SECONDARY TEACHERS' UNDERSTANDING OF STATISTICAL MODELING AFTER TEACHING A SIMULATION-BASED STATISTICAL INFERENCE COURSE

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Simulation-based methods for teaching statistical inference have been touted as effective for helping students develop authentic understanding of statistical modeling and inference. This research used problem-solving interviews designed to explore teachers' understanding of statistical models and the connections between model, study design, data, and inference. The interviews were conducted with four secondary teachers who were teaching an undergraduate-level simulation-based introductory statistics course to secondary students. The results suggest that these teachers were able to use the simulation-based methods effectively, yet had several misconceptions that may stem from their non-simulation-based statistics education. This research has implications for the professional development of instructors who plan to teach statistics using simulation-based methods.

BACKGROUND

Recently, introductory statistics curricula have been developed that focus on statistical modeling and simulation (e.g., Lock et al., 2016; Tintle et al. 2015; Zieffler & Catalysts for Change, 2017). These curricula have been used at both the college and secondary levels. Previous research indicated these curricula are associated with improved student understanding of statistical concepts and student attitudes towards statistics (e.g., Beckman, delMas, & Garfield, 2017; Chance, Wong, & Tintle, 2016). To date, little of this research has been focused at the teacher-level.

Models and Statistical Models

There is not a single agreed upon definition of a model. Lesh and Fennewald (2010) point out the "fuzziness" in defining the term *model* should not stop investigations into models and modeling. A seminal framework by Wild and Pfannkuch (1999) suggested that statistical thinking is dependent on modeling skills. "The main contribution of the discipline of statistics to thinking has been its own distinctive set of models, or frameworks, for thinking about certain aspects of investigation in a generic way" (p. 227).

Statistical models, considered extensions of mathematical models, contain systematic (deterministic) processes plus crucial additional stochastic elements represented by probability distributions (Davison, 2003). Variability from the stochastic processes can arise from both random and systematic sources (Davison, 2003). Statistical models can model this random variation through theoretical probability distributions or through simulated data generating devices (McCullagh, 2002).

Knowledge for Teaching Statistical Models

Teacher knowledge has been identified as an important predictor of students' knowledge and understanding. Research from the Learning Mathematics for Teaching Project has found that student achievement in mathematics is related to teachers' *pedagogical content knowledge* (PCK) in mathematics (Hill, Rowan, & Ball, 2005) and *mathematical knowledge for teaching* (MKT) (Hill, Ball, & Schilling, 2008). Recent research into *statistical knowledge for teaching* (SKT) shows similar findings (e.g., Callingham, Carmichael, & Watson, 2016; Godino, Ortiz, Roa, & Wilhelmi, 2011; Groth, 2007; Leavy, 2015). Unfortunately, many secondary teachers lack the SKT for teaching statistics (e.g., Lovett & Lee, 2017).

While there is some research that examines teachers' conceptions and understanding of mathematical modeling (e.g., Baumert, 2010; Cetinkaya et al., 2016), research related to teachers' knowledge of statistical models and modeling is more sparse. Evidence suggests that teachers have more difficulty modeling phenomena involving stochastic processes (Doerr & Lesh, 2011; Lee &

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Mojica, 2008). For example, in studying preservice teachers' understanding of modeling to carry out statistical inference, Biehler, Frischemeier, and Podworny (2015) found that one of the teachers' greatest difficulties was in understanding and generating data under a null model, a process entirely based on modeling a stochastic process.

Given this background, the current study explores the following research questions with regard to secondary mathematics teachers who teach a simulation-based statistics course.

1. How do teachers view statistical models, particularly in contrast to mathematical models?
2. When designing or evaluating statistical models in the context of simulation-based methods, what competencies and misunderstandings do teachers demonstrate?

Answers to these questions have implications for the professional development that both pre- and in-service teachers may require to effectively teach simulation-based statistical concepts.

METHODS

Participants

Four secondary mathematics teachers participated in this study. Each participant had taught one year of an introductory statistics course as part of the College in the Schools (CIS) program at the University of Minnesota (Zieffler & Huberty, 2015). The CIS program allows students to earn college credit for taking university-level courses in their secondary schools. The CIS statistics course uses the CATALST curriculum (Zieffler & Catalysts for Change, 2013), which is based on pedagogical principles founded in research (see Garfield, delMas, & Zieffler, 2012). CIS teachers are experienced secondary school faculty who have had their qualifications vetted for teaching the university-level course. These teachers are also provided with ongoing professional development. The four teachers were purposefully selected based on their varied experiences teaching statistics. Table 1 summarizes the educational and teaching backgrounds of the participants.

Table 1. Background of study participants

Participant	Bachelor's degree	Master's degree	Total years teaching	Prior statistics teaching
1	Math education	None	18	Probability & Statistics
2	Mathematics	Education	16	Advanced Placement Statistics
3	Math education	Education	12	None
4	Math education	Education	25	Advanced Placement Statistics

Interview Tasks

Each participant completed an individual problem-solving interview designed to elicit their reasoning about statistical models. The interview included a series of tasks, adapted from research by Pfannkuch, Budgett and Arnold (2015), in which the participant compared pre- and post-differences in a set of blood pressure measurements. These measurements were taken from subjects that were randomly assigned to a fish oil diet or a regular oil diet. The goal was for a participant to decide whether they were convinced that one diet was more effective than the other in reducing blood pressure, given the data. A participant was then asked to use TinkerPlots^{TM} to model the amount of experimental variation and use that to answer the same question.

In a second task, a participant was shown a series of TinkerPlots^{TM} samplers (visual depictions of a model generating process) that either *did* or *did not* correctly model the experimental variation in the previous task. The participant was then asked to discuss whether or not each sampler could be used to provide an answer to whether one diet was more effective than the other in reducing blood pressure.

RESULTS

In the first task, three of the four participants were able to correctly model the experimental variation in the data for using TinkerPlots^{TM} in the initial task. Participant 4 incorrectly set up a model that assigned treatments with replacement, which resulted in an overestimation of the experimental error. In the second task, the participants performed poorly when asked to identify if each TinkerPlots™ sampler correctly modelled the experimental variation. In contrast, their reasoning about *why* a particular model would correctly generate data to quantify the experimental variation was more informative. It was common for participants to comment about needing to pay attention to whether a sampling device was set up *with* or *without replacement* and for them to consider this while evaluating data generation devices used in the model. For example, two participants noted that a spinner device, sampling *with replacement*, may not assign exactly half of the values to each condition. However, this recognition did not extend to understanding that the goal, regardless of device, was to model the experimental variation, a process that requires sampling without replacement. Even when participants recognized this in a model, the broad ideas seemed to be isolated to that model and did not necessarily carry over to other models.

DISCUSSION

Most secondary mathematics teachers have very limited training in statistics, and usually even less with pedagogy related to teaching statistics. In particular, Participant 2 described how his statistics coursework taught him to conduct statistical tests using rote procedures without learning how to think and model statistically. With minimal training these secondary mathematics teachers were able to design a model that closely mimicked the models taught in the CATALST curriculum. The teacher participants all seemed to show an understanding that statistical modeling requires accounting for randomness and uncertainty.

While the participants all seemed to show an understanding of some elements of statistical modeling (e.g., randomness and uncertainty), they were inconsistent in their own use and understanding of statistical models. Evidence from this study indicates that these teachers were able to design a model that closely mimicked the models they taught in the curriculum. However, when they encountered models that were different from those in the curriculum, they were unable to abstract and transfer the important characteristics for modeling experimental variation, despite having worked with the data generating devices in other contexts. Furthermore, participants who had taught statistics using normal-theory based methods (e.g., Advanced Placement Statistics) often incorrectly fused ideas of these methods with simulation-based models (e.g., trying to impose normal-theory assumptions on the simulation methods without clearly understanding why they were necessary).

The results of this study suggest that mathematics teachers need additional experiences with modeling and simulation to complement their preparatory experiences in statistics. Without this development, a teacher's misunderstanding or limited understanding of statistical modeling could have detrimental effects on their students' understanding and reasoning. Future studies could explore the types of tasks that can be used to reveal gaps in teachers' understanding and also study what professional development experiences can help develop the necessary statistical content knowledge for teaching.

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