THE MODELLING PATH FOR INFORMAL INFERENTIAL REASONING

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Exploration, construction, validation and use of statistical models are foundational aspects of students' modelling process for informal inferential reasoning. In this study, a class of middle school students (aged 12-13) was posted a problem rooted in a crisis of society (obesity) to examine the nutritional value of their lunch boxes. The context of the problem offered an opportunity to engage students in the exploration, construction, validation and use of statistical models with real and authentic data to support inferences and to reflect on their own nutrition. The main sources of data were students' discourse in the classroom, students' artifacts and researcher journal. Analysis of the data gathered provided insight into students' process of modelling and insight into the influence of context in informal inferential reasoning.

INTRODUCTION

Models are a foundational part of statistical thinking and a big element to support inferences. Modelling is usually understood as the route of constructing models that can explain reality (Pfannkuch, et al., 2016; Zapata-Cardona, 2015) and it is an essential activity of statisticians' daily practice. Typically, professional statisticians use models either to generate simulated data or to fit statistical models to existing data (Garfield & Ben-Zvi, 2008). In spite of the importance of modelling in the statistics activity, it has not received the proper attention in school statistics (Garfield & Ben-Zvi, 2008). The modelling process, as a teaching recourse, should focus on the very foundational aspects of the model construction such as exploration, construction, validation and use. In this article, students' processes of exploring, constructing, validating and using models to support their informal inferential reasoning is investigated. Consequently, the research question addressed here is: "what is the modelling path students follow for informal inferential reasoning?"

THEORETICAL FRAMEWORK

Statistical Model and Modelling

Some statisticians, from the field of statistical practice, conceive a statistical model as a set of probability distributions (McCullagh, 2002). This conception helps little in the educational practice. In the field of statistics education, however, the conception of a model is much wider and the modelling process considers all the previous stages of the creation of the model as essential components of such a process. In the modelling process, the modeler has to explore data, construct representations that explain the reality, validate the model and use it to make conclusions or predictions (English, 2012). In that sense, a simple tool like a graphical representation of data from a real-life phenomenon, a summary measurement, or a creation to structure data could be considered a model (Konold, Finzer & Kreetong, 2017).

Another school of thought considers models as simplified abstractions representing a phenomenon, which could be verbal, visual, symbolic, and gestural (Gould, Johnson, Moncada-Machado & Molyneux, 2015). Thus, a model is "a representation of a real-life process that explicitly or implicitly addresses variability" (p. 181). Variability seems to be a core component of the modelling activity which is closely related to inference. Watson (2006) has stated that data modelling involves fundamental components of inference such as variation and prediction. Variability also appeared as a key element in a study with practicing statisticians, Wild and Pfannkuch (1999) made evident that *reasoning with statistical models* is closely related to measuring and modelling variability for the purpose of prediction, explanation, or control. Models allow summarizing data in meaningful ways to facilitate the interpretation and reasoning that leads to inference based on evidence; they could be used for representation, information, explanation or prediction.

English (2012) stated some core components of data modelling: (1) Generating and selecting attributes (creation, analysis and revision of data classification models); (2) Structuring and representing data (invent or design new representations, explain their creations, and understand the role they play); (3) Identifying variation in data (data are collected, graphs are created, and averages are computed to manage variation); and (4) Making predictions from data (draw informal inferences based on various types of data). This conception of modelling has the potential to engage students in extended and integrative experiences in which they generate, test, revise, and apply their own models in solving problems that are meaningful to them.

Informal Inferential Reasoning

Informal inferential reasoning in statistics has been understood as "the way in which students use their informal statistical knowledge to make arguments to support inferences about unknown populations based on observed samples" (Zieffler, Garfield, delMas & Reading, 2008, p. 44). It includes making judgments based on data without using formal methods, integrating prior knowledge, and articulating evidence-based arguments for claims or predictions. Similarly, other authors have defined it as "the process of making probabilistic generalizations from (evidenced with) data that extend beyond the data collected" (Makar & Rubin, 2009, p. 83). A statistical inference —as a product— is a claim inferred from a sample to a population that embraces a certain level of confidence. According to Makar and Rubin (2009), such claim should include three key components: a *generalization* that goes *beyond the data*, use of data as *evidence* —claims beyond anecdotes and believes— and articulation of uncertainty through *non-deterministic language*.

METHODOLOGY

The research question for this study was: "what is the modelling path students follow for informal inferential reasoning?" To address this, a series of lessons was analyzed from a seventh-grade classroom (43 students, ages 12-13) in a public school of a major city in Northwest, Colombia in which the teacher participated in the design.

A researcher and a middle school teacher met to plan the lesson, which was inspired by a problem the teacher had previously observed in her community: the rapid increase of obesity among youth. The lesson took place over four ninety-minute sessions one week apart. The lesson activities included the watching and discussion of a video, methodological design to solve a given problem, design for data collection, data analysis and oral presentation.

The researcher and the teacher met after each session to discuss their observations and make recommendations and adjustments for the direction of the next session. The researcher supported and accompanied the teacher in the implementation of the lesson, but the teacher taught the lesson and made all the decisions about the content and teaching of the lesson.

For the design of the lesson, connecting a relevant context —critical situations of society— to the educational process was essential to promote critical citizenship (as described by Skovsmose 1999). Students started by watching and discussing a 3-minute long video that debates changes on nutrition habits over time and the impact on obesity. The statistical question students studied was "how nutritious is the food we bring to school in our lunchbox?" To explore this statistical question, students gathered information from their lunchboxes on a particular day. They made a list of 31 products they brought from home or bought from the school cafeteria and then they, working in groups, researched the nutritional information of each product such as the amount of proteins, carbohydrates, fats, sugars, sodium. The students created a database with the information collected (10 variables for 31 products) and analyzed it using different strategies.

For the analysis, information was gathered from oral and written registers from video recordings of the lessons, observations, students' artifacts and researcher's journal. During the implementation of the lesson, notes were taken to be discussed afterward with the teacher. The focus of the analysis was on students' discussions. The units of analysis were key episodes during class discussions that indicated exploration, construction, use, and validation of models and the connection with students' informal inferential reasoning related to the statistical question. Those episodes were selected to illustrate the path followed by students in their informal inferential reasoning. Because of space constraints in this study, only one episode is discussed.

RESULTS

Results will be illustrated with an episode of students' discussions. The discussion took place on the third day of the lesson. The teacher asked volunteers teams to present preliminary results to the class. The team formed by Kenny, Cristian and Andrew presented their initial findings (names are pseudonyms to protect the identity of participants under Colombian research law). The team developed a strategy to summarize the information from the database by adding up the calories reported in each product to find out "the amount of calories consumed in a day". However, students' interchange revealed that the strategy was insufficient to describe the behavior of the whole class and a student from another team suggested a more efficient strategy.

Kenny: What we did was to add all the quantities in the columns to find out the amount of calories we are consuming in a day. We concluded that we are overtaking calories [adding up the amount of calories in the products 4454]. We are consuming 4454 and an individual's calorie limit should be between 2000 and 2500.

Teacher: What do you [the rest of the class] think about what they did?

Samuel: That is what the whole class consumes. So a person does not consume 4454 calories, of that 4454, a person might consume around 200.

Kenny: Since the analysis is for the whole class, that is why we are putting the class together as one person. [...] It is like saying that we are all in one.

Samuel: When he [Kenny] said that we were all one, I imagined an answer. I figured that with those 4454 calories you divide them by the number of students […] getting a complex average.

In this excerpt, Kenny's team explored the data and constructed a model to summarize the amount of calories consumed by the whole class. However, for Samuel —a student from a different team— that strategy felt short on describing the class behavior. His intervention suggested that the actual measure of adding up the calories column was not the best summary to reflect the calories of the lunchboxes. He proposed a measure that took into account the contribution of each member of the class to the calories, which he called "*complex average*". Samuel's intervention functioned as a request for the team to validate the model and the mean emerged as a better model to summarize the calories intake behavior.

DISCUSSION

The episode of the interaction between Kenny and Samuel will be used to orient the discussion of the findings in different aspects. A discussion about the different stages students go through in the modelling process is presented.

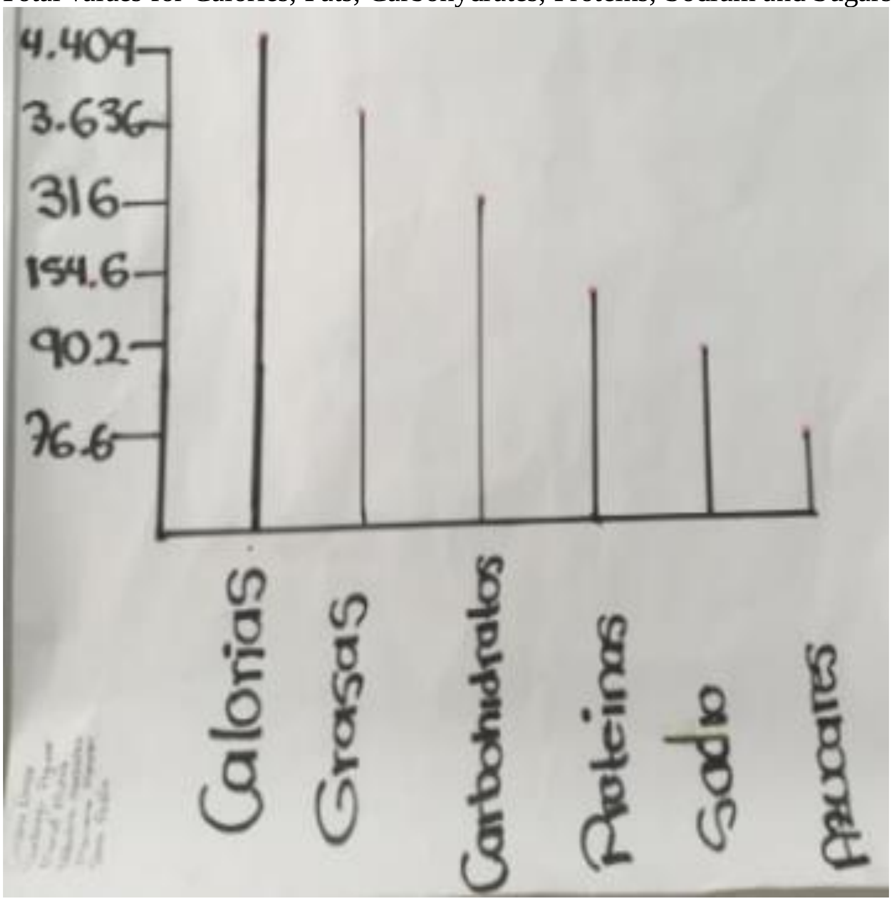
Exploration of Data

To answer the statistical question "how nutritious is the food we bring to school in our lunchbox?" students had to understand the scenario under study and make decisions about what to consider and what to overlook. They also need to discriminate the essential information from the secondary one. In the experience described here, students decided to construct a database with 31 products. If a product appeared more than once in the lunchboxes, students only considered it once. The decisions students make in this organizational stage are crucial because they reflect the way they think about the situation and about the modelling process (English, 2012). That decision of considering a product only once prevent students from exploring the snack preference of the class. In the excerpt presented, students took into account information from the world "an individual's calorie limit should be between 2000 and 2500". With this statement, Kenny went beyond the database and integrated information he had from the world outside to make sense of the situation. In the modelling process, the modeler goes back and forth between the mathematical world and the contextual world (Pfannkuch, et al., 2016; Pfannkuch, 2011).

Constructing Models

Structuring data is an approximation to the construction of the model. Students in this class structured data in different ways. Kenny's team calculated a summary measurement but other teams created graphical representations like the one shown in Figure 1. Some authors have shown that in creating representations of data, students frequently either ignore crucial information or use redundant information (English 2012). However, by creating their own representations, analyzing them, explaining them to others, and refining them, students exhibit their own comprehension of the data and polish their interpretations. Presenting and explaining those representations might function as a metacognitive activity for students to polish their thinking (Radford, 2006) and might help teachers to understand students' thinking process in order to offer them the proper support to address difficulties.

Figure 1: Graphical representation of Sandra's team
Total values for Calories, Fats, Carbohydrates, Proteins, Sodium and Sugars



Use Models for Inference

Totaling the values of the calories column was used as a model to make predictions. The modelling process involves aspects related to inference, which includes variation and prediction. Variation is an essential characteristic of statistical reasoning and permeates every component of the statistical practice (Cobb & Moore, 1997). Some authors consider that the collection and organization of data and calculation of measurements are tasks carried out precisely to manage variation from phenomena characterized by variability (Watson, 2007) in order to make predictions with the less possibility of error. However, dealing with variations is challenging. Research has shown that students have difficulties in looking at data as a whole and tend to focus on individual values (Rubin, Hammerman & Konold, 2006). Students discourse in the class interaction revealed different approaches in which they took into consideration variation. Some examples include the followings:

- Adding up all the values displayed in the table and recording the total
- Using summary measurements to refer to the data as a whole
- Giving a range of possible values for the calories consumption
- Pointing out that the total value of calories only gives aggregated information but not particular information
- Making graphical representations that display the variation of the data

The goal of the lesson reported in this article was to promote informal inferential reasoning in students through modelling within statistical investigation environments. Students' discourse and interactions showed claims that suggest they were *on the route* of informal inferential reasoning. Claims like the one stated by Kenny illustrate this fact: "We are overtaking calories. We are consuming 4454 calories". This claim suggests a *generalization* that Kenny built *based on evidence from data* after the data modelling process. However, the claim failed to use *non-deterministic language* to offer a degree of uncertainty. According to Makar and Rubin's framework (2009) for informal inferential reasoning, this claim is incomplete to be considered an inference, but the claim reveals that the lesson, within a relevant context, put students in the path of informal inference reasoning. Although this claim is very deterministic to be considered an inference it discloses that in the inferential thinking students suggest ideas and hypothesis that help them explain the contextual situation under study. Those associated processes are crucial to support informal inferential reasoning.

Validation of Models

The excerpt of students' interaction shows that Kenny's team added the values of the calories column and used that measurement as a model to describe the behavior of the calories intake of the class. However, Samuel pointed out the lack of efficiency of the proposed model and suggested a more refined one that considers the contribution of each member of the class to the calories consumption. The mean emerged as a model that could possibly solve the inefficiency of the model previously proposed. The students' interaction suggests a request for validation of the model. Models need to be validated to be able to use them for prediction. Pfannkuch and colleagues (2016) have shown that professional statisticians in the statistical practice are aware that good decisions cannot come from models that are not valid.

CONCLUSIONS AND IMPLICATIONS

To develop skills in modelling, students need to be exposed to different ways to deal with data from relevant situations. The modelling process is not reduced to the construction of the model, all the previous stages related to exploration of data and the posterior stages related to validation and use of the model are also crucial. It is important that early on in schooling students have opportunities to deal with messy data from relevant contexts in which they have to make decisions about the data collection process, structure of the data, validation of models and use for making predictions and conclusions. According to English (2012) when students are exposed to the tensions of data modelling, they have to make decisions about what to privilege and what to ignore. This decision making is fundamental to refine the modelling process and consequently the informal inferential reasoning.

Relevant contexts could help improve the teaching and learning of statistics. In this experience, the students articulated statistical knowledge with knowledge of the critical situation to create models and make claims and conclusions. The results show that a relevant context inscribed in a crisis of the society could engage students in interesting discussions. Students are part of a society and schooling needs to integrate school knowledge with real-life knowledge (out-of-school knowledge). The purpose of schooling is the development of the critical citizen and it could be done if statistical lessons consider in their design the empirical study of the crisis of society.

This experience opens the door to point out several misconceptions introduced by students during the class such as: omitting repetitions in data collection, interpretation of aggregate data, and problematic scale in the graph. However, the main purpose of this article was to make evident the path students follow in the process of making informal inferential reasoning. This does not mean that those misconceptions were ignored but were not the data to support the intended discussion in this article.

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