DATA AND CHANCE WITH FATHOM: TEACHING MATERIAL FOR IMPLEMENTING COMPUTER-BASED STOCHASTIC COURSES

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Students often have inadequate primary intuitions when dealing with random phenomena at the beginning of stochastic courses. This experience gap between misleading intuitions and a sustainable perspective cannot be closed by physical random experiments alone, because the feasibility is limited in classrooms. In fact, the implementation of a statistical software tool like Fathom opens new opportunities for teaching and learning stochastics. According to our research, teachers need support by didactical materials for implementing computer-based stochastic courses. In our working group we have developed, evaluated and optimized our didactical concept through various empirical studies. Our textbook "Data and Chance with Fathom", written for high school students in grades 6-12, is the result of this work. This paper gives an introduction to the outline and the structure of the textbook. Examples and teacher experiences are discussed.

INTRODUCTION

The new content domain *Data and Chance* (KMK education standards 2004, 2012) and the availability of computer technology (CT) have had a great impact on content, pedagogy and course format in teaching and learning stochastics at school in Germany over the past ten years. A contemporary course in stochastics, which adequately takes data and chance into account, is hardly feasible without the use of technology. However, an appropriate use of CT in classrooms is loaded with many challenges like (Chance et al. 2007):

- Modification of learning goals,
- Lack of familiarity with new technologies,
- Lack of support for teachers,
- Necessity for providing instruction time for students,
- Error rate of CT,
- Modification of teacher and student roles.

As a dynamic educational software tool, FATHOM enables teachers and students to use, modify, and develop embedded microworlds for themselves. In such microworlds, the students have access to multiply-linked representations, as well as the possibility to design and run simulations, and use interactive explorative features, like sliders. Complex applications of stochastic phenomena can easily be prepared by the teacher. Dealing with data is intuitive and concrete.

For the implementation of using FATHOM at schools, the research group of Rolf Biehler has developed the GESIM concept as a design-based research concept over the past ten years. The term GESIM means "**g**eneral **e**ntry in learning stochastics with **sim**ulations". A lot of didactical material for classrooms was designed, tested, analyzed and redesigned in different types of schools and grade levels. Besides important aspects of simulation, CT can never replace thinking about stochastic problems. Therefore students' activities in modeling random phenomena and the interpretation of simulation results become more important in classroom experiments. At the same time teachers and students need didactical and instructional support for using computer-based simulations in order to sustain knowledge acquisition in a more differentiating, more interesting and more motivating way (Hofmann, Maxara, Meyfarth, & Prömmel 2014).

CT helps to bridge the experience gap, but that cannot substitute haptic experiences and thinking about stochastics. But it allows sustainable concrete approaches to understand stochastic phenomena and to generate procedural and conceptual knowledge in a more appropriate way (Biehler et al. 2013). One of the key concepts is the thinking in distributions, which is built up step-by-step as a spiral curriculum in teaching and learning stochastics (Wild 2006; Prömmel 2013).

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OUTLINE AND STRUCTURE OF THE TEXTBOOK

A lot of didactical materials and teaching concepts for schools and universities were developed and tested within the last 10 years by the working group of Rolf Biehler. Some of the well-designed materials are summarized in Biehler et al. (2011). They include worksheets and FATHOM microworlds especially for teaching at schools. The textbook provides ideas for teaching and learning stochastics in different grades to realize the central idea of d*ata and chance* with the software tool FATHOM. The textbook is especially designed for use by teaching staff which have no previous experience with computer use in the teaching of stochastics. An introduction to FATHOM is done by the multimedia learning environment eFATHOM, which is also explained in the textbook. The four modules of eFATHOM provide a gently start into using FATHOM. Short tutorial videos are one of the central ideas in this teaching concept. To complete each module takes about one hour. EFATHOM can be used both at home and in the classroom providing the opportunity for more individual learning time (Hofmann 2012).

Stochastics is interesting and exciting if it is supported by appropriate software. Learners can analyze, visualize and modify complex data sets according to their own questions. Students generally have little experience with random phenomena when they come into stochastic courses. Simulations can be used to visualize phenomena and to develop appropriate mathematical models. Experiments can be repeated by simulations as often as needed to make an estimate of unknown probabilities which is sufficiently accurate and reliable. Appropriate use of software can positively affect the learning process of the students. Students can experiment with methods by changing options, modifying parameters and see how the result changes. Complex mathematical relationships can be visualized and an experimental manipulation is accessible to students.

The textbook consists of three major parts. Part A is an introduction into data analysis, Part B presents stochastic simulations with FATHOM and Part C deals with probability distributions and inferential statistics.

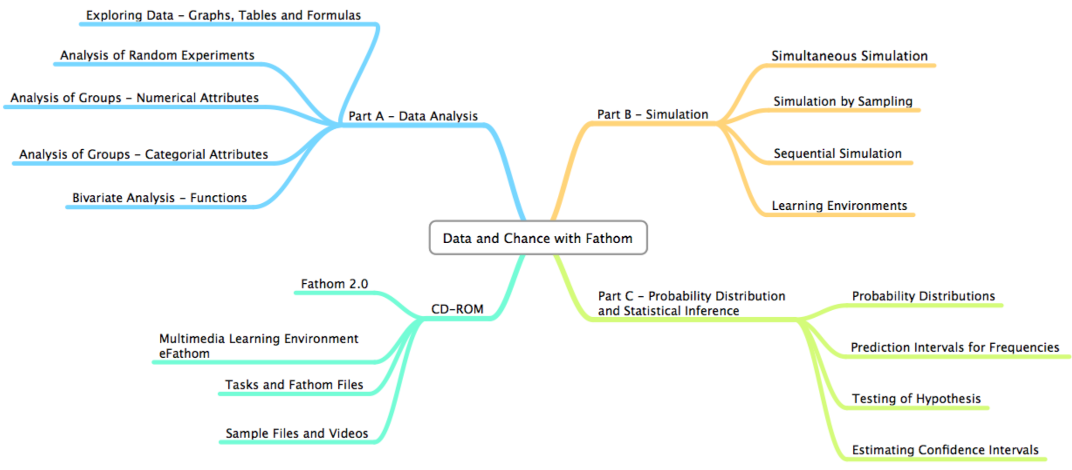

Figure 1. Structure of the textbook

Each chapter is structured into two parts: FATHOM basics are discussed in the first section, and further sections present teaching ideas, worksheets and FATHOM microworlds. At the beginning of each instruction the appropriate grade level is given. Beginners should first look at the section *FATHOM basics*. Then they are well prepared to deal with the following teaching examples. The required FATHOM skills are always mentioned at the beginning of the sections. The CD-ROM includes all FATHOM files and tasks which are used in the textbook.

In the next subchapters a deeper insight into the three parts of the textbook is given, especially in Part A. Here, the reader gets a better idea of the content of the textbook.

Part A - Data Analysis

The American statistician John Tukey speaks of "detective work with data". The examples in this part of the textbook are characterized by this type of data handling. The first two modules of eFATHOM are a very good start for learning data analysis with FATHOM.

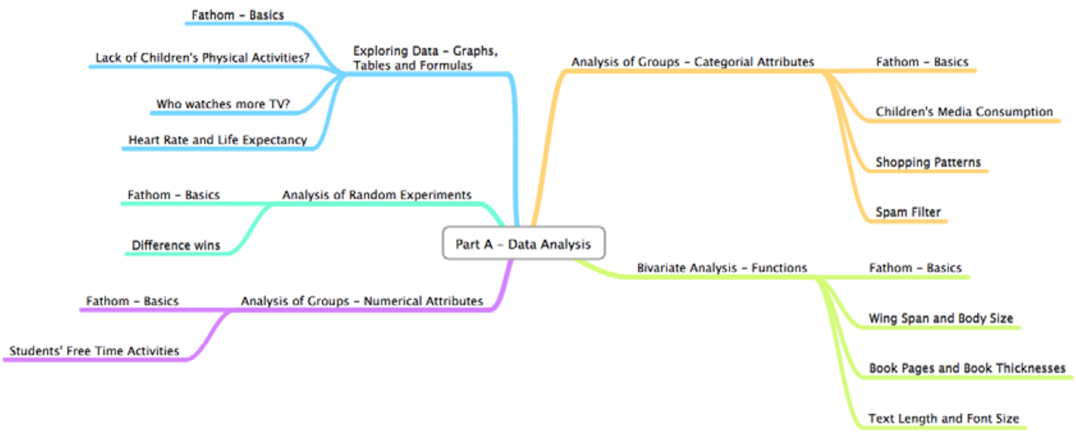

Figure 2. Structure of Part A - Data Analysis

In the five chapters of Part A we use a growing repertoire of methods for analyzing data. In each of the first subchapters the FATHOM basics are introduced. The subsequent subchapters contain concrete examples which we have tested in classroom experiments, teacher training courses and university students' education. We have successfully used FATHOM in various school types from grade 6 on.

Chapter 1 introduces standard representations of distributions (bar graphs, histograms, box plots) and parameters (location and spread parameters). FATHOM allows generating statistical reports with just a few clicks and can flexibly be adapted to each research question. The focus is on the data set *muffins* (Biehler, Kombrink & Schweynoch 2003), containing data about media use and spare time activities of students.

Chapter 2 deals with analyzing methods for random experiments. For the introduction of probability it is important to relate experimental and theoretical aspects to each other. The students model random experiments, explore relative frequencies and compare them with hypotheses of theoretical probabilities. In Chapter 2 some options are presented to implement such random experiments with FATHOM, and to calculate theoretical probabilities and to perform elementary comparisons.

Chapter 3 takes up the important issue on if and how girls and boys differ in spare time activities and if the availability of electronic devices in adolescence has an impact on the media consumption. While in public media a distribution is often reduced to a mean value, we show students how they can use the whole distribution for a comparison.

Chapter 4 takes up contingency tables. In the simplest case, values of two categorical attributes are measured, such as computer ownership and gender. It divides the data into four groups: female computer owners, male computer owners, female non-owners and male non-owners. The proportion of girls among computer owners is often confounded with the proportion of computer owners among girls. Fallacies are the result. Dealing with such 2x2 tables in statistics is important to understand conditional probabilities and the Bayes' Theorem.

Chapter 5 deals with relations between two numerical attributes in different contexts. In the modeling context the real data are important for the set up and the verification of functional models. The examples show how students can use FATHOM-functions like moveable line models, and how residuals further help to improve the fit of the function to the data. FATHOM also provides the ability to curve fitting with the method of least squares or the use of the median-median line. But a first look at the data in the scatterplot is always more important. The examples can be used in the treatment of functions or as part of a lesson on the topic of regression.

Part B - Simulations

FATHOM provides several random functions to simulate random devices (such as a coin). In this textbook, three different types of simulation are distinguished. For supporting students' simulation competencies some guiding material was developed in the working group of Rolf Biehler, e.g. a general 6-step simulation plan (Biehler & Maxara 2007) and a simulation plan scheme (Biehler & Prömmel 2010).

These didactical supports assist students when they implement simulations by themselves. There is an adaptation of the simulation plan scheme available for all three types of simulation. Working with such a simulation plan scheme has some facilitation for students and teachers. Students can use the scheme for planning, guiding and documentation. The simulation plan scheme gives teachers the opportunity to help students and to identify students' difficulties in the implementation.

Part C - Probability Distributions and Inferential Statistics

The chapters of Part C also deal with data analysis and simulation. We want to use the students' experiences with data analysis and integrate them into the methods of inferential statistics, as currently required in the educational standards for secondary education (KMK 2012). New aspects get into focus, because FATHOM is used even more to visualize and experiment with mathematical models, particularly with probability distributions and functional relationships. The simulation is more often used for the illustration of stochastic phenomena and for the experimental inquiry of theoretical contexts, e.g. a more direct approach than with formula.

We emphasize stochastic problems based on real examples to compare binomial distribution and normal distribution models in settings with real data. This is rarely done in school, even if it is a central point of application-oriented courses. The visualization and simulation have a great didactic potential to provide students with conceptual knowledge. It would optimize stochastic courses if hypothesis testing and confidence interval are discussed in the classroom. The textbook shows some important relationships between these two topics.

The understanding of the ideas of prediction intervals is important for the concepts of hypothesis testing and confidence intervals. Only when one has realized what might happen under different assumptions, one can make statements starting from an observation. The insight into the 1-by-root-n-law is essential for the understanding of inferential statistics. We discuss this topic in embedded microworlds from different viewpoints. It completes the knowledge of the law of large numbers and explains the rules of thumb for the precision of the simulation.

DISCUSSION OF SELECTED EXAMPLES

In subchapter 2.2 we start with the game *difference wins*. The basic idea of the game is that each of two players has 18 coins which they distribute on a game sheet with six fields (0 to 5 for the possible differences) before the game starts. It is played with two dice. The difference in the outcomes is documented. Each player takes one of his or her coins from that field of the game sheet which is labeled with the same difference, if one of his or her coins is still available there. The player who has taken all coins from the game sheet first is the winner. The students have to find an optimal winning strategy in order to win.

Characteristics and Classroom Implementation

In the textbook we first give some advices for implementing the game in the classroom. All materials - game sheets, protocol sheets and FATHOM files - are available on the CD-ROM.

The aim of the task is that students develop an optimal winning strategy on the basis of their own experimental results and theoretical considerations. In a first phase, the students are asked to play the game in groups and record their starting positions of the 18 coins and their outcomes on a protocol sheet. According to our experiences the students change their strategies within the game by evaluating the frequencies or by considering combinatorial ideas. If students note down the outcome differences in a protocol sheet, they have the possibility to deal with the concept of probability in the classroom at different levels (real model and mathematical model) and they can better understand the relationship between relative frequencies and probabilities. Therefore simulations are extremely useful to generate large datasets under different assumptions and they support the visualization of stochastic phenomena, in particular those for the empirical law of large numbers.

After finishing the gaming phase (with data generation and data recording) the winning strategies of the groups are always collected on the black board. We have observed three strategies for coin distribution: equal strategy 3-3-3-3-3-3, proportional strategy 3-5-4-3-2-1 and disproportional strategy, e.g. 2-9-5-2-0-0. We also try to answer the question of an optimal game strategy on the basis of the data. Therefore we collect the results of the groups. We use the fact that a larger sample generally estimates an unknown probability better than a smaller sample. We combine the numerical data analysis already when entering the data in FATHOM with a bar graph of the frequency distribution of the differences. So the students can already see the changes in the frequency distribution with increasing sample size. The law of stabilization of the frequency distribution for growing sample size is an important aspect in the phenomenon complex of the empirical law of large numbers.

During the gaming phase students should get a sense of the randomness in a single outcome and of the structures in a series of outcomes. Applicable stochastic terms can be introduced or resumed in this phase: random experiment, relative and absolute frequencies, outcomes and probability. The frequency distributions obtained by the experiments can be analyzed and visualized by using the software FATHOM and represent an essential aspect on the law of large numbers. The experimental phase should be followed by a theoretical phase of counting possible outcomes of the game in a table sheet. With the theoretical ideas, the coin distribution at 3-5-4-3-2-1 in the fields at 0-1-2-3-4-5 can be regarded as an optimal strategy. It can be discussed with the students that although the proportional strategy has the best winning chances, there is no winning guarantee with the optimal strategy. So the data may be compared with each other on two levels: first on the level of the relative frequencies and theoretical probabilities, and second on the level of the optimal coin distribution.

Further Considerations

The experience of this game motivates students to study the question how well one can estimate probabilities by relative frequencies in a more detailed way. Therefore we have developed different embedded microworlds (Biehler & Prömmel 2011). These microworlds have been used in our classroom experiments for demonstration. For most students, the large variation at n = 50 is surprising because the number of different outcomes in a real game is between 30 and 70, which is less reliable than expected. With these activities, a first step is made on the way to a deeper understanding of the precision of relative frequencies as estimates of probabilities and the role of sample size.

The law of large numbers is also discussed on different levels in module 3 of eFATHOM and in other subchapters. Hence the textbook gives a well-designed concept for a step-by-step construction of an appropriate conceptual knowledge about different aspects of distributions and the law of large numbers (Biehler & Prömmel 2013).

DISCUSSION AND CONCLUSIONS

One of the most important aims of our research group was the design-based implementation of the educational tool software FATHOM for stochastic courses at German schools and universities. Therefore we have developed well-designed didactical material for instructional support, evaluated this material in classroom experiments and improved it on the basis of research outcomes. The results of our studies have shown that it is feasible to implement computer-based stochastic courses with FATHOM at schools and universities. The didactical materials presented in the textbook, eFATHOM and the simulation plan scheme may support teachers and students in using computer technology (CT) for enhanced teaching and learning of probability and statistics in a more effective way (Hofmann, Maxara, Meyfarth & Prömmel 2014). We emphasize some best practice conditions, which are important for this success:

- TSPD-concept methodical framework (task familiarization phase, students' working phase, presentation phase, and discussion phase),
- Well-designed process-oriented worksheets,
- Cooperative learning on the PC (habit of FATHOM),

- Discussion of misconceptions,
- Availability of appropriate tasks (task design),
- Support of teachers through available learning and training materials,
- Easy use and learning opportunities of tool software.

But we have also observed a gap in the sustainable use of FATHOM in classrooms in Germany. Teachers are more familiar with software tools like EXCEL, GeoGebra and TI-Nspire. Teachers who use CT in the classroom usually look for a software tool which they can easily use at different grades, levels and for various mathematical topics. FATHOM has not the leadership as in the U.S. but it is in direct competition with other software like GeoGebra, which is for free. In addition, in many federal states the use of a GTR or a CAS is compulsory. Handhelds and apps for tablet PCs provide more mobility. Students need not use of a special PC room for dealing with CT. Therefore, there is a great interest in a mobile FATHOM version. Especially the data input and the rapid access to data are crucial advantages compared to other software. In addition, it seems reasonable to establish a FATHOM community that develops and implements didactical materials. This can be accomplished in professional learning groups with teachers who utilize FATHOM in a useful way in their stochastic courses.

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