EFFECTS OF STATISTICAL WORDS ON THE WAY STUDENTS VIEW DATA

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As statistics become increasingly important so does the need to enable students to draw reasonable conclusions about uncertain events. When students analyze and interpret data they need to be able to take into account not only the center but also the variation in the data. This paper reports the effects of statistical words on the way students view data by investigating the students' written arguments. The practices were conducted in two different classes. The students had the same learning experience and the same teacher gave them the same task, but the teacher's utterances concerning the statistical words were different between the classes. This result suggests that statistics teacher need to introduce the statistical words that represent the variation in data.

INTRODUCTION

Statistical inference for drawing reasonable conclusions from data is a crucial skill in modern societies filled with statistical data (Makar & Rubin, 2009). It is essential for exploring uncertain events as well as reading statistical information, so it forms an important part of statistical literacy (e.g., Gal, 2004). We must help students develop the skill in statistics education.

It is not appropriate to pay attention only to the center in data in drawing reasonable conclusions from the data. It is important to consider not only a measure of the center of data but also the extent of variation around it keeping in mind the existence of error and randomness included in the data set (Konold & Pollatsek, 2002; Reading & Reid, 2006). Formally speaking, the variation in the central part in data is quantified using the theoretical distribution and the confidence interval. The idea is generally taught at the end of upper secondary school or in introductory statistics course in college and university. However, in previous studies such as Garfield, Le, Zieffler & Ben-Zvi (2015), it is pointed out that it is significant to introduce the informal way of handling the variation to students before teaching the formal way and then help students shift gradually to the formal one. As the first step, when drawing conclusions from data, students are expected to qualitatively evaluate the variation in the central part in data with some words such as "approximately" and intuitively quantify it in interval representation such as "A or less", "B or more", "from A to B".

However, it is known that it is not easy for students to take into account the variation in data. Studies on the development of distributional reasoning (e.g., Reading & Reid, 2006) point out that students tend to depend on only the center in data. The tendency is also the obvious fact in Japan. The 2017 National Assessment of Academic Ability in mathematics for Japanese students in the third year of lower secondary school (NIER, 2017) revealed that most of them could not consider data set as distribution. They represented the data with only a specific value and ignored the variation in data, despite the visual shape of the data distribution being presented. The possible cause for this result is that they could not use statistical words to represent data variation. Introducing informal statistical words for doing it into students is expected to prompt them to view data in the way which the words itself have and to help them shift to the more formal statistical inference. Therefore, in this paper, the effects of introducing statistical words on the way in which students view data is explored. Can students conclude with the variation in data taken into account by introducing informal statistical words which represent it?

METHOD

The task context for the investigation must be accompanied by the necessity for students to conclude with considering variation in the central part in data. In this paper, the task "estimating the likelihood of the square face of a rectangular solid dice" is used. In stochastics teaching, an asymmetric randomizer is only used to confirm that the limit of relative frequency converges to a

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certain value, and activities for inferring its probability or likelihood are not generally implemented. The rectangular solid dice (30 mm × 30 mm × 15 mm), unlike cubic dice, cannot evaluate the probability which a face will occur a priori, so the likelihood of a square face has to be inferred directly or indirectly through experiments. Since the inference depends on data, students are expected to consider the variation in the central part in data to draw more reasonable and persuasive conclusions.

In the experiment activity, to avoid overly focusing on the center in data, the Central Limit Theorem is set as the potential background, and an upper limit is put for the number of trials. Specifically, the experimenter shakes well a transparent box which contains the 10 rectangular solid dices, then records the number of the dice whose upward face is square, and repeats this one trial many times. The maximum number of trials is 100. Experiment data is recorded in tally graph and then transformed to histogram. Using the experimental method, it can be confirmed that the data distribution becomes mound-shaped with its peaks around 0.7 ~ 0.8 (see Figure 1).

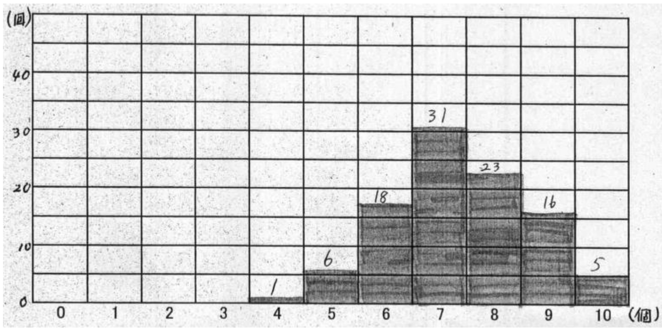

Figure 1. A histogram created by students group

Practices were implemented in two different classes of the third year (14- or 15-years-old) of public lower secondary school in Japan. The teacher was one of the authors. The all students had almost the same learning experience. In both classes, the same task was given. The students were asked to work on the task with a small group of 3-4 people (total 6 groups in Class A and 4 groups in Class B). In addition, in only Class B, the teacher introduced the informal statistical words representing the variation in data.

The teacher's and the students' utterances in the both classes were videotaped. The arguments made by each students group were analyzed in terms of the kinds of words and inference methods they actually used. Regarding the inference methods, the "variation" code was assigned to the arguments which describes the variation in the central part in data by the statistical words and the "center" code to the arguments which ignores the variation. If it is difficult to identify from the described arguments, the utterances of the student in the recording was referred to.

TEACHER'S UTTERANCES AND STUDENTS' REACTION IN CLASS A

In Class A, the teacher explained the task and asked the students to work on it as follows.

- "(in a scene to explain the problem) I'd like you to experiment, then quantify what rate a square face occurs at."
- "(after experimenting) I'd like you to create a histogram first of all, look at it, and consider about what the likelihood of this square face is. Consider for each group. Look at and what?"

The informal statistical words representing the variation in data were not intentionally introduced by the teacher. The attention to the variation in data was not prompted in the teacher's utterance after experimenting. Therefore, the way in which students view data reflected their learning experience. Their reactions in the task context were as follows (see Table 1).

Group A1 represented the lower likelihood part with using the word "4 or less". This implied that the higher likelihood part in the data was not that part, but they could not express it linguistically. The same was true for Group A5. They used the word "0.7 or more" to represent the likelihood of the mode value. On the other hand, Groups A3 and A4 were able to explain the higher likelihood part linguistically. Taking into account the teacher's utterances, it seemed that they had been able to spontaneously acquire the desirable way of viewing data.

Table 1. Students' reactions in Class A

Group	Argument	Words	Method
A1	As a result, 7 [out of 10 dices] was the most likely to occur, followed by 8 and 6. The likelihood that a number of 4 or less occurred was low. We turned out that the mean was about 7.6.	most likely to occur, likelihood, 4 or less, mean, about	Center
A2	When shaking 50 times, the number of times a square face occurred was 17, so if shaking it 100 times, we expected that 8 [out of 10 dices] will occur 34 times. This is 30% of the total, which is more than the other numbers occur, so our group have got the result that 8 is the most likely.	30% of the total, more than, most likely	Center
A3	7 [out of 10 dices], 23 times, 23/80 8 [out of 10 dices], 27 times, 27/80 more 7 and 8 [out of 10 dices] are 50/80, which account for more than half (5/8). If we shake 8 times, 7 or 8 [out of 10 dices] will occur 5 times.	more, 7 and/or 8, account for more than half	Variation
A4	5 ~ 10 is likely to occur, 0 ~ 4 is unlikely to occur. 0 did not occur.	~, likely, unlikely	Variation
A5	Since the mode is 7, it occurs at a rate of 0.7 or more, but it does not always occur with absolute probability.	mode, rate, 0.7 or more, absolute probability	Center
A6	Since the mode is 9, 9 [out of 10 dices] occurs with a 20/80, 1/4 likelihood.	mode, likelihood	Center

TEACHER'S UTTERANCES AND STUDENTS' REACTION IN CLASS B

In Class B, the teacher explained the task and asked the students to work on it as follows.

- "(after explaining the task and the experimental method) So I'd like you to infer how much a square face is likely to occur. I'd like you to argue that in this group a square face will occur with some likelihood."
- "(after experimenting) I'd like you to write how many, not the only how many, if you cannot represent only how many, then from a number to a number, from 6 to something, on the contrary, you can write this part does not occur. What I mean is that I'd like you to be able to talk about the likelihood of the square face with using numbers, such as how many, from a number to a number, … my word choice is inappropriate, or more, or less, and so on."

The difference from Class A was that the teacher intentionally prompted to express the likelihood of the square face with using interval representation such as "or more", "or less". The attention to both the center and the variation in data was paid by the teacher's utterances after experimenting. Therefore, the utterances should have had some effects on the viewpoint of the students for the data. Their reactions were as follows (see Table 2).

All groups in Class B were able to turn their attention to the variation in the central part in the data. Group B2 was not able to use the word "or more", "or less", and so on, but they were aware that more than half of the total trials gathered in the parts of 7 and 8, and they calculated the mean to confirm that the part was central. The other three groups focused on specific intervals and also referred to its likelihood. They could do an informal statistical inference which may be ideal.

DISSCUSSION

In the two statistical practices, the students with almost the same learning experience worked on the same task in the same way, but the teacher uttered different words between the classes. As a result, obvious differences in the students' reactions were confirmed. The students in Class B who are intentionally introduced an interval representation by the teacher were able to conclude with the variation in the data taken into account. This means that introducing the statistical words by the teacher has the positive effects on the viewpoint of the student for the data.

Like some students in Class A, a student with a potential ability may take into account variation in data without the help of a teacher. However, what is important is that all students can view data in such desirable way. The role of teachers is crucial to develop all students' views on data.

Table 2. Students' reactions in Class B

Group	Argument	Words	Method
B1	We shake the box 42 times, so that 8 [out of 10 dices] was the most frequented, and 35 out of 42 times got 6 to 9. In other words, about 85% of the total was 6 ~ 9 [out of 10 dices]. The median is 8, the mode is 8, the mean is about 7.5	most frequented, 6 to 9, about, %, ~, median, mode, mean	Variation
B2	As a result of shaking 100 times, 7 and 8 [out of 10 dices] occurred frequently. When we calculate the mean, it turned out that 7 [out of 10 dices] are likely to occur.	frequently, mean, likely	Variation
B3	2 or less [out of 10 dices] did not occur at all. Mostly 7 to 9 [out of 10 dices]. 70 out of 100 times was 7 to 9 [out of 10 dices]. It was about 70% likelihood. It is likely to occur a square face, which is over half probability.	2 or less, mostly, 7 to 9, about, %, likelihood, likely, probability	Variation
B4	5 ~ 8 occurred 55 times. Since the total is 80, it is about 70% at 55/80. 0 ~ 2 occur less frequently.	5 ~ 8, about, %, less frequently	Variation

It is pointed out that for teaching statistical inference it is useful to pay attention to its informal version in the previous studies. However, informal statistical inference may not be adequately taught or may be just attributed to students' spontaneous development due to its informality, even though the existence and significance of the informal version are recognized. The result of this paper suggests that appropriate support by teacher, which is direct or indirect, is obviously necessary for students to develop their statistical inference skill. Especially, since the means which statistical words have are different from deterministic ones obtained through mathematics learning, it is necessary that teacher intentionally introduces it and their students need to master it. The role of teacher is to finally develop and refine it to expression of formal statistical inference. For example, association between center and variation in data is very important but it is hardly considered in this paper, so the next step is to make it possible for students to pay attention to an interval part centered on a specific representative value.

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