THE USE OF A STORY-BASED TASK TO EXPLORE SAMPLING DISTRIBUTIONS

Collette Lemieux
Bissett School of Business
Mount Royal University, Calgary, Canada
clemieux@mtroyal.ca

This study, framed in a narrative learning perspective, investigated how an innovative story-based task facilitated students' understanding of sampling distributions. The story was situated in a real-world context about quality control in manufacturing. Students were required to write dialogue for the characters in the story to explain key concepts of the sampling distributions. The thirteen participants were enrolled in a post-secondary introductory business statistics course. Data, consisting of students' written responses to the story-based task, were analyzed qualitatively. Results suggest that the story approach was effective in helping the participants to develop conceptual understanding of the key concepts of the sampling distribution and how these concepts were meaningful to the real-world story context.

INTRODUCTION AND BACKGROUND

Many authors have argued that a distinguishing characteristic of statistics from mathematics is that context is front and centre (e.g. Cobb & Moore, 1997). Thus context-rich problems are a natural way to present statistics to students. Yet the results of using context-rich problems to learn statistics are mixed. For example, Neumann, Hood, and Neumann (2013) found that context-rich problems helped students remember the material. But Pfannkuch (2011) found that the context could, on occasion, take students' attention away from learning statistical reasoning. These findings suggest that further research is needed in this area. Additionally, research on simulations as an effective tool for students to explore concepts such as sampling distributions has recently gained prominence (e.g. Garfield, delMas, & Zieffler, 2012). This paper contributes to both of these areas by reporting on a study that investigated how a narrative learning perspective involving the use of a story-based task that incorporates simulations impacted student understanding of sampling distributions.

From the narrative learning perspective, students can learn from being presented with a story (story as a tool for learning) and by "storying" their understanding of a concept or idea (narrative process of learning). When students are presented with a story, they learn about the concept from a context-rich scenario that aids in learning by engaging the whole brain, and by grounding the concept in a memorable and believable situation (Rossiter & Clark, 2007). When we are learning about something new, we try "to make sense of it, to figure out its internal logic, and how it's related to what we know already … we work to story it, to make this new idea coherent to ourselves" (Rossiter & Clark, 2007, p. 77). It is through this process of struggling to create this narrative that students learn. The story-based task in this study combined stories as a tool and as a process of learning.

METHODS

A qualitative research method was used for this study to capture the participants' understanding as reflected in their written work.

The story-task

The story used for this study is called *The Dragon Lady.* In the story, a company owned by Jed makes solar-powered electric scooters that are significantly faster than the competition's. The company has secured an investment from "The Dragons' Den" (a reality TV show for entrepreneurs seeking investment capital), but it is contingent on a consultant, Reema (aka the "dragon" lady), finding ways to cut costs. She finds that the company is overspending on testing individual scooters in a bid to meet a contract when, in fact, the actual requirements of their contract is that the *mean* speed of a batch of 30 scooters is within the tolerance interval of 47.5 and 52.5 kmph. This leads the consultant to suggest looking at the sampling distribution of sample means to determine the probability that a batch satisfies the contractual obligations. Throughout the

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story Reema explains what a sampling distribution is to Jed and uses it to address the problem. She also uses a simulation (ww.rossmanchance.com/applets/OneSample.html) to generate an empirical sampling distribution from a parent sample of peak speeds for individual scooters. At key points, the story has been left intentionally incomplete so that students can write their own dialogue explaining important aspects of sampling distributions and resolving the problem presented.

Participants

The participants in this study were students enrolled in a section of a first-year algebra-based business statistics course in the winter 2017 semester at a university in southern Alberta, Canada. This study received ethics approval from the institutional review board.

Data Collection and Analysis

Data sources were the students' written work in the class for the story-based task. Students completed the task in groups of two or individually and choose their own groups. Only submissions where all members of the group agreed to be in the study were included. This resulted in the inclusion of eight submissions submitted by three individuals and five groups of two. For this study, the participants' dialogue for the story-based task were analyzed. There were twelve questions in the story-based task. This study focused on the questions that specifically examined how sampling distributions are constructed (#5), and the shape (#11), centre (#11) and variation (#9, 11) of sampling distributions in general.

Table 1. List of questions used from story-based task for data analysis

Q5	Have Jed and Reema discuss what a sampling distribution is. In particular, comment on how the data in the sample and the data on the sampling distribution are different and how the process of collecting a sample is different from the process of creating a sampling distribution.
Q9	Have Jed and Reema discuss why the standard deviations [of the empirical sampling distribution vs. the parent sample] are so different.
Q11	[Prior to this prompt, the students explored sampling distributions for different parent populations using a simulation: onlinestatbook.com /stat_sim/sampling_dist/index.html] Have Jed and Reema discuss the properties of a sampling distribution of the means. In particular, explain what happens to the mean, the standard deviation and the shape of the sampling distribution for different sample sizes. This question is the crux of this assignment. Spend most of your time answering this question. If you really get this, you've really got this concept.

The analysis was done qualitatively by coding the dialogue, which was guided by the research focus of the impact of the story-based task on the students' understanding of sampling distributions. This resulted in two broad categories of findings involving how the dialogue demonstrated the development of the participants' understanding of the relevant concept and how the context was used in their reasoning. The results focus on these two categories.

RESULTS

The results are presented to highlight: (1) the understanding of sampling distributions demonstrated by the participants and (2) the use of contexts in their reasoning. Because the data consisted of a mix of individual and group work, when referring to this work, pseudonyms of Submission 1, Submission 2, etc., will be used to identify the participants involved.

Understanding demonstrated by participants

In more than half of the submissions, the participants correctly articulated how a sampling distribution of sample means is constructed (Q5). For example, Submission 1 wrote the following dialogue between Reema and Jed:

> *Reema*: Yes! Sampling distribution is when we take multiple samples and then find the mean of those samples.
>
> *Jed*: What's the difference between a sample and sample distribution?
>
> *Reema*: Well a sample is a single collection of individual numbers, while sampling distribution you are collecting multiple samples and then measure their means.

Most of the students could explain why the standard deviation of the sampling distribution was less than the standard deviation of the parent sample (Q9). For example, Submission 8 provided the following dialogue:

> *Jed*: Would this relate to the law of large numbers? The parent sample is smaller and the sampling distribution is much larger, therefore it would be more representative of the population?
>
> *Reema*: No, this isn't necessarily the reasoning behind the difference we are seeing in the standard deviations. It is because the parent sample will have extremes in it, and these extremes have an impact on the standard deviation as it is measuring the average distance to the mean. The standard deviation of the parent sample is 3.403. The standard deviation of the sampling distribution, is much smaller at 0.589 because it is comprised of the means which smoothes out the extremes we see in the parent sample.

Lastly, in Q11, students were asked to discuss sampling distributions of sample means in general. Most of the students accurately described the shape of the sampling distribution when the sample size is greater than 30. More than half of the participants understood that the mean of the sampling distribution is the same as the mean of the parent population and that the variation in the sampling distribution is less than the variation in the parent population. Having said that, there were mixed results with the details of the central limit theorem. In particular, some submissions did not fully explain what happens to the shape if the sample size is less than 30 and none of the submissions stated that the sampling distribution is *approximately* normal. To illustrate, Submission 3 wrote the following dialogue (note: they incorrectly use "sample distribution" instead of "sampling distribution"):

> *Reema*: The Central Limit Theorem states that if our sample size is sufficiently large, for our case it was 30, the sampling distribution will be normally shaped no matter what the parent sample looked like! So the crates need to have at least 30 scooters for this to work. If they had 20, then our sample distribution would not have been as normal as we wished.
>
> *Jed*: Okay I see what you're saying…
>
> *Reema*: If the parent population was normally shaped, then automatically the sample distribution would also be normally shaped. Also, the mean will always stay the same as the parent population; the standard deviation will always change. This is because when we take a sample from the parent sample, it's just a smaller representation, so it would make sense that the mean would stay the same.
>
> *Jed*: Okay so basically you're saying that the standard deviation changes because when we increase our sample size from the parent sample, the sample distribution becomes narrower and narrower. This makes the standard deviation become smaller and smaller while the mean becomes more true to the entire population!

Use of context in answers

Comparing the dialogue for Q5 and Q9, the context was used more pre-dominantly in Q5. Here most participants used the context to explain how a sampling distribution is constructed. In particular, they used the context to motivate why the sampling distribution was appropriate in this context and explain the details of the construction by relating the size of each sample to the batch of scooters and that mean peak speed was measured for each sample (e.g. from Submission 3: "So basically, since our contract states the average speed of a batch of scooters needs to be 50KMPH, if we use this bootstrapping technique, we can figure out if the scooters are fulfilling the requirements of the contract.").

In Q9, the context was used by more than half of the submissions but most of them used it minimally. That is, it was not an integral part of their answer. For example, Submission 5 had the following response:

> *Reema*: To understand this concept, we should go over what each graph is measuring. The Parent Sample is measuring individual scooters, with a large variation in speed, while the sampling distribution measures means of a group of 30 taken from our original sample. The speeds recorded in the Sampling distribution will be much closer therefore because averages, with far less extreme numbers are used in the applet. The standard deviation is narrower or smaller for the sampling distribution because this is using the mean of groups

of numbers taken from the entire sample. There will be far less representation of extreme numbers. While the parent sample is a collection of individual values, and will therefore by skewed by extreme outliers.

The response uses the context of scooters at the beginning, but most of the reasoning used to address the overall question is decontextualized.

For the Q5 and 9 submissions that used the context, most of the dialogues demonstrated an accurate understanding of the context being explained. That is, for most of the participants, when the context was used, the context did not interfere with their articulation of their understanding of the concept.

As Q11 is about generalizing the properties of the sampling distribution, there would be no expectation that the students used the context. Yet, some of the submissions still did. In particular, they related why it was important for the batches to have 30 scooters as it ensured the central limit theorem could be applied to this context (e.g. from Submission 2: "That's why we tested the scooters in batches of 30, so that we would get a normally shaped histogram and would be able to calculate probabilities.").

CONCLUSIONS

Based on the results, it appears that the story-based task was most effective in the development of understanding why the standard deviation is different for the sampling distribution versus the population distribution. But it was also useful for participants for developing understanding of how the sampling distribution is constructed and the shape and centre of the sampling distribution of sample means, in general. The participants connected sampling distributions to re-sampling and most recognized that the sampling distribution is for a statistic (in this case, the sample mean) and could distinguish between the type of data used for the parent sample and the sampling distribution.

In Pfannkuch's study (2011), students on occasion had problems separating the context and the theory. In this study, most of the participants who articulated an accurate understanding of the concepts of how a sampling distribution is constructed and why the variation is smaller in the sampling distribution used the context to some extent in their explanation. Further in Q11, which asked for students to discuss sampling distributions in general, most of the submissions showed an understanding of the general properties of sampling distributions of sample means. That is, most of the participants used the context to help with their understanding when appropriate, but when they needed to expand the theory beyond the context, they were able to do so.

Overall, the study suggests that a story-based task that incorporates simulations can help students develop an understanding of difficult abstract concepts within a context-rich scenario and that the real-world context can add to their understanding without distracting from the theory.

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