for learners to understand the concepts of *spread* and *standard deviation*, they have to experience different data sets (vary the spread) with the same mean (do not vary the mean). Figure 1 displays an activity that shows such patterns of variation.

Activity

Three classes A, B, C, each has 9 students. The following data represent number of books read by each student last semester.

Question 1: find the mean for each class.



mean (class A)= ______ mean (class B)= _____ mean (class C)= ____

Question 2: of the three, which line plot's data has the least variation from the mean?____

Explain why_________________

Which has the most variation from the mean?____ why?___________

Question 3: From working on this activity, you probably got an intuitive sense of the variation in the three data sets. But is there a way to measure exactly how much the values in a line plot differ from the mean? Any suggestions?

Figure 1. Activity for varying the spread but not the mean

SUSTAINABLE LEARNING OF STATISTICS

Several studies have provided evidence that when people learn something, they may use it in situations they have never seen before. Using systematic patterns of variation and invariance has shown positive effects in delayed assessments in many cases (e.g., Fülöp, 2019; Holmqvist & Lindgren, 2009; Marton & EDB Chinese Language Research Team, 2010; Marton & Pang, 2007; Pang, 2010, 2019; To & Pang, 2019).

Marton (2006) explained the positive results of these studies: When students interacted with patterns of variation and invariance, they were likely to develop more powerful ways of seeing the object of learning and become better at discerning its critical aspects. Furthermore, every time they encountered instances of the object of learning after the end of the learning occasion, the likelihood of discerning those aspects grew higher. The studies suggest that every time learners discern critical aspects, they become better at discerning new critical aspects (Marton, 2006), which supports the idea of variation theory as a powerful pedagogical tool for improving learning.

On the other hand, the statistics education community has already acknowledged the importance of students working with variability in data (e.g., Bargagliotti et al., 2020). Many examples from the literature have shown how considering variability in data can promote stochastic reasoning (see, for example, Meletiou-Mavrotheris & Lee, 2002; Pfannkuch & Brown, 1996; Prodromou & Pratt, 2013). From the perspective of variation theory, when learners are exposed to variability in data, this automatically includes variations in learners' experiences of what they are being asked about.

This paper offers an invitation to consider the possibilities that variation theory offers with regard to designing instruction in a more systematic way.

CONCLUSION

To support students in reasoning stochastically, teaching has to make it possible for them to experience uncertain situations and handle random variables. Our point is to open students' eyes to stochastic ways of reasoning. The main tool for bringing about this change is to place greater emphasis on variability in data, and hence, according to variation theory, variation in the learners' experience.

Variation theory can serve as a pedagogical tool for researchers and teachers when planning and enacting teaching to facilitate the sustainable learning of statistical reasoning, which requires both deterministic and stochastic reasoning.

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