Teaching Materials Using Board Game and Classifying Table for Helping Understand Binomial Distribution

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Difficulty in distinguishing two levels of trials in binomial distribution

Binomial distribution is one of most important discrete distributions just like normal distribution is in continuous distributions. Many phenomenon are described using binomial distribution model and we can expect frequency of specific outcome. But learners have difficulty understanding in applying binomial distribution in realistic situations even if they understand in simple situations such as coin-tossing or dice-rolling situations.

One of the reasons of this difficulty may come from that an experiment of binomial distribution includes the following two levels of trials: (1) Each Bernoulli trial. (2) A sequence of n independent repeating Bernoulli trials. It seems to be difficult for many learners to distinguish among two levels of trials, events, and probabilities correctly. To solve this problem, I developed materials that help learners classify them and apply binomial distribution model in realistic situations.

Introductory materials using 'SS Board Game'

As the introductory materials of binomial distribution, I designed a board game named 'Sue-hirogari Sugoroku' Board Game ('SS Board Game) based on the Galton board(See Cochran(2001) for example). By using the Galton board, we can explicitly show process of each Bernoulli trial and the result of sequence of Bernoulli trials to learners simultaneously, but learners have difficulty understanding two levels just by looking balls falling between pegs of the board. In 'SS Board Game' I devised, learners move their tokens in accordance of die rolling results instead of just looking balls, and these activities help learners become aware of two levels.

Rules of 'SS Board Game'

- 2 players.
- Preparation: the gameboard (Figure 1), one die, and 10 tokens for each player.
- (1) Player A put a token on the starting circle. Player A roll a die. If the outcome is 1, then A moves the token to next circle of the ↘ direction. If the outcome is NOT 1, then A moves to next circle of ↙ direction. A moves the token only one step per one roll regardless of the outcome of the die. Player A rolls 6 times and moves the token 6 steps.
- (2) Player B guess where A's token will reach BEFORE player A performs a set of 6 die rolling and put B's token.
- (3) If B successes in guessing the destination of A's token, B gets A's token. If not, A gets B's token.
- They record destinations of tokens.
- They play 10 sets.

Effects of 'SS Board Game'

By playing this game, players are expected to be able to distinguish two levels of trials naturally: each die rolling of player A and the destination of A's token after a sequence of 6 die rolling. I observed four mathematics classrooms of 12th grade using this game and found that learners seemed to be able to distinguish two levels (Narita 1991).
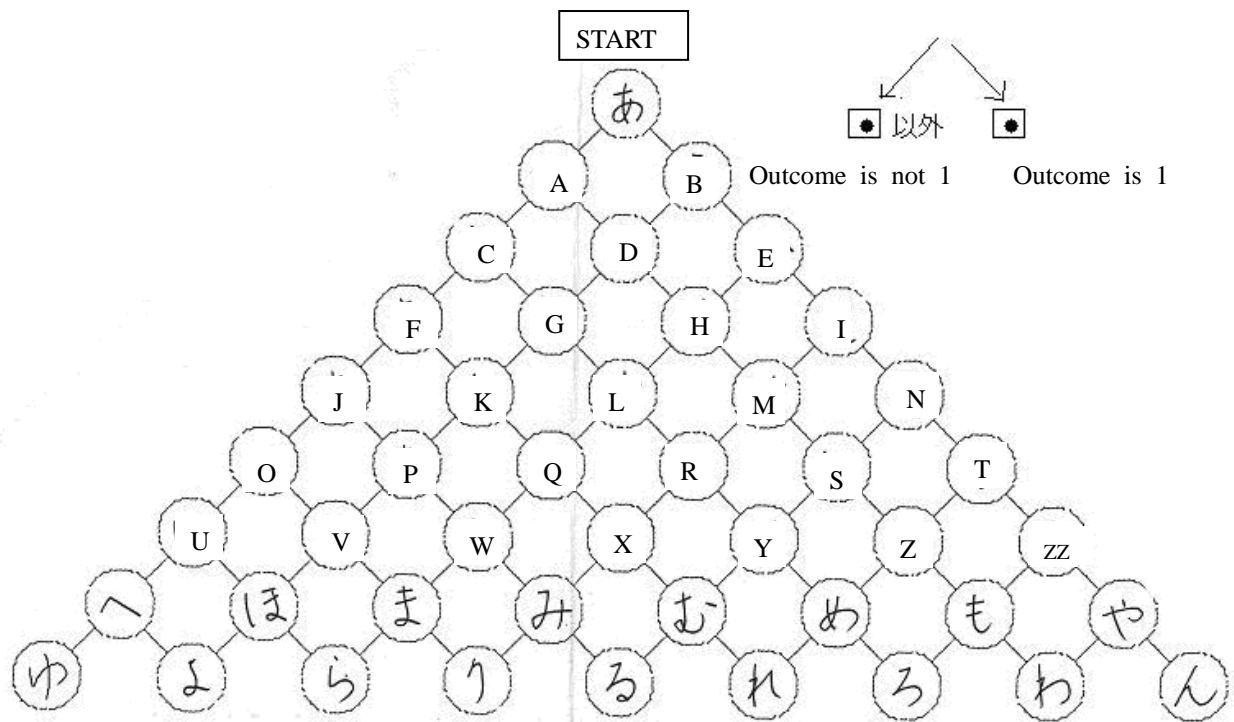
START

Outcome is not 1   Outcome is 1

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z ZZ

Figure 1   The Gameboard for introduction of binomial distribution
Labels of each circle of the original board are in Japanese.

START

X='the number of the outcome is 1'

$X \sim B(6, 1/6)$

Outcome is not 1   Outcome is 1

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z ZZ

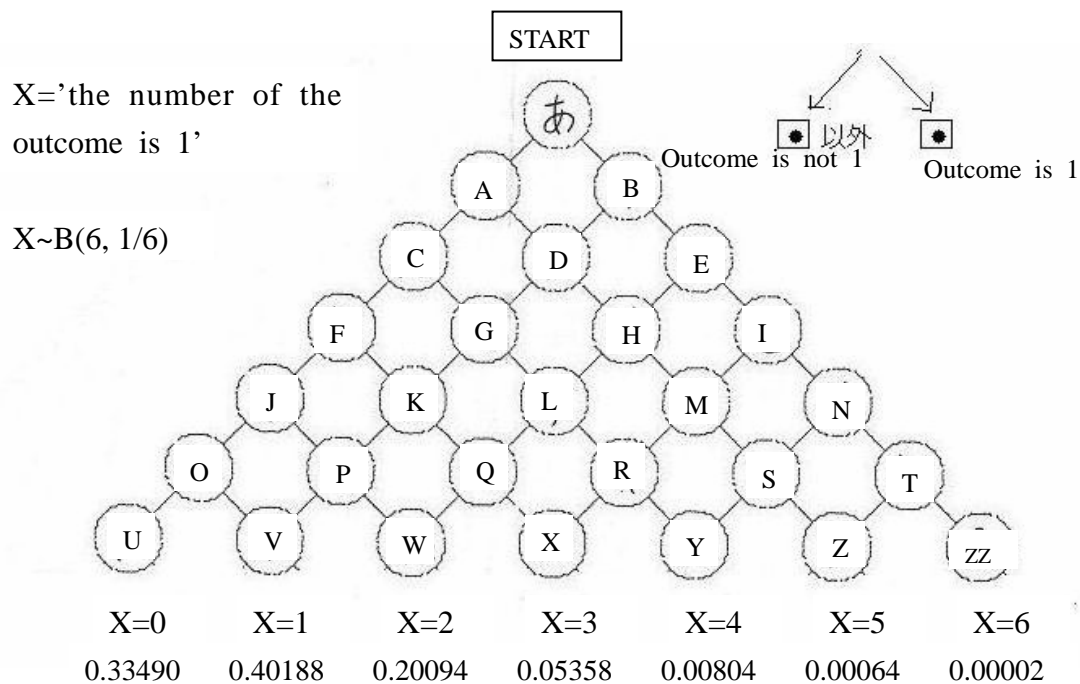| X=0	X=1	X=2	X=3	X=4	X=5	X=6
0.33490	0.40188	0.20094	0.05358	0.00804	0.00064	0.00002

Figure 2   Probabilities by destinations of tokens

Table for helping arrangement of two levels of trials

   After playing the SS Board Game, definition and formula of binomial distribution are introduced. Methods and procedures for calculating the probabilities of each value of a random variable are explained. Following above explanation, learners are presented problems in various situations and required to analyzing by using binomial distribution model.

   Example 1. When a baseball player who has a batting average of 0.3 is at bat five times in a game, how much is the probability for him to hit three times in the game?

   Example 2. If a success rate of certain type of operation at a hospital is 90%, How much is the probability of succeed 90 operations or more in 100 operations?

   In order to classify two levels of trials, events, and probalilities, the following tables are explained and encouraged to use them repeatedly.

Table 1   Classifying Table for 'BB Board Game'

	Trials	Events	Probalilities
Bernoulli Trial.	Roll a die once.	Outcome is 1. (Move to ↘)	$p(1)= p(\;↘\;)= \dfrac{1}{6}$

Repeating 6 times

	Trials	Events	Probalilities
A sequence of 6 independent repeating Bernoulli trials	Roll a die 6 times.	The number of the outcome of 1 is x times. (Move x times)	$p(x) = \dbinom{6}{x}\times\left(\dfrac{1}{6}\right)^{x}\times\left(\dfrac{5}{6}\right)^{6-x}$ =BINOMDIST(x,6,1/6,FALSE)

Table 2   Classifying Table for Example 1: Hitting in a baseball game

	Trials	Events	Probalilities
Bernoulli Trial.	The player is at bat.	The player hits.	$P(\text{ hits })= 0.3$

Repeating 5 times

	Trials	Events	Probalilities
A sequence of 5 independent repeating Bernoulli trials.	The plyer is at bat 5 times.	The player hits x times.	$p(x) = \dbinom{5}{x}\times(0.3)^{x}\times(0.7)^{5-x}$ =BINOMDIST(x,5,0.3,FALSE)

Table 3   Classifying Table for Example 2: Operations success number in a hospital

	Trials	Events	Probalilities
Bernoulli trial	One operation.	Success in operation.	$P(\text{success})= 0.9$

Repeating 100 times

	Trials	Events	Probalilities
A sequence of 100 independent repeating Bernoulli trials.	100 operations performed.	Success in opearations x times.	$p(x) = \dbinom{100}{x}\times(0.9)^{x}\times(0.1)^{100-x}$ =BINOMDIST(x,100,0.9,FALSE)

Conclusion

The introductory 'SS Board Game' and using the table above seem to help learners arrange of two levels of trials and understand binomial distribution from the records of observation of the classroom. But the records do not have enough reliability on effects of the materials so I would like to plan classroom experiment and records more carefully.

REFERENCES

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RÉSUMÉ (ABSTRACT)

Binomial distribution is one of most important discrete distributions. We can apply binomial distribution model to various phenomenon and make inference on frequency of specific outcome. But learners have difficulty understanding in applying binomial distribution to specific realistic situations even if they can understand in simple situations such as coin-tossing or die-rolling situations. One of the reasons is that binomial experiment includes two levels of trials : (1) each Bernoulli trial and (2) a sequence of n independent repeating Bernoulli trials. It is difficult to distinguish and classify the two levels for many learners. I developed materials that help learners understand binomial distribution model and classify Bernoulli trial and repeating Bernoulli trials in realistic situations. The characteristic of the materials are as follows: (1)In the introductory stage, learners play board game with which learners experience and become aware of the differences between among two levels of trial described above. (2)The materials provide table for classifying trials, events, and probabilities. In this paper, some examples of the materials are described and discussed the effects of the materials.

KEYWORDS

teaching materials, binomial distribution, board game, trial