TEACHING STATISTICS FOR SUSTAINABILITY

Hanan Innabi
University of Gothenburg, Sweden
hanan.innabi@gu.se

The concept of sustainability is viewed as spirally shaped, it continues to grow and expand upon its original meaning of remaining and continuing. The variation theory of learning has been adopted to explain teaching statistics for sustainability. The concern is the generative learning that continue and prepare for the future. For this to occur, a focus on variation thoughts in teaching and learning statistics is a necessary condition for all grades and levels. To teach variation in a powerful way, students' awareness levels should be considered in regard to instruction and assessment. Dynamic visual tools are significant for helping students see variation . Also, Statistics has to be taught not as isolated discipline but rather within a context of complex real life issues.

BACKGROUND

Sustainability can be defined as the propensity of an object or programme to continue and grow over time (Selvanathan, 2013). Its roots that can be traced to ancient times (Du Pisani, 2006). However, it became visible by the end of the twentieth century, with population growth and an increasing awareness of the need to use resources in a sustainable way. When the world realised that economic development at the expense of ecological health would not lead to long-lasting prosperity, a mode of international thinking regarding the adoption of sustainable development arose. In addition to ecological and economic issues, social issues had to be discussed. In 1987, the United Nations (UN) adopted an agenda for sustainable development, in which the three pillars for sustainability were environmental preservation, social equity, and economic viability (The World Commission on Environment and Development, 1987). Thereafter, global concern for sustainability development grew, and it continued to expand, with the appearance of an additional pillar related to the promotion of cultural vitality and diversity (Mebratu, 1998). The concept of sustainability continued to expand. Selvanathan (2013) suggested that in addition to viewing sustainability as economic, social, environmental, and cultural, there was a necessity for a fifth pillar: educational sustainability. The objective of educational institutions is to ensure that the knowledge that is transferred is sustainable.

The concept of sustainability is growing, expanding upon its original meaning—that is, things keep going and sustain themselves. Sustainability can be observed as a spiral shape that keeps expanding with human development. Besong and Holland (2015) called for the promotion of more sustainable values, behaviours, and lifestyles that can make the world safer, healthier, and more prosperous for all.

In 2015, the UN adopted a new and ambitious sustainable development agenda to be achieved by 2030. This agenda contains 17 goals related to 17 issues, and their aim is to change our world. These goals seek a decent life for all: no poverty; enough food on the table; health and well-being; empowerment for women; sustainable industry innovations in science, technology, and transport; and healthy oceans and seas (UN, 2015). One of the noteworthy goals among these is the fourth sustainable development goal (SDG4), which is related to education. The aims of this goal are to 'ensure inclusive and equitable quality education and to promote lifelong learning opportunities for all' (UN, 2015, p. 17). There is agreement that education influences how the other SDGs function.

While reviewing the literature related to education and sustainability, two different forms or categorize appeared: 1) sustainable learning, related to learning that continues beyond the end of formal instruction, meaning preparation for future learning, and 2) education for sustainable development (ESD), meaning how to educate people to be aware of sustainability issues, such as climate change, equity, and global citizenship.

The focus of this paper is the first form which is sustainable learning. It aims to bring attention to the helpful role of teaching statistics—when preparing the necessary conditions for powerful learning—in sustainable learning. The theoretical framework upon which this paper is

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based is the variation theory of learning. The following section includes a brief clarification of this theory.

Variation Theory

Variation theory is a learning theory that was developed from the phenomenographic research approach. Phenomenography is a qualitative, non-dualistic, ontological, and explorative research approach that aims to describe how students experience and perceive a phenomenon (Marton, 2015). It is derived from the relationship between the student and the phenomenon. Phenomenography is based on the fact that individuals understand phenomena in the world differently. Each way of experiencing (understanding) can be understood in terms of which aspects of the phenomenon are discerned and not discerned in the learner's awareness (Marton & Booth, 1997).

In variation theory, being good at learning the object of learning means discerning its critical aspects simultaneously (Marton, 2015). The differences in how students experience the same object of learning depend on which aspects of this object they discern (Lo, 2012). According to variation theory, the discernment of critical aspects occurs during systematic interactions between a learner and a phenomenon (what is to be learned), and variation is the agent that generates such an interaction. Variation, therefore, is a necessary condition for learning. The most effective way to help students learn is to focus on providing opportunities for them to experience variation in terms of the concepts they currently take for granted. A dimension of variation is an emerging aspect of a phenomenon that can take on different *values* while some aspects of the phenomenon vary (Marton & Tsui, 2004).

Four patterns of variation were proposed: contrast (seeks to distinguish different and dissimilar), separation (the awareness of critical features and the dimensions of variation), generalisation (a variation interaction that is inductive in nature and explores whether an observed pattern can occur while certain aspects vary), and fusion (integrates the separated critical aspects or dimensions of variation into a whole) (Marton, 2015).

STATISTICS FOR SUSTAINABLE LEARNING

This section discusses Statistics and sustainability from the perspective of learning sustainability in that learning continues beyond the end of formal instruction (Knapper, 2013; Tractenberg, FitzGerald, & Collmann, 2017). It is what the SDG4 of the UN's agenda describes as *lifelong* learning or what the variation theory of learning refers to as *generative* learning and *learning to learn* (Marton, 2015). The question now is as follows: 'Preparing students for an unknown future is increasingly seen as one of the major ambitions of the educational system. But how can the educational system, which is supposed to promote what is known, achieve that aim? How can the students be prepared for the unknown by means of the known' (Marton, 2015, p. 67).

According to variation theory, for this to happen—that is, for learning to be sustainable—students must interact with powerful patterns of variation and invariance. If we want to prepare learners to handle future novel situations in powerful ways, we have to help them learn to see these situations in powerful ways. This assumes that the likelihood of opening dimensions of variation in the future is increased by having opened them in the past (Marton, 2015). Therefore, by learning to discern some values in one situation, the learner learns to discern other values in the same dimension of variation in another situation. By perceiving differences and similarities between situations, learners, when learning to do something in one situation, will be able to do something different in other situations. Marton (2006) identified the importance of widening the focus of attention to include how situations are related through differences (and similarities). To be able to identify differences, the learner must have encountered differences. To see how something differs, the learner must have previously seen something from which it differs. Seeing one thing affects how the learner subsequently sees another thing; this is not because of the sameness between the two, but because of both the similarities and differences (Marton, 2006).

Marton (2015) explained what Holmqvist, Gustavsson, and Wernberg (2007) identified, which is called *generative learning*. They found that, in certain classes, the outcomes of learning related to language content turned out to be better sometime after the learning occasion than immediately thereafter. The explanation was that the students interacted with more powerful patterns of variation and invariance than students in the other classes. Accordingly, they got better at discerning the critical aspects of object of learning. This means that after the end of the learning occasion, every time they encountered instances of the object of learning the likelihood of discerning those aspects was higher than for students from other classes. And every time they discerned critical aspects they became better at discerning critical aspects. Every encounter with instances of the object of learning was thus a new learning occasion. This was a *learning to learn* effect. They had learned the object of learning better, but also learned to learn it even better again. (Marton, 2015, p. 228)

In this paper, we are talking about statistics, the science based on *variation* and *uncertainty*. In any set of data, the elements are not usually alike, and the extent to which they vary among themselves is of basic importance in statistics. Variation in statistics appears among individual and repeated measurements. Shaughnessy (2007) stated that students' reasoning about variability in statistics is focused in three areas: variability within data, variability across samples, and variability between distributions.

Even though the term *variation* in regard to statistics is not the same as in relation to the theory of learning, both are based on the same idea, which is differences. In his theory, Marton calls for teachers to help students to see differences in all sorts of objects of learning so that they can learn in a powerful way. In statistics, variation is explicit, and all we need to do is give it attention in the curriculum and teaching.

With the increasing calls to acknowledge the importance of statistical reasoning, variation in statistics education is becoming increasingly acknowledged. In the 1990s, a movement that touted the importance of variability in statistics education began. Moore (1990) describes statistical thinking as recognising the omnipresence of variability. Snee (1999) stated, 'If there was no variation, there would be no need for statistics and statisticians' (p. 257). Several scholars (e.g. Cobb & Moore, 1997; English & Watson, 2016; Garfield & Ben-Zvi, 2008; Wild & Pfannkuch, 1999) consider variation the fundamental phenomenon underlying the entire statistics field.

Many statistics educators realised there was a lack of attention being paid to variability in the curriculum and in national assessments (e.g. Batanero, Godino, Vallecillos, Green, & Holmes, 1994; Green, 1993; Shaughnessy, 1997). Accordingly, notable changes were made to several curriculum documents around the world. This includes the common core state standards for mathematics in the United States, which include standards such as 'develop understanding of statistical variability' (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). In the Australian mathematics curriculum in 2015, a clear concern for variability starting from the first grade is noticeable (English & Watson, 2016).

Despite awareness among statistics educators of the importance of focusing on variation in statistics education, much effort is still needed in relation to teaching and learning statistical reasoning. There is a need to emphasise variation from the beginning of a child's experiences with chance and data throughout all learning stages (Watson & Kelly, 2007). Students need to consider appropriate ways to realise, understand, and model the variability of data (Carter, Brown, & Simpson, 2017).

In a recent study that can be explained by the variation theory of learning, it was found that by using non-collinear data points in the real world, students transferred slope conceptualisations when reasoning covariationally—that is, reasoning about how variables change together (Nagle, Casey, & Moore-Russo, 2017).

In aiming to generate the benefit of the variation theory and its tool, phenomenography, and to teach variation thoughts in statistics in a powerful way, three points must be considered: *students' awareness levels*, *dynamic visualisation*, and *complex real life context.*

Students' Awareness Levels

In statistics education, many voices are calling for acknowledgement of the importance of, and the need to better understand, students' development of variability concepts in different statistical contexts (English & Watson, 2016; Noll & Shaughnessy, 2012). Such an understanding is not associated with a specific single answer, it is associated with a description, a range of numbers, or a sketch that is assessed qualitatively. Assessment tasks must give students opportunities to display a range of levels of developing understanding in a phenomenographic way.

Watson and Kelly (2007) mentioned the need to discard traditional assessment tasks that ask for a right or wrong numerical answer.

The following are two examples of research findings related to students' reasoning about variability in statistics. Such findings can be useful tools for instruction and assessment that seek to achieve sustainable learning.

Example 1: Noll and Shaughnessy (2012) found three hierarchal categories for students' reasoning about empirical sampling distributions that were generated from a variety of repeated sampling tasks. These categories were as follows: a) additive (based upon frequency information), b) proportional (proportional reasoning focused on percentages and relative frequencies), and c) distributional (integrating multiple aspects of expected sampling distributions [e.g. centre, shape, and variability] when making predictions about or estimating population proportions from a sampling distribution).

Example 2: Konold, Higgins, Russell, and Khalil (2014) grouped how students view numerical data into four categories: a) pointers (focusing on issues outside the data set itself), b) case values (focusing on particular data points, such as the largest value), c) classifiers (counting individual data values having the same attributes), and d) aggregate (paying attention to the overall distribution of the data, including the centre and variability).

For students to discern variations in statistics, they need explicitly related classroom experiences. For example, to help students to integrate multiple aspects of distribution and to become distributional reasoners, instructional experiences that focus students' attention on multiple aspects of sampling distribution (centre, shape, and variability) are necessary (Noll & Shaughnessy, 2012). By focusing on fixed, specific cases, such as what you expect to get if you select 10 gumballs from 100 mixed gumballs—50 red and 50 green—we limit the opportunities to open the dimensions that might help students with learning to learn or generative learning. In sampling situations, Shaughnessy (2012) identified the need to consider variability in our teaching and not simply focus on expecting the value from one sample, as this makes students focus on centres and ignore variability. We must consider what type of variation we expect if more than one sample is selected. Therefore, questions such as 'What if we select 50 gumballs or 5 or 2?' must be considered.

Dynamic Visualisation

According to the variation theory of learning, students must experience dimensions of variation and invariance. One powerful tool that can help in doing so is dynamic visualisation, which is being increasingly developed. By giving students of all ages and levels a proper dynamic visual tool that they can use to answer questions related to change and differences—such as what is happening in the present, what might happen in the future, and what would happen if this or that variable changed—we help them to open up new dimensions when learning to learn for the future.

Substantial studies show that dynamic visual methods can be highly powerful analytic tools in teaching statistical reasoning (e.g. Konold & Miller, 2005; Marton & Pang, 2006; Paparistodemou & Meletiou-Mavrotheris, 2008; Ridgway, 2015). For example, Ridgway, McCusker, and Nicholson (2003) use Rasch scaling to show that computer-based problems involving three variables can be easier to solve than those involving one variable problem presented on paper. Another example is Ekol (2015), who found that the dynamic sketch and dragging action mediated students' informal understanding of the meaning of standard deviation and variability, and it helped students to check their conjectures, which was not possible in a static environment.

Complex Real Life Context

Statistics is a powerful tool to help people to see the world and understand its complexities. This is because statistics presents a fact and evidence base for all issues related to sustainability.

Based on the variation theory, learning proceeds from a vague undifferentiated whole to a differentiated and integrated structure of ordered parts (Marton, 2015). Thus, moving from parts to a whole in teaching does not work. Statistics must be taught not as an isolated discipline, where its contents are presented in separated bits and pieces, but rather taught in a comprehensive and coherent manner that reflects the complexities of world issues. We need to provide enough space to help students to discern critical aspects simultaneously. A good example of such an approach is case-based approaches, such as the CASE project (Fawcett, 2017), which is based on stimulating interest in a particular area of a student's curriculum using complex, real-life scenarios.

CONCLUSION

Teaching statistics for sustainability demands focusing on *variation* through all school grades and at all levels thereafter, including undergraduate and postgraduate. To teach variation in a powerful way, three issues should are important: 1) students' awareness levels should be considered with regard to instruction and assessment. Phenomenography has to be used not only for research purposes but also for teaching. 2) Dynamic visual tools are a great opportunity to help students see variation and to learning to learn for the unknown future. 3) Statistics better to be taught not as a separate discipline but as an integrated tool that deals with life's complex issues.

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