RETHINKING THE STATISTICS CURRICULUM:
HOLISTIC, PURPOSEFUL AND LAYERED

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Bakker and Derry (MTL 13:1, 2011) challenged the field to address three persistent problems in statistics education—students' difficulty in applying what they learn due to inert knowledge, lack of coherence from the students' perspective due to an atomistic approach, and sequencing of curriculum which does not build students' understanding in a coherent way. To address these challenges, a rethinking of school statistics curriculum is needed. This paper outlines three principles to move forward: focusing on holistic learning through informal statistical inference, purposeful tasks which use an inquiry approach, and a layered curriculum that provides students with experiences that informally access powerful statistical tools years before they are formalised. The principles align with recent studies in statistics education.

INTRODUCTION

Student achievement and interest in school statistics has been below expectations. Bakker and Derry (2011) articulated three persistent challenges in statistics education that need to be addressed in order to improve students' understanding and valuing of statistics. (1) The first challenge is inert knowledge, where students are unable to apply what they know. (2) The second challenge is the atomistic approach in how statistics is taught. This means that students' knowledge is developed by teaching individual topics, rather than focusing on their relations to one another. Finally, (3) the sequencing of the curriculum from simple to complex hierarchies of concepts (rather than building reasoning) is a third challenge that further aggravates a lack of coherence of the curriculum. This sequencing does not prioritise coherence. It takes years for students to learn enough statistics to be able to access its power; most are not around long enough to experience this power.

This paper outlines developments in statistics education research that directly address these three challenges, such as informal statistical inference, applications of the theory of inferentialism, progress in understanding statistical inquiry and statistical modelling, the critical role of purpose, and new perspectives on informal learning. From this literature, three principles for re-thinking curriculum are proposed to address Bakker and Derry's challenges in statistics education.

THREE PERSISTENT CHALLENGES IN STATISTICS EDUCATION IN SCHOOLS

Inert knowledge. Statistical knowledge is critical to solving problems involving data, but much of our formal knowledge is inert. Even students who demonstrate that they know statistical concepts (by performing well on a test, for example) do not necessarily draw on their knowledge when they apply it. Transfer literature has provided strong evidence that people often do not draw on their knowledge when solving an unfamiliar problem (e.g., Day & Goldstone, 2012). Multiple studies in statistics education research have documented that students can compute a mean, but do not think to draw on their knowledge to compare groups (e.g., Konold & Pollatsek, 2002; Mokros & Russell, 1990). Textbooks tend to "cue" students about which statistical concepts to apply simply by placing their exercises at the end of each chapter. Unless students have opportunities to practice applying their knowledge beyond the textbook, their knowledge will remain inert.

Atomistic approach. Students often do not see the "big picture" of statistics and miss how ideas are connected. For example, rather than see centre, spread and shape as properties of a distribution, they experience these concepts atomistically. However, one key purpose of working with distributions is for data interpretation within the context of solving a problem. When topics are taught separately, students don't see the purpose or utility of what they are learning. Teaching concepts through a purpose (i.e. solving a problem requiring interpretation of data) rather than only as a skill provides coherence from a student's perspective while allowing for skill development.

Coherent curriculum sequencing. Sequencing of curriculum relates to what content is taught, the order that topics are introduced and how ideas build from one year to the next. "When [curriculum is] … put together in disconnected fragments there appears to be an un-stated expectation that the

learner themselves will achieve a coherence which, if it exists at all in the syllabus, does so only very covertly" (Burton, 2004, p. 26). Current curriculum relies on behaviorist theories developed in the 1960s, in which concepts are broken into components and taught from simple to complex (National Research Council, 2000). For example, calculating average requires division, so averages are taught after students can master the computation. However, children encounter typicality long before they have a need to calculate averages. Young children can recognise people of typical or atypical height based on experience; they know that as they get older they grow taller, informal signaling association of variables; and they may recognise that what is typical in one country (Australia) may not be typical in another (Vietnam), suggesting that data can come from different populations. Relationships between these statistical ideas, although informal, are coherently related to one another in investigating typical heights of primary children (Makar, 2014). Therefore, investigations can introduce relationships between complex statistical ideas long before being introduced formally.

The implications of not addressing these three challenges are stark. If we continue in the current tradition of producing curriculum documents sorted into separate topics (e.g. graph construction, calculations) rather than emphasise statistical reasoning and inference, then students will continue to develop inert knowledge, abandon sense-making within learning experiences that lack coherence, and suffer from sequencing that does not build connections. In the next section, I identify recent research in statistics education that address Bakker and Derry's (2011) challenges.

DEVELOPMENTS IN STATISTICS EDUCATION RESEARCH

Over the past decade in particular, developments in statistics education have had a substantial impact on the development of conceptual tools for engaging with reforms to the teaching of statistics that have long been promoted but not realised (Cobb, 1991). In this section, I briefly overview informal statistical inference, the theory of inferentialism, emphases on purpose and utility of concepts, and new approaches to statistical inquiry and statistical modelling.

An *informal statistical inference* is an uncertain claim or prediction based on data (Makar & Rubin, 2009). It is intentionally broad to allow for those with little formal background in statistics to access the power of statistics—making predictions based on data. Formal approaches to statistical inference are typically introduced at university, but most people don't reach this level and even those who do struggle. This is due to the large and abstract foundation of mathematics and probability that underpins hypothesis testing. Numerous cases of informal statistical inference have appeared in the literature over the past 15 years from young children (Leavy, Meletiou-Mavrotheris, & Paparistodemou, in press) through to university level.

Bakker and Derry (2011) used the theory of *inferentialism* to explicitly address their three challenges in statistics education. Inferentialism is a philosophical theory in which concepts are understood in terms of their inferential connections to other concepts, meanings of words are based on their social use and claims are interpreted in terms of the inferences that can be made from them (Noorloos, Taylor, Bakker, & Derry, 2017, p. 446). Inference in this philosophical context is broader than statistical inference. The benefit for curriculum in statistics is that an inferentialist perspective promotes the use of concepts in context to highlight their connections with other concepts.

Ainley, Pratt and Hansen (2006) argued that students experience an impoverished curriculum because they do not see the *purpose or utility* of the what they are learning. By purpose, they refer to the tasks that genuinely solve problems from students' perspectives. Utility, on the other hand, refers to how content adds value to the problem solution. Tasks need not be "real world" for students to see them as beneficial. By developing purposeful tasks, students can be involved in meaningful sense-making because they care about solving the tasks. By ensuring that the statistics used in the tasks add value, students are able to see statistics as worth learning. These two ideas—purpose and utility—therefore give students a sense of the usefulness and power of statistics to solve meaningful problems.

Statistical inquiry has been described as a process of addressing complex, ambiguous problems with statistical evidence (Makar, 2014). Wild, Utts and Horton (2018) highlight that in statistical inquiry, "People start with very vague ideas about what their problems are, what they need to understand, and why. … [Developing] questions that can realistically be answered using statistical data always involves a lot of hard thinking and often a lot of hard preparatory work" (p. 11). Much of the newer classroom research in mathematics and statistics is set in an inquiry context in order to focus more on the reasoning that develops, including problem definition and 'a need to know'

something about the world. To engage in statistical inquiry, the norms of classroom learning environments need to encourage meaning-making and collaboration (Ben-Zvi, Gravemeijer, & Ainley, 2018). Inquiry underpins many of the recent perspectives on *statistical modelling*, which emphasise the whole investigation process, "ranging from posing researchable questions to deciding what about a system is worthy of measure and designing investigations that will generate a sample of observations" (Lehrer & English, 2018, p. 231).

PRINCIPLES FOR STATISTICS CURRICULUM: HOLISTIC, PURPOSEFUL AND LAYERED

Using the ideas above—informal statistical inference, inferentialism, purpose and utility, and statistical inquiry and modelling—the final section of this paper sets out three principles for statistics curriculum: Holistic, purposeful and layered. As a framework, these principles can be used to design or review curriculum and learning activities for students.

- *Holistic curriculum.* Informal statistical inference is one approach that allows for a holistic curriculum by placing the focus of learning on the relationships between concepts in the course of solving problems. Even young children can draw on a web of reasons in justifying their ideas (Leavy et al., in press). Because informal statistical inference requires that students both acknowledge and reason beyond their data, they necessarily draw on concepts informally that might not be encountered at an early age. For example, students aged 5-6 used fundamental ideas about data, representation, distribution, evidence, typical/atypical in making an informal statistical inference about the typical shoe size of children in first grade (Makar, 2016). Young children often learn how to collect and represent data, but their learning activities often neglect relationships to other key ideas.

- *Purposeful curriculum.* Statistical inquiry can give students a purposeful use for statistical knowledge. A greater focus on informal statistical inference and statistical modelling further uses the power of statistics to make predictions. If students simply describe data, then it becomes difficult for them to see the purpose of statistics. This is where the theory of inferentialism, focusing on relationships, reminds us to consider how a representation or piece of knowledge fits into a larger purpose.

- *Layered curriculum.* A layered curriculum acknowledges that rather than order topics by their complexity, curriculum can foreground formal content with early exposure to informal content to emphasise reasoning. Introducing powerful statistical ideas early at an informal level, then allows students to develop statistical reasoning and build on concepts multiple times over the years until concepts are formalised. Research has illustrated exemplars of classrooms where children address problems related to their world, with informal statistical concepts and involve rich yet simple modelling or informal statistical inference (Leavy et al., in press). Children in primary can undertake statistical investigations that involve collecting, organising and presenting data as evidence and that draw on informal ideas of sampling and making predictions from data. This will introduce them to the ill-structured nature of statistical questions, develop their language and reasoning as they seek and discuss data as evidence, and see representations as a resource for analysing and presenting data aligned with the question and their conclusion. In the secondary years, young people can build on these experiences to undertake more complex contexts and methodologies, formalising some concepts (like distributions) and informally introducing multivariate models, for example. By the time students finish school, they will have undertaken a variety of statistical investigations that involve inquiry, informal inference and statistical modelling.

CONCLUSION

If you teach statistics, you will have experienced Bakker and Derry's three challenges that plague learning in the discipline. Coherence as a target of curriculum design is improved when it attends to the complexity and idiosyncratic nature of learners' reasoning and experiences as well as the canonical body of knowledge (Hammer & Sikorski, 2015). "In the intended and implemented curriculum, coherence has a purpose; it aims at developing coherent knowledge in the attained curriculum. In the attained curriculum, coherence is the purpose: students' coherent knowledge" (Verschut & Bakker, 2010, p. 2). Building coherence for students acknowledges their sense-making, which may yet not align with canonical knowledge; within an inquiry-based environment learners

can face rather than avoid these tensions and work through explanations in the discipline and their experience to resolve or challenge the lack of alignment. These experiences can develop understanding over time, allowing opportunities in multiple contexts to strengthen connections between concepts. Pfannkuch (2018) posed these questions to consider when reimagining the statistics curriculum to prepare students for the future: "What learning experiences will prepare students to deal with complexity and ambiguity, to be statistically literate citizens, and to challenge statistically based arguments? What thinking, concepts, and patterns of reasoning are essential to provide cognitive infrastructure that will endure despite rapidly changing technological tools?" (p. 389). This highlights the urgency in developing coherence through a holistic, purposeful and layered curriculum that takes advantage of innovations in statistics education research.

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