YEAR SIX STUDENTS' REASONING ABOUT RANDOM 'BUNNY HOPS' THROUGH THE USE OF TINKERPLOTS AND PEER-TO-PEER DIALOGIC INTERACTIONS

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Students' intuitions play a major role when making inference about uncertain events and they are often inconsistent with the accepted theoretical understanding of statistics and probability. These intuitions need to be challenged to develop more powerful, formal understandings of stochastic ideas. This paper describes how the reasoning of a pair of 11-year-old students develops as they conduct simulations of random 'bunny hops' using TinkerPlots software for a large number of trials. We found that promoting dialogic talk, which involves questioning or challenging any claims and seeking reasons in response to challenges, facilitated students' reasoning while they used TinkerPlots to test their conjectures and explain the outcomes. The study shows how the interaction of a software tool and talk between students can develop their understanding of statistical ideas and highlights some of the processes leading to conceptual change.

INTRODUCTION

Previous research shows that in the absence of well-developed schemas for probabilistic thinking, students tend to use certain intuitive conceptions and strategies when judging the likelihood of uncertain events. Some of these might be plausible in some situations but often they are in conflict with the principles of probability theory. Among these intuitions is the heuristic of representativeness, which implies that people often evaluate the probability of an uncertain event based on the degree to which it represents some essential features of its parent population (Kahneman & Tversky, 1972). For instance, when flipping a coin six times, students often consider the sequence TTTHHH more likely to happen than HHHHHH because the sequence HHHHHH might appear less representative of the expected proportions of heads and tails (50-50 distribution) in the population. On the other hand, they might see the sequence TTTHHH significantly less likely than THHTHT since the sequence TTTHHH seem to be less random in terms of irregularity in the sequence. However, all three sequences are equally likely to occur based on the theoretical model of assigning probabilities.

In another study by Konold, Pollatsek, Well, Lohmeier, and Lipson (1993), when asked to choose the 'most likely' result among the following possible sequences from flipping a coin five times, that is, "(a) HHHTT (b) THHTH (c) THTTT (d) HTHTH (e) All four sequences are equally likely", the majority of the students correctly responded that the sequences are equally likely to occur. However, in a follow-up question where students were asked to select the "least likely" outcome, only 38% of these students again answered that all four sequences were equally likely. Konold and his colleagues conjectured that this inconsistency in students' responses stemmed from a change in perspectives, from an outcome approach to the representativeness heuristic. When asked about the 'most likely' result, students interpreted it as to predict what would happen and thus judged all the sequences were 'equally likely' because 'anything could happen' which was an indication of the outcome approach (Konold, 1991). In the 'least likely' case, however, students switched to the representativeness heuristic by basing their response on how well a sample represented the randomness of the process that generated it.

From an educational perspective, these intuitive strategies or shortcuts that students have need to be challenged in order to develop normative probabilistic reasoning. Thus, it is essential to investigate how students make a shift from using intuitive reasoning to probabilistic reasoning. There have been many research studies focusing on supporting students' learning of probability through various pedagogical approaches and the use of computer tools drawing on various theoretical frameworks (see Jones, Langrall, & Mooney, 2007). Our research aim is to expand our view of conceptual shifts in students by bringing dialogic theory into the learning of probability. The emerging new dialogic theory of conceptual change and conceptual development argues that children change their conceptions as a result of seeing as if from a different point of view, which is something that they learn to do in dialogues (Wegerif, 2011). Real dialogues in the classroom do

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not only provide the opportunity of seeing as if from the point of view of the specific other people that they are talking to, but also of seeing their own point of view as if from the outside.

CONTEXT AND METHOD

Our research is part of a larger design study (STATSTALK project- http://socialsciences.exeter.ac.uk/education/research/projects/projectlinks/statstalk) investigating how to develop young students' conceptual understanding of key ideas in statistics and probability in the context of informal statistical inference and the mediating roles of technological tools and students' talk. In one of the earlier iterations of the study, a total of six 11-year-old (Year 6) students, two boys and four girls (pseudonyms: Ozzy, Jake, Keyna, Flora, Gabby, Blair), from a local primary school in Exeter, UK, participated in the study. The students were recruited through their classroom teacher for the mathematics enrichment class. The participants attended three sessions, each of which was about three hours long, and investigated a variety of probability events through working in small groups and using *TinkerPlots* software during the summer term in 2013.

TinkerPlots 2.0 (Konold & Miller, 2011), a data analysis tool with simulation capabilities (see Figure 1), is used to explore various chance events during the study. The software builds on learners' intuitions about data representations and analysis, and enables students to construct their own graphs when progressively organizing their data by ordering, stacking, and separating. One of the new features in *version 2* is the probability simulation tool that expands its focus from data to incorporate probability. With the Sampler tool, students can build their own chance models using a range of devices, including mixer, spinner, bars, stacks, curve, counter, that can be filled with different elements to sample from. This tool then allows students to collect outcomes and conduct a large number of trials very quickly.

In addition to the use of *TinkerPlots* to model and reason about various random events, the participants were introduced to a dialogic way of talking in a group work (Dawes, Mercer, & Wegerif, 2000). More specifically, the expectations we discussed in class involved: 1) making sure that each person had an opportunity to contribute ideas, 2) asking each other 'why?' questions, listening to the explanation, and trying to understand, 3) asking others what they thought, 4) considering alternative ideas or methods, and 5) trying to reach an agreement before they did anything on the computer.

Each pair working around a computer in the sessions was videotaped. The qualitative analysis of data incorporates the video recordings and the written artifacts each group produced to document any conceptual shift in students' reasoning during the interactions between participants through the use of computer tools.

TASK DESCRIPTION

In this paper, we describe the results of one pair's work while they explored random binomial bunny hops using the probability simulation tools in *TinkerPlots*. The random bunny hops task was adapted from Wilensky (1997) and used to investigate younger students' reasoning about the way various distributions were shaped in different chance situations through physical experiments and simulations in the *NetLogo* environment (see Kazak, 2006; Kazak & Confrey, 2006). In the current study, the task was extended by focusing on the combined role of the talk between paired students and the use of *TinkerPlots* modeling/simulation tools.

Prior to this task, students were engaged in data modeling by building 'data factories' using the Sampler tool in *TinkerPlots* and modeling of other random events including coin flipping and some chance games in the previous two sessions.

On the third session, students were introduced to the following problem: "Suppose there are a number of bunnies on land and each bunny can choose randomly to hop only right or left. For each hop, bunnies are just as likely to hop right as left. We want to know where a bunny is likely to be after 5 hops." After a class discussion about how we can decide which way the bunnies might hop, students were given the following instruction to make their initial predictions: "Imagine that a bunny is standing on a number line at 0. You spin a coin to decide which way the bunny hops. If the coin lands heads up, it hops one step right (that is, one step along the positive direction). If the coin lands tails up, it hops one step left (that is, one step along the negative direction)." We also showed them a demo for five random hops in *TinkerPlots* (Figure 1). In the example below, using the Sampler tool, a single mixer device (on the left) is set to model equally likely random hops to right (R) or left (L). Next to that the table displays the results of each repeat in the Outcome column and the Position attribute created with the formula option indicates the location of the bunny after each hop on the number line. The graph on the right shows each individual hop by the trial number. In this example, the bunny hops 'right-right-left-right-right' and ends up at 3 on the number line.

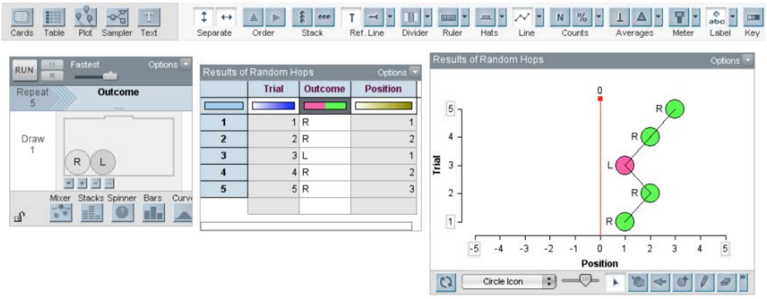

Figure 1. A model of five random bunny hops in *TinkerPlots*.

After the demo of random bunny hops, each pair was asked to make a prediction about where the ten bunnies are likely to end up after 5 random hops and show them by marking their final locations with X on the given number line. Next, students in pairs simulated five random hops of 10 bunnies by spinning a coin to see where they end up. They were also asked to record the path each bunny takes and where it ends up on the number line (e.g., the path "R,L,R,L,L" leads to -1) in the table on the worksheet, and to make a graph of their final locations by marking X for each bunny on the number line (Figure 3). Based on the simulation results, students were asked again to decide where they think a rabbit is most likely to be after 5 hops and to discuss together and give an explanation on the worksheet. Then they used *TinkerPlots* to simulate random bunny hops to explore where the bunnies are most likely to be after five hops. The *TinkerPlots* model was given to them since displaying the final positions of the bunnies involved formulas. In Figure 2, the Sampler at the top is set to watch 5 random hops of an individual bunny (used for the demo). The Sampler below is currently set to make 100 bunnies and the plot on the bottom right corner shows the results from one trial. Pairs used this Sampler to conduct a total of 5 trials and record the percentage of bunnies on each final position in a table on the worksheet. They were then expected to discuss where the most of the bunnies end up after five hops and why.

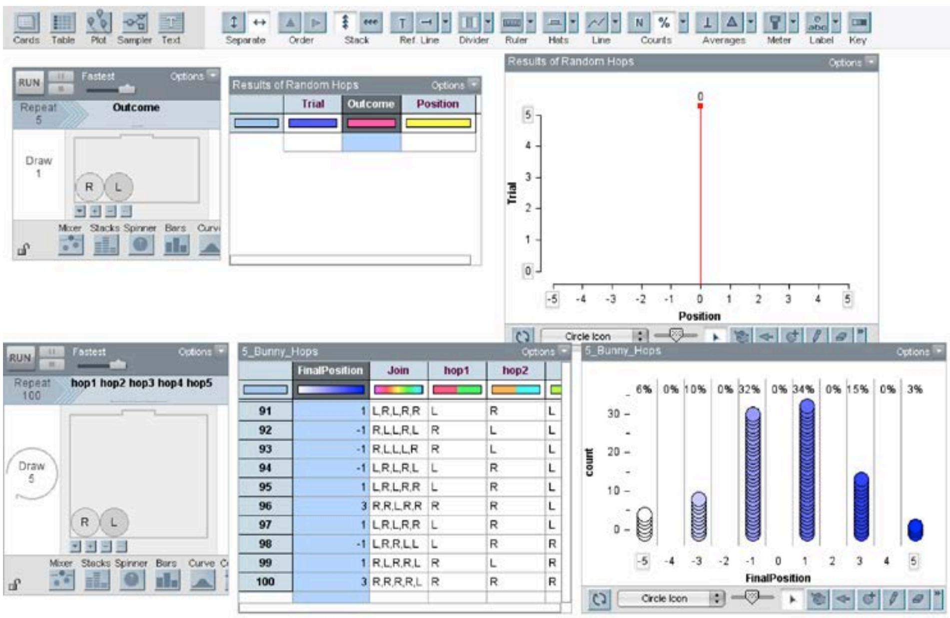

Figure 2. TinkerPlots document that students used to simulate 100 bunnies hopping randomly to right or left five times and an example of results from a trial from the bottom Sampler.

A PAIR'S REASONING ABOUT RANDOM BUNNY HOPS

In this paper we focus on one pair (Gabby and Blair) working together to explain why the bunnies are distributed in a symmetric mound-shaped way after they ran several simulations in *TinkerPlots* and looked at the resulting distributions. First, students' initial predictions for 10 bunnies showed that they expected five bunnies on 1 and five bunnies on -1 on the number line after five hops. Their expectation revealed symmetry around 0 and the data were only on 1 and -1. When asked to explain their prediction, they mentioned that it would be possible to get bunnies from -5 to 5 but they interpreted the 50-50 chance of hopping either right or left as the results being most likely to be like 'left-right-left-right-left' (LRLRL). This intuitive reasoning is consistent with the representativeness heuristic (Kahneman & Tversky, 1972) similar to the findings in previous research (Kazak, 2006) and suggests that students tend to think LRLRL is more representative of the expected 50-50 distribution of flipping a coin. When all the pairs completed their coin simulations and displayed their data on the plot, we looked at each plot and discussed how the results were distributed. While the other pairs got sort of a mound-shaped distribution, Gabby and Blair had even results on every possible outcome, except -5 (Figure 3). The results, especially the 'unusual' flat distribution of Gabby and Blair, generated a discussion about 'most likely' and 'less likely' outcomes in the class. Perhaps this led to Gabby and Blair's next prediction about 100 bunnies: 3 for -5, 10 for -3, 37 for -1, 37 for 1, 10 for 3, 3 for 5 on the number line. Their explanation indicated that they considered both the results being symmetric around 0 and the most likely to be around that and less likely to be on the sides.

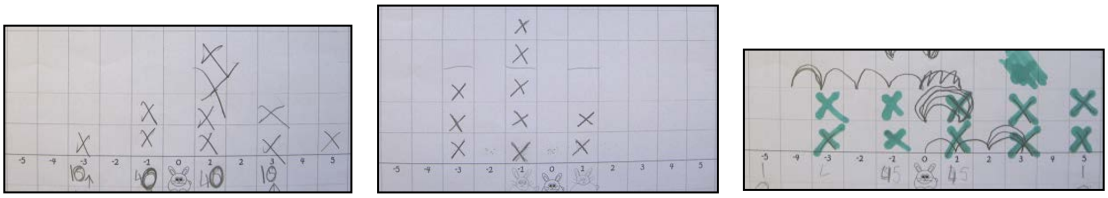

Figure 3. Each pair's coin simulation results (the plot on the right hand side is Gabby and Blair's).

When they began to run their *TinkerPlots* simulations and record the results in the table, they seemed to compare the results with their predictions and judge the fit between the two, for example, Gabby: 'it is quite close'. As they increased the number of repeats in *TinkerPlots* (n=1000 and 10000), both students strengthened their expectation of the most likely outcomes, that is, 1 and -1, since the frequencies of outcomes began settling.

When the teacher asked them to explain why they think they get more on 1 and -1 than the others and less on 5 and -5, Blair started to use the Sampler at the top as seen in Figure 2, which shows the path of five hops of an individual bunny on the plot. She ran it several times to get, in their terms, the 'perfect' and the 'worst' examples for the most likely and the least likely outcomes (Figure 4). This seems to be consistent with the representativeness heuristic as well.

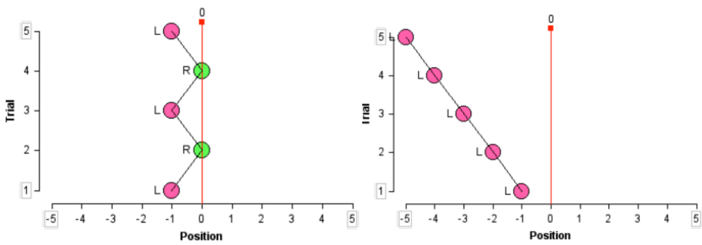

Figure 4. According to the students, the path on the left is the 'perfect' example whereas the path seen on the right is the 'worst' one.

After another prompt by the teacher researcher ("Did you see any other example of landing on 1 [referring to the path they got in the plot]?"), the generation of the path of five random bunny hops with the *TinkerPlots* simulation tool became a shared space for the students to explore their idea of 'more chance of getting bunnies on 1 and -1 than on 3 and -3' by counting the number of different paths leading to those positions on the number line. First, they ran the Sampler until they thought they had found all the possible ways to get to -1. By trial and error, they were able to list only 6 different paths on the worksheet. Later, when they were looking for the paths for landing on 3 in *TinkerPlots*, they realized a systematic way of counting all possible ways. For instance, Blair used the path on the left in Figure 5 to show that they can have the single left hop (the pink circle labeled L on the plot) in five different places on the path "because there are five dots [*showing on the plot*]" she says. In a brief exchange around that plot on the screen, Gabby began to see the problem from Blair's perspective and eventually became convinced that there were five different ways to land on 3.

> B: If you have, okay, you have got it [*pointing to the pink circle for L*] up there, you should have it there, there [*moving her finger on the dots*], you can have it in five different places for one left dot.
> G: So ten
> B: So there is
> G: No, there is five that way
> B: Yeah because you have
> G: Wait [pointing to the plot] yeah I get it, I get it [leaning back in her chair and smiling].
> B: There is five different ways because there is five different places. Weee!

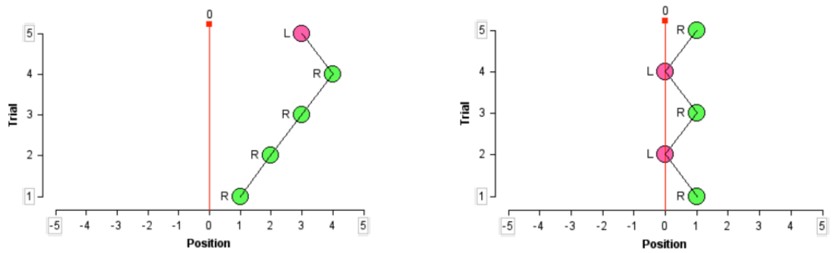

Figure 5. Representations that helped students count all possible ways to get to 3 (on the left) and 1 (on the right).

Next, using the same strategy of counting the dots, they together figured out the 10 different possible ways to get to 1 on the plot to the right in Figure 5. When asked about the distribution pattern, Blair quickly made the connection with their findings about the number of possible paths and the shape of the random bunny hops distribution they got from the sample of 10000: "Because there is only one way to get each five and then there is five ways to get three and then there is ten ways to get one."

CONCLUSION

In the above section, we described how Gabby and Blair's intuitive reasoning about the distribution of random bunny hops evolved into considering the number of different paths to explain the distribution shape with the use of *TinkerPlots* and the prompts by the teacher researcher. This shift towards more conceptual reasoning was facilitated largely by the affordances and tools provided by the software and the students' talk in that shared environment as well as some prompting questions by the teacher getting their attention to the different examples of paths in five random hops. The students' emotional investment in the task was shown by their use of terms like the 'best' (or 'perfect') and 'worst' examples of five hops. This emotional engagement led to a consideration of other examples of paths in the random event and became a starting point for seeing a pattern in the stability of the distribution shape in long run. Their conceptual thinking was mediated by seeing the different paths of five random bunny hops on the screen and recording those on paper. The screen representation became a shared space in which they could point to alternatives and discuss together as they were figuring out ten different ways to end up on 1 by showing the number of possibilities on the plot. In the extract of talk, we can see how Gabby moves from external dialogue, learning from the perspective of the other (Blair), to the personal appropriation in internal dialogue of 'I get it now'. Her conceptual advance is mediated here by the gaze of the other. Consequently, we need to pay more attention to the combined role of computer tools, students' talk, and teacher prompts in developing their understanding of probabilistic ideas.

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