EXPLORATION OF PRESCHOOL TEACHERS' REPRESENTATIVENESS NOTIONS OF STATISTICAL GRAPHS

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In this article, we present research whose objective is to explore the interpretation of statistical graphs by preschool teachers in service. We particularly sought to understand their notions on the representative value of a graphically displayed data set. The participants in the study were 20 teachers who were asked to analyze a bar graph and two histograms. To analyze the information gathered, we used the levels proposed by Curcio (1989). The results show that the participants have difficulties extracting information from the graphs; additionally, as representative value of a graph, they focus on the mode or a point in-between values of it. Preschool education demands specialized knowledge to promote graph interpretation in and outside the classroom.

INTRODUCTION

Statistics allows us, among others, to analyze data from samples or populations at a large scale to have (statistical) references to characterize such data and, if it necessary, make decisions. Statistical graphs are part of this analysis and when they are carefully read, they help to identify typical or representative values, such as mode and median, as well as dispersion and how data are distributed. Being able to read and understand a graph is a desirable skill in citizens and information users. However, this skill represents a requisite for teachers, who must be able to pose situations to deal with and interpret statistical graphs in the classroom.

Research related to the use of statistical graphs has dealt with a number of objectives; for instance, Batanero, Arteaga and Ruiz (2010) evaluated graphical competence: interpretation, evaluation, and communication of information regarding statistical graphs. For their part, Jacobbe and Horton (2010) studied the comprehension of graphs with respect to the expectations described in the GAISE (Guidelines for Assessment and Instruction in Statistics Education). Additionally, Alacaci, Lewis, Brien and Jiang (2011) carried out a research on the teacher's ability to adequately choose a graph. In contrast, Sanoja and Ortíz (2013) studied the knowledge of statistical content and found that data visualization is part of it. All these studies included the participation of elementary-school teachers in training and service. One of the most relevant studies in the group of works concerning graph interpretation is that by Aoyama (2007), who proposed to clarify a hierarchy of graph interpretation. Leavy and Noreen (2006) studied the understanding of the mean by future teachers, specifically the conceptions and use of the arithmetic mean. A six-task questionnaire was administered to future teachers and the results show that 25% of the participants have a conception of the mean and confusion between arithmetic mean and mode. Leavy (2004) studied the strategies employed by teachers in training when they build representative measures for different data distributions. He used 5 tasks to do this and, according to the results, found that the mean was the most used measure by the participants, regardless of the form of the distribution. The study also shows little attention to measures of variability, particularly when data were not presented graphically.

The aim of this research is to explore the notions that preschool teachers in service have about statistical representativeness in graphs, understood as what is typical, representative or average (Mokros, & Russell, 1995). To do so, we posed the following question: Which are the notions on representativeness from data in statistical graphs that preschool teachers in service show?

CONCEPTUAL FRAMEWORK

To study the comprehension that the teachers have about representativeness when reading graphs, as theoretical reference, we propose the concept of Common Content Knowledge, CCK (Ball et al., 2005; Hill, Ball, & Schilling, 2008), the levels of graph compression (Curcio, 1989)

In M. A. Sorto, A. White, & L. Guyot (Eds.), *Looking back, looking forward. Proceedings of the Tenth International Conference on Teaching Statistics (ICOTS10, July, 2018), Kyoto, Japan.* Voorburg, The Netherlands: International Statistical Institute. iase-web.org

and the approaches used by Mokros and Russell (1995) to describe the approaches to problems of averages.

The teacher must have mathematical knowledge to read graphs. Hill et al. (2008) introduce the concept of Mathematical Knowledge for Teaching and define it as "the mathematical knowledge that teachers use in classrooms to produce instruction and student growth" (p. 374). There are six subdomains in this knowledge among which CCK stands out. Ball, Thames, and Phelps (2008) consider that the six subdomains correspond to the knowledge acquired by a person at school and along daily life. In statistics, according to the CCK, the teacher must be able to visually represent data, recognize and interpret them, make correct use of notation, and identify imprecisions in the graphs. The notion we seek to explore in this article is that of representativeness, characteristics of a data set which summarizes and allows for making sense of them (Mokros, & Russell, 1995). The specialized literature states that it is highly important for the teacher to understand graphs, which can be understood as reading and interpreting them. To analyze the reading levels of the in-service teachers in this report, we consider the levels described by Curcio (1989), who defines them as *Reading data*, *Reading between data*, and *Reading beyond data*. Below, each level is described:

- *Reading data,* this refers to the literal reading of the graph. The reader simply copies the facts expressed explicitly in the graph. There is no interpretation at this level of comprehension.
- *Reading between data,* this includes the interpretation and integration of the data in the graph. It demands the ability to compare quantities and the use of other abilities and mathematical concepts that allow the reader to combine and integrate data.
- *Reading beyond the data,* this refers to the reader to predict or make inferences from the data to obtain information that is neither explicitly nor implicitly expressed in the graph.

In addition to the levels reading, Curcio, Mokros and Russell (1995) suggest five approaches to describe the responses to problems of averages as a referent to organize the notions on representativeness that teachers show. Below, we show part of those approaches:

- *Average as mode,* students use mode to interpret a distribution; they see mode only as "the most", not as a representative value of a data set.
- *Average as algorithm,* students consider finding an average as carrying out the procedure learned at school for finding the arithmetic mean. They often show a variety of useless and circular strategies that confuses total, average, and data; they have limited strategies for determining the reasonableness of their solution.
- *Average as reasonable,* the students choose an average that is representative of the data from both approaches mathematical and of common sense; they can use the algorithm to find the mean. If so, the result of the calculation is analyzed for its reasonableness. They consider that the mean of a data set is an approximation that can have many values.
- *Average as a midpoint,* the students choose an average that is representative of the data from a mathematical and a common sense perspective. They look for a "middle" to represent a data set. This middle is alternatively defined as the median, the middle of the *x* axis, or the middle of the range.
- *Average as a mathematical point of balance,* the students look for a point of balance for representing the data. They consider the values of all the data points and use the mean with a beginning comprehension of the quantitative relationships among data, total, and average. They are able to work from a given average to the data, from an average to a total, and from a given total to the data. Additionally, they divide problems into smaller parts and find "submeans" as a way to solve more difficult problems of average.

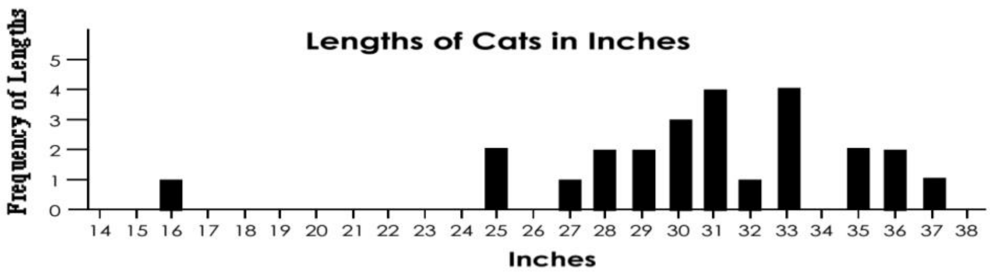
METHOD

The participants in the study were 20 in-service preschool teachers in Mexico City, Mexico, between the ages of 25 and 50. Their experience ranged from 2 to 20 years of service. A third of the participants had master studies at the moment of the study.

To understand how the 20 teachers interpreted statistical graphs, we designed and implemented a questionnaire that included three problems related to statistical graphs. The questionnaire was solved in approximately an hour. In this article, we only report the results of two of problems, as shown below:

Problem 1. Task: Lengths of Cats

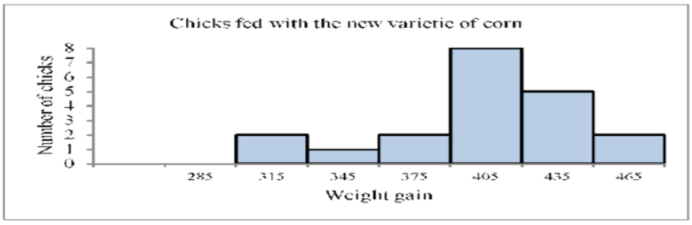
A group of students has been investigating information about their pets Several students have cats. They decided to collect some information about each of the cats. One set of data they collected was the length of the cats measuring from the tips of the cats' noses to the tips of their tails. Here is a bar graph showing the information they found:

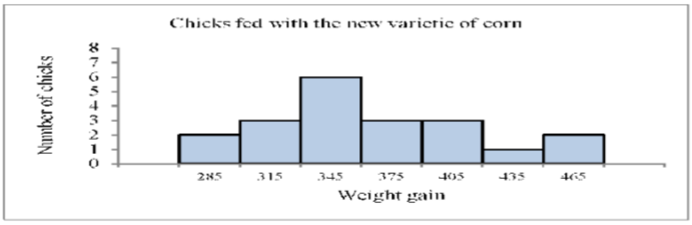


1. How many cats are 30 inches long from nose to tail? How can you tell?
2. How many cats are there in all? How can you tell?
3. If you added up the lengths of the three shortest cats, what would the total of those lengths be? How can you tell?
4. What is the typical length of a cat from nose to tail? Explain your answer.

Problem 2. Task: New corn variety

Corn is an important animal food. Normal corn lacks certain amino acids, which are building blocks for protein. Plant scientists have developed new corn varieties that have more of these amino acids. To test a new corn as an animal food, a group of 20 one-day-old male chicks was fed a ration contained the new corn. A control group of another 20 chicks was fed a ration that was identical except that it contained normal corn. Here are the graphs of that groups after 21 days.





a) What does your plot show about the effect of the improvement corn variety on weight gain?; What difference, by weight (g), was there between the two types of chickens observed in this experiment? Explain your answer.

To analyze the information, first we coded the answers them based on identification of words or group of words (Birks, & Mills, 2011), and we later categorized based on the levels proposed by Curcio (1989). The analysis was descriptive and aimed to identify, through categories, the notions of representativeness that the teachers demonstrated in their responses, as a product of their reflection of their common knowledge of the graphs.

RESULTS

Problem 1

In this problem, the teachers were asked to answer the question: What is the typical length of a cat from nose to tail? Asking the teachers to provide the typical value of a data set evidences their shortcomings when interpreting the information from a graph. According to the reading levels by Curcio (1989), the teachers displayed the following:

Reading data. In two responses, we identified a reading that demonstrates the absence of the notion of typical value: "It goes from 16 to 37 inches, according to the registered data". This justification makes reference to the data range represented in the graph, dispersion, a complementary idea to that of middle or typical. In four of the justifications, the typical value was represented by the teachers as a recurring characteristic of the graph. In the length representation of the cats, five of the bars had a frequency of 2. For example: "considering that the typical one is the most frequent 2". These strategies agree with the approach of *average as algorithm*.

Reading between data. Three teachers interpreted an idea related to the median of the data set, represented in the graph, as a typical value. For example, "If it refers to the average length of the cats measured, it is 31 inches. If the total number of cats is 25, the ½ [middle] is 12.5, and corresponds to 31 inches". In this response, we observe that the teacher interprets the *average as midpoint*. At this level, we grouped other 11 responses which consider the *average as mode,* for example: "Between 30 and 33 inches because there is a higher repetition in these measures; that is, these measures are repeated more frequently."

Problem 2

The teachers answered two questions in this problem: 1) What does your plot show about the effect of the improvement corn variety on weight gain? and 2) What difference by weight (g) was there between the two types of chickens observed in this experiment? In both questions, the participants provided different justifications. Below, we represent the reading levels which arose from the explanations to question 1.

Reading data. The *average as mode* was a characteristic that made sense for two of the teachers, who considered it to justify their response: "That with the new variety of corn, they obtained a greater weight, 405 grams." Value 405 corresponds to the mode of the distribution shown in the graph.

Reading between data. Two more teachers used the mode, but supplemented this value with a description of the proportion of chickens that reached that weight. For instance, "There is an increase in the frequency of chickens with a higher weight, 8 out of 20 chickens and 6 out of 20 chickens, and the maximum weight increased from 345 to 405 grams" (*average as mode*). This evidence shows the participant is reading between data and uses key characteristics (middle and proportions) of the graph. At this level, three other teachers based their responses on the range of "most" frequent values (*average as mode*). For example, "The effect on the weight of the chickens is greater because there are more chickens reaching a weight from 405 to 465 grams."

Thirteen of the responses did not show a clear reading level of the graphs. As an example, "That with the new corn, there are more chickens that weight; although there is variation in the weights." The response does not show whether the chickens weigh more or less; therefore, we do not know what the participant considered. In other examples, the participants expressed the chickens increased their weight but they failed to justify that based on a quantifiable difference or with numerical references read in the graph. An example of this type of responses is: "The [chickens] fed with the new variety [of corn] had a higher weight". From this response, we observed that the participant identified the increase in weight of the chickens, however, it is unclear how much weight the chickens gained or whether the participant considered a reference from reading the graph to produce the justification.

Reading levels from the responses to the question: what difference by weight (g) was there between the two types of chickens observed in this experiment?

Reading data. Eight of the participants compared the labels on the bars in the graph (315, …, 465 and 285, …, 465). For example: "20 to 30 grams, approximately, among the lower, and 30 to 60 grams in the higher". In this explanation, we observed that the teacher read and compared the extreme values shown in the labels, an *average as algorithm* approach.

Reading between data. Two participants provided a response to this question by making operations with the values of each data set. For example: "780 grams. I multiplied the weight of the chickens times the number of frequency and added the totals of each graph. At the end, I subtracted the lowest number from the highest one and obtained the difference". We observed that the reading of the variable (weight) and frequencies was correct, however, multiplying the variable times the frequency and adding the product does not result in a representative value that would lead to the approach of *average as algorithm*. This question proved to be the most difficult for teachers-t only three of them gave an appropriate response to the problem. For example, “In the maximum weight, there is a difference of 60 grams and the frequency increased from 6 to 8 out of 20 chickens.” From this justification, we determined that the teacher compared the modal values besides the proportion of chickens that increased their weight. This demonstrates an approach of *average as mode*.

Three of the responses only state there was an increase in the weight of the chickens, but they do not state the difference between the groups of animals. For example, “There was a difference; [it] increased in the new maize.” Four participants did not provide a response.

CONCLUSIONS

The question posed in this research was: Which are the notions on representativeness from data in statistical graphs that preschool teachers in service show? From the results and based on the approaches by Mokros and Russell (1995), we found that the notions expressed by the teachers are the following: average as mode, average as algorithm, and average as midpoint.

In the problem of length of cats, which included a bimodal graph, the most frequent notion of representativeness was that of average as mode, based on the approaches by Mokros and Russell, which considers that the participants in this approach see mode as “the most”. We also identified the notion of average as an algorithm in which the participants show inadequate or circular strategies when resorting to isolated data. Only three teachers showed the notions of average as midpoint.

In Problem 1, participants were expected to use the typical value to justify the effect of the corn. 13 teachers provided a confused response and displayed the approaches of average as algorithm and average as mode.

In section Problem 2 Section B, future teacher participants were expected to find the difference in weight from the typical value. The approaches they displayed correspond to inadequate strategies. They also resorted to some data in the graph.

The CCK reading and interpretation of the graphs plays a key role in reading data and between data in the graphs shown in Problems 1 and 2 (Ball et al., 2008). This knowledge leads participant-teachers to only establish relationships between the data of the graph; however, these are not enough for the teachers to make predictions or inferences from them. We deduce that the CCK related to statistics, acquired during teacher training and daily life is not sufficient for the participants to make more abstract readings of the shown graphs. In this respect, teachers showed some shortcomings that we consider evident in terms of relative knowledge of graph interpretation. This also shows the importance of carrying out investigations characterized by the intervention to construct basic knowledge in statistics.

Teachers in service must make a better reading between data and be able to make calculations as arithmetic mean, besides the inclusion of measures of dispersion. Teachers in the early school years must be conscious of providing experiences that set the basis for later work with formal measures of center (Groth, 2005). When teachers have the necessary guidelines to interpret a graph, they will be able to display the competences indicated by the programs of the Public Education Secretary, designed for preschool level (SEP, 2011).

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