TIME SERIES – ITS PLACE IN THE SECONDARY SCHOOL CURRICULUM

Rachel Passmore
University of Auckland
New Zealand
r.passmore@auckland.ac.nz

Time series has been a topic in the New Zealand curriculum since 1996. New Zealand is one of the few countries that include time series specifically in the secondary school curriculum. This paper discusses the motivation for its inclusion and examines the changes in the teaching and learning of time series that have occurred since its introduction over 20 years ago. Changes include the integration of data visualisation software into the teaching, learning and assessment of this topic as well as a shift in emphasis to encourage the development of higher levels of reasoning. Challenges and opportunities for teachers and students are identified and pedagogical solutions are presented. The transition between time series at secondary and tertiary levels is discussed.

INTRODUCTION

Time series appear regularly in the media and traverse numerous disciplines. Specialists in economics, biological sciences, medicine and social sciences, to name a few, will encounter time series frequently in their subject areas. Given their prevalence it is therefore not surprising that time series as a topic in its own right was introduced into the final year of the NZ secondary school curriculum, but it is perhaps surprising that other countries have not elected to follow suit. The University of Sydney's website notes "*One of the fastest growing areas of employment for mathematical graduates is the high-tech end of the financial sector. Banks, insurance companies and investment houses are busy setting up quantitative analysis groups and departments, staffed by highly trained mathematical graduates."* A course covering time series analysis is a compulsory component for all their undergraduates majoring in Financial Mathematics and Statistics. A Google search on an American website on one day revealed just fewer than 6000 positions that required time series skills. Vacancies were for data scientists, economists, investment analysts, oceanographers, operational researchers as well as positions in big companies such as Amazon, CBS and IBM. In short this is a much sought after skill that may substantially improve the employment prospects of students. In my own experience as a Government Statistician in the UK, I was involved in the analysis and projection of numerous time series including the prison population in England and Wales, hospital waiting lists, and volume of prescriptions dispensed.

In this paper I outline the development of the time series topic in the New Zealand curriculum over the last 20 years. Particular mention is made of a curriculum change implemented in 2013 that significantly changed the focus of this topic from one of calculation to one of interpretation and raised some issues that are discussed. Analysis of this curriculum change was the subject of my Master's thesis (Passmore, 2016). A framework is described that was used to categorise the types and levels of reasoning required for time series. This framework was used to analyse the 2013 curriculum changes but it also offers pedagogical guidance for the teaching of time series. Finally I consider the transition between time series as a topic in secondary schools to current tertiary offerings.

BRIEF HISTORY OF TIME SERIES IN THE NEW ZEALAND CURRICULUM

The topic of time series has been assessed under three different systems in New Zealand. The most recent and arguably the most substantial change was implemented in 2013. In 1996 it was taught as part of the Bursary Statistics course, an external assessment completed in the last year of schooling. In 2003, the National Certificate of Educational Achievement, NCEA, which is a standards based assessment system, was introduced. At the same time three levels of achievement (Achieved, Merit and Excellence) replaced marks and grades that had previously been used. For the first time in 2004, an Achievement Standard for the topic of time series was internally assessed. This was a popular Achievement Standard in which large numbers (*$n \approx 15500$)* of students enrolled. However, although the time series were presented in a contextual setting, students were not required to engage with the context but rather focused on producing sets of calculations with little

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or no requirement for interpretation. Submission of an EXCEL spreadsheet and a one sentence comment were sufficient to pass this time series assessment. Unfortunately, as well as a lack of interpretation, 'successful' students were also permitted to fit linear trends to time series that were clearly non-linear, a practice I found difficult to endorse. In this format the topic of time series did not deserve a place in the secondary school curriculum. It became little more than an exercise in entering formulae into a spread sheet and assessment methods meant authenticity of student work was difficult to monitor.

In 2013, this and other standards were replaced following a curriculum change which aimed, amongst other objectives, to close the gap between the statistics being practised by professional statisticians and the statistics being taught in schools. The new time series Achievement Standard, (*http://www.nzqa.govt.nz/nqfdocs/ncea-resource/achievements/2017/as91580.pdf*) shifted the focus away from repetitive calculations and firmly towards the interpretation of data visualisations produced using technology. For example, a typical task required students to analyse the area of polar sea ice to assess the impact, if any, of global warming. Students submitted a full report on their findings including references to other articles or studies on trends in sea ice as well as any conclusions drawn from their analysis. The fundamental principles of exploratory data analysis, first espoused by Tukey as long ago as 1977 were finally reflected in this standard, as was the call by Gal and Garfield (1997) for students to handle, interpret and communicate about data sets from meaningful contexts. Moore's (1997) insistence on the automation of calculations and production of graphics is also present with the requirement that this standard be taught and assessed using technology such as *iNZight*. Finally, as Cobb (2007) predicted, computing power had enabled the time series curriculum to be freed from tedious calculations and shift toward deeper interpretation of and insight from the data. These changes were in line with the direction being taken by the statistics curriculum as a whole, to promote higher order thinking through the investigation and analysis of real data.

THE IMPACT OF CURRICULUM CHANGE

In order to analyse the impact of the 2013 curriculum change on the teaching and learning of time series, I analysed data from three different sources; 34 exemplars of student work from before and after the curriculum change, questionnaire survey of teachers (18 respondents) and in depth interviews of five lead statistics teachers. Changes were identified in not only student outcomes, but also assessment management, software used and pedagogical practice.

The development of free software such as *iNZight* was absolutely crucial. It offered a whole new set of data visualisations to support interpretation and analysis of a time series. Integration of new technology in schools is often treated with caution and to shift a whole cohort of teachers to a new software package can take a considerable length of time. In my research the schools of all my participants had shifted to one of the new software options within a two-year time frame. Oates (2009) identifies a number of key factors that must be in place for effective implementation of new technology including:

- Integration of new technology in teaching, learning and assessment;
- Access must not be restricted by either software cost or availability of computers;
- Professional development of teachers to alert them to the pedagogical value of the new technology and to promote effective use in teaching and learning activities;
- Official documents such as exemplars posted by the national qualifications authority must model use of new technology;
- Teachers need professional development to cope with any new mathematical content.

iNZight addressed these issues by providing the software free on all devices, with technical support provided via an on-line forum. The techniques used by *iNZight*, Holt-Winters for prediction and the seasonal trend LOWESS for series decomposition was new to many teachers, so extensive resources were provided to support teachers. The New Zealand Qualifications Authority (NZQA) utilised outputs from *iNZight* in model student exemplars posted on their website and also used the new software in workshops held around the country. In the past availability of computers has been an equity issue in New Zealand, but my participants did not report this as a concern. Professional development workshops were held throughout the country to support teachers to cope effectively with this curriculum change.

The change in mode of assessment from an internal assessment completed under time restricted exam conditions, to a more project-style assessment allowed students to engage with and research the context of their time series in order to inform their interpretation and analysis. This change in focus and method of assessment resulted in a number of challenges and also concern from schools that retained a more formal assessment approach. Teachers with little time series content knowledge coped with the previous procedural calculations but struggled to cope with the new requirements unless supported by professional development. New technology was not embraced by all teachers; in fact some actively distrusted the software ‘black box’. Some teachers felt that student understanding was decreased by the absence of procedural calculations.

It was interesting that some teachers in my study perceived the repetitive calculation of moving averages and seasonal effects as a higher-level skill than interpretation of the outputs from *iNZight*. This viewpoint is not supported by established hierarchical frameworks which purport to describe mathematical or statistical thinking, or by my framework which will be discussed in the following section. A pedagogical solution was proposed for this issue. Strong research evidence (e.g. Arnold, Pfannkuch, Wild, Regan & Budgett, 2011) suggests that by-hand activities conducted before the introduction of new technology is beneficial to students. Hence, a short tutorial was developed as part of professional development workshops which covered a variety of smoothing techniques to show how students could compare the results by hand before engaging with the new software. Another issue was the use of real data sets which, when compared to the well-behaved synthesised data sets used before the curriculum change, meant that description of the long-term trend posed greater difficulties for teachers and students. At a recent workshop, a task involving the description of the long-term trend was posed to teachers resulting in a great diversity of opinions. Although some further workshops on this topic have been provided it remains an on-going challenge and reflects the need for professional development of secondary school teachers in additional statistical content, since few have studied time series at any level.

Development of framework of types and levels of reasoning with time series

A number of established but different frameworks were examined in my study in order to develop a synthesised framework to examine the types and levels of student reasoning with time series. These included frameworks:

- developed for levels of reasoning in assessment tasks (Verhage & De Lange,1997);
- characterising development of mathematical thinking (Pirie-Kieren,1994);
- characterising development and dimensions of statistical thinking (Wild & Pfannkuch,1999);
- for assessing interpretation of data and data displays (Konold et al., 2015; Curcio,1987).

Aspects of these frameworks which were deemed important for reasoning in time series were incorporated into the synthesised framework. Wild and Pfannkuch (1999) describe four dimensions of statistical thinking which are essential for the statistically literate citizen. All four dimensions have a part to play in the analysis of time series but some aspects play a more prominent role. For example, the types of thinking mentioned in dimension 2 – transnumeration, consideration of variation, integration of statistical and contextual knowledge are particularly important for time series. Transnumeration is the process by which students view a data set through a number of different representations in order to gain deeper understanding. Integration of statistical and contextual knowledge is also essential if the variation in the time series is to be meaningfully interpreted. Konold et al. (2015) describe different levels of understanding students have about data with one level being seeing *data as an aggregate*. In time series students must be able to view the data set as a whole, differentiating between small and large deviations from overall long-term trends, with only larger deviations warranting further investigation. Curcio’s (1987) framework for graph comprehension comprising three levels; reading a graph, reading within a graph and reading beyond a graph are particularly relevant and reflected in levels 1, 2 and 4 of the synthesised framework. Shaughnessy (2007) extended Curcio’s framework to a fourth level, adding reading behind the graph and this is reflected in level 5, interpretive reasoning.

The resulting synthesised framework, Table 1, describes a hierarchy of six levels of reasoning.

Table 1. Synthesised framework of types and levels of reasoning with time series

Level of Reasoning	Types of Reasoning	Description	Possible Student Response
1	*Vertical reasoning*	Reading individual items of data from a time series graph	Identification of peak/trough
2	*Horizontal reasoning*	Reading across the whole time series, identification of features	Description of long term trend
3	*Procedural reasoning*	Performing calculations using the time series data	Calculation of moving average
4	*Extended procedural reasoning*	Using calculations to predict beyond current domain	Calculations of predictions
5	*Interpretive reasoning*	Interpreting key components and features of a time series through integration of statistical and contextual knowledge	Interpretation of a short term trend using contextual knowledge
6	*Interrogative reasoning*	Critical analysis of statistical model and interpretation offered	Comparison with a similar or related time series

RESULTS

Each sentence and calculation from exemplars of students' work completed before and after the curriculum change was coded against the framework and results analysed. Table 2 shows a mean level of reasoning by level of achievement from student exemplars completed before and after the curriculum change. Four student exemplars were graded as Not Achieved and were not included in the results. After the curriculum change the mean level of reasoning is greater at every achievement level, with the biggest increase (3.40 to 3.99, or 17.4%) in level of reasoning found at the Excellence achievement level. In fact examples of the highest level of reasoning, *interrogative reasoning*, were only found in student work completed after the curriculum change, confirming that the goal of moving towards higher order thinking tasks had to some extent been achieved. At the Achieved level, examples of level 4 and 5 reasoning appeared after the curriculum change demonstrating a broader range of reasoning skills even at the lowest level of achievement. Given that this was a topic covered in the last year of schooling it is not surprising that no examples of level 1 reasoning, an elementary level, were found.

Table 2. Comparison of mean levels of reasoning in student exemplars
before and after curriculum change

		Number of Examples of Reasoning Levels found in Student Exemplars					
Achievement Level	No. Exemplars	Reasoning Levels (Table 1)					Mean
		2	3	4	5	6	
Before							
Achieved	4	1	15	0	0	0	2.94
Merit	6	9	34	30	8	0	3.46
Excellence	5	3	25	16	3	0	3.40
After							
Achieved	6	35	28	27	16	0	3.23
Merit	6	29	20	38	34	0	3.64
Excellence	3	28	18	26	29	22	3.99

However, an increase in cases of horizontal (level 2) reasoning appears to contradict other evidence that showed a shift towards higher levels of reasoning. Verhage and De Lange's pyramid (1997) helps to explain this apparent paradox. The pyramid, Figure 1, shows that lower level reproductive reasoning assessment tasks can range from simple to complex; horizontal reasoning fits into this category. A time series whose overall trend is clearly linear represents a relatively simple task to describe; this was a common feature of time series prior to the curriculum change. After the curriculum change, teachers were encouraged to use real data sets and real data tends not to follow well-behaved linear trends, resulting in this low level reasoning task becoming more complex.

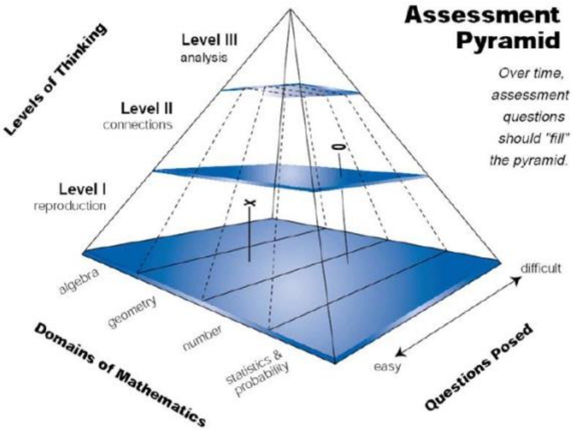

Figure 1. Assessment Pyramid
Reprinted with permission from Verhage, H. & De Lange, J. (1997).
Mathematics education and assessment. *Pythagoras, 14-20.*

After this curriculum change the topic of time series had a much improved focus, demanding that students integrate their statistical and contextual knowledge in a way that had not been demanded of them before, but crucial for the development of a student's statistical reasoning skills. However, it was disappointing that the curriculum change did not fully exploit the potential that the new software data displays offered. For example, one option on *iNZight* is to compare related time series on the same screen, which permits a student to examine similarities and differences between related series and to speculate on whether there are any links between them. Comparisons of related time series can provide useful insight into the context of the data sets and aid interpretation of features. Although some teachers might include this in their teaching and learning it was not a requirement of the standard at any level and was not included in publicised students exemplars. If this had been included as a requirement at Merit level and above, I believe that time series would truly deserve a place in any secondary school curriculum. It would have had the added benefit of clarifying Achievement Level boundaries, an issue that had confused some teachers.

PREPARATION FOR TERTIARY AND BEYOND

As I mentioned in the introduction, skills in time series analysis are much sought after by employers, and these skills can be further developed at the tertiary level. Despite the growing demand for students with these skills, undergraduate courses that include or focus specifically on time series are not numerous. Four of the teachers who participated in my research ($n = 18$) had studied time series as part of a degree in Mathematics, and another two had studied it at a post-graduate level. At the University of Auckland, time series is not covered at all in Stage 1 courses, is the subject of about three hours of lectures in a Stage 2 course, and finally by Stage 3 it qualifies as a course in its own right. This means that there is a disconnection between when a student studied time series at school and when they next meet the topic at University. It is I believe a missed opportunity that skills students develop in secondary school are not consolidated and developed until, for most students, their final year of undergraduate study.

In response to demands from employers, some Universities are providing financial mathematics pathways that will equip students with the skills required to operate at a high level in the financial sector. One of the compulsory skills is the ability to analyse time series. At the University of Auckland, a financial mathematics pathway is currently under development and will involve collaboration between the Departments of Mathematics and Statistics. I hope the structure of this and similar undergraduate pathways will ensure that time series skills are developed each year, building on skills covered at secondary school, rather than have time series as a topic that is only covered at Stage 3 as has been the case in the past.

CONCLUSION

I have argued that time series as a topic in its own right deserves a place in the secondary school curriculum. As such it covers important preparatory work for the topic to be studied at undergraduate and postgraduate level and is a skill much in demand in the workplace. However, its introduction into the secondary school curriculum is not without challenges as evidenced by the experience in New Zealand. The teaching and learning of time series has been enhanced substantially by the development of free software giving students access to data visualisations which aid interpretation and liberate them from the tedium of repetitive calculations. However, for student outcomes to benefit from this new technology a programme of professional development is essential for teachers, many of whom may have never studied time series as part of their own undergraduate degrees. Also the new technology must pervade all aspects of teaching, learning and assessment of time series if practitioners are to be persuaded to switch from the technology or methods that they have previously used.

Teaching time series in a secondary school environment offers a wonderful opportunity to develop the statistical reasoning of students. However, care must be taken over how the topic is implemented. In New Zealand the acceptance of an incorrect statistical technique did not endear statisticians to the inclusion of time series in the curriculum, but the more recent version has made a number of substantial improvements, particularly shifting the focus of the topic from procedural calculations to one of interpretation and integration of statistical analysis in context. In the future I hope that the New Zealand curriculum will exploit all the opportunities afforded by the new software and provide an example to others of the potential of time series in the secondary school curriculum.

REFERENCES

Arnold, P., Pfannkuch, M., Wild, C., Regan, M., & Budgett, S. (2011). Enhancing students' inferential reasoning: From hands on to movies. *Journal of Statistics Education, 19*(2), 1-32.

Cobb, G. (2007). The Introductory Statistics Course: A Ptolemaic curriculum? *Technology Innovations in Statistics Education, 1*(1), 1 - 15.

Curcio, F. (1987). Comprehension of mathematical relationships expressed in graphs. *Journal for Research in Mathematics Education, 18*(5), 382-393.

Gal, I., & Garfield, J. (Eds.). (1997). *The assessment challenges in statistics education.* Amsterdam, The Netherlands: International Statistical Institute.

Konold, C., Higgins, T., Russell, S., & Khalil, K. (2015). Data seen through different lenses. *Educational Studies in Mathematics, 88*(3), 305-325.

Oates, G. (2009). *Integrated technology in the undergraduate mathematics curriculum: A case study of computer algebra systems.* Doctoral thesis, University of Auckland, NZ. Retrieved from https://researchspace.auckland.ac.nz/handle/2292/4533

Moore, D. S. (1997). New pedagogy and new content: The case of statistics. *International Statistical Review, 65*(2), 123- 165.

Passmore, R. (2016), *The impact of curriculum change on the teaching and learning of time series*. Master's thesis, University of Auckland, NZ. Retrieved from http://hdl.handle.net/2292/29102

Pirie, S., & Kieren, T. (1994). Growth in mathematical understanding: How can we characterise it and how can we represent it? *Educational Studies in Mathematics, 26*(2-3), 165-190.

Shaughnessy, M. (2007). Research on statistics learning and reasoning. In F.K. Lester (Ed.) *Second handbook of research on mathematics teaching and learning* (Vol. 2, pp 957-1009). Charlotte, NC: Information Age Publishers.

Tukey, J. (1977). *Exploratory data analysis.* Boston, MA: Addison-Wesley.

Verhage, H., & De Lange, J. (1997). Mathematics education and assessment. *Pythagoras*, 14-20.

Wild, C. & Pfannkuch, M. (1999). Statistical thinking in empirical enquiry. *International Statistical Review, 67*(3), 223-265.