ABOUT CENTRAL ISSUES OF MENTAL MODEL THEORY IN CONTEXT OF LEARNING STATISTICS

Markus Vogel[1] and Andreas Eichler[2]
[1]University of Education, Heidelberg, Germany
[2]University of Education, Freiburg, Germany
vogel@ph-heidelberg.de, andreas.eichler@ph-freiburg.de

Learning to draw right conclusions and to make well-founded decisions on base of data is a goal being at the core of statistics education. Because of its central characteristics and underlying concepts the theory of mental model is well suited for describing and reflecting on these processes of statistics learning in two respects: On the one hand the concepts of structure and function of mental model theory allow for adequately capturing data-based situations which require drawing conclusions. On the other hand the concept of mental model is an essential part of widely accepted information processing theories which are currently at the forefront of research. Thus, students' processing of data-based problem situations and making decisions can be highlighted in terms of this theoretical approach. Beside a survey of fundamentals of mental model theory the presentation focuses on these aspects. Theoretical insights will be underlined by empirical research results.

INTRODUCTION

In this paper, we want to highlight central concepts of a theory of mental models with regard to their power when describing and analyzing processes of students' learning statistics in an interdisciplinary approach. Firstly, we refer to central aspects of statistical thinking following the seminal work of Wild and Pfannkuch (1999) and embed exemplarily one of the data-based modeling activities of statistics learning which we focus on. Afterwards, we expose basic characteristics of mental model theories from a psychological point of view and connect them to the structure of the statistical thinking processes mentioned before. Then, we consider mental models in terms of the information processing model of Schnotz and Bannert (1999) and reflect on consequences for designing adequate representations of problem situations concerning basic modeling of uncertainty and well-founded data-based decision making. Finally, we connect these insights to current research approaches on informal inferential reasoning (e.g. Ben-Zvi et al., 2012).

STATISTICAL THINKING WITH REGARD TO ACTIVITIES OF MENTAL MODELLING

Modelling uncertainty that goes beyond a purely subjective (maybe spontaneous intuitive) estimation should be based on an analysis of available data as well as on available information about the situation of uncertainty. Thus, predictions of possible future outcomes – resulting from working and reasoning with available information, data, and probability in the situational context – will become more serious and plausible. The five types of statistical thinking according to Wild and Pfannkuch (1999) are to be seen as being fundamental for such modelling processes: recognition of the need for data, transnumeration, consideration of variation, reasoning with statistical models, and integrating the statistical and contextual information, knowledge, conceptions. "Constructing models and using them to understand and predict the behavior of aspects of the world that concern us seems to be a completely general way of thinking." (Wild & Pfannkuch, 1999, p. 230). This means that modeling of future outcomes in a situation by drawing inferences is not only an experts' activity, but an activity of everyone. The underlying (mental) models only differ with regard to different levels of expertise in regarding and processing external sources of context reality, in building statistical models to gain insights from this information and in applying personal available statistical knowledge and experience (cf. ibid., p. 230). While the five aspects of the statistical thinking represent an individual's statistical acting beyond specific statistical topics, several topics were declared to be central referring to statistical knowledge, i.e. sampling, central measurement and variation, distribution, graphical representation of data, or regression and correlation (e.g. Curcio, 1989; Mokros & Russel, 1995). The tasks we have used in our studies (Eichler & Vogel, 2011; 2012) were designed with regard to these central topics and processes of statistical thinking: central measurement, variability of statistical data, proportional reasoning involving students' consideration of variability, interrelation of bivariate data, and simple random experiments.

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Mental modelling of a statistical situation can be roughly described as three-part structured: recognizing the initial problem situation –working on the problem by mentally simulating and by making inferences – deriving answers from the resulting situation. Of course, this structure should not be interpreted as something like a “record of mental modeling”: the underlying thinking processes will surely being more complex not being one-dimensional. It is only to be seen with regard to the principle structure of every process which starts, runs and finally ends. We will illustrate this principle structure by means of the so called M&M task (Eichler & Vogel, 2012):

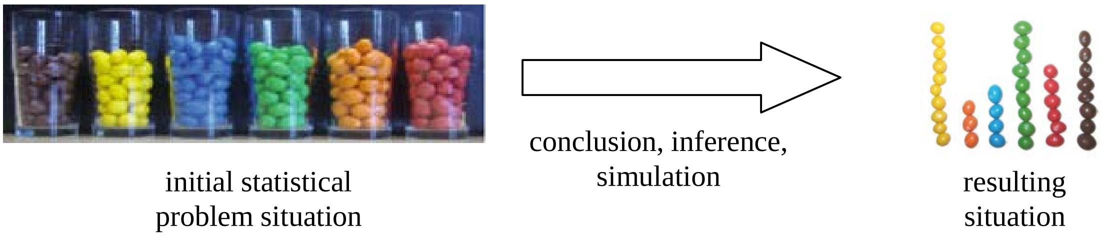

Figure 1. Three components of mental modeling of a statistical situation

- Initial situation involving the statistical problem:
  The students see a picture which shows the candies (M&Ms) of 20 bags being sorted by color in different jars. Additional, they were told that in one bag contains about 19 candies on average and that the candy bags are filled up independently from each other.
- Process of modeling involving drawing conclusions and mentally simulating:
  The students are asked to estimate the numbers of colors referring of two further candy bags (and to give a rational).
- Resulting situation involving a solution:
  Here, one example of a student’s expected color distribution of two candy bags is shown above.

It is obvious that between the initial situation and the resulting situation something significant has occurred in the student’s mind, because the shapes of the color distributions in the initial situation on the one hand and the resulting situation on the other hand differ totally. We will try to analyze and describe this interpretation in more detail. For this, we firstly outline some issues of mental model theory being important for our research approach before recurring to this finding and highlighting it in terms of mental model theory.

ESSENTIALS OF MENTAL MODEL THEORY

The concept of mental models is closely connected with Johnson-Laird (e.g. 1983), or with Seel (e.g. 1991) or Dutke (1994). Mental models are defined as representations of an entire situation in contrast to both the semantic representation of isolated propositions of a situation and the representation of superficial features of a situation (Kintsch, 1998).The theory of mental models suggests that, when interacting with demands of a specific situation, the learner builds a mental model in order to deal with relevant aspects of the situation to be cognitively mastered (cf. Seel, 1991). Such mental models can differ with regard to their kind of abstraction: The spectrum ranges from mental models that are closely connected to the perception of a situation from a specific point of view to mental models that are to be seen as constructs consisting of abstract entities of purely thinking and their relations. Johnson-Laird (1983, p. 156) states: “A mental model […] plays a direct representational role since it is analogous to the structure of the corresponding state of affairs in the world – as we perceive or conceive it.” Mental models are hypothetical internal quasi-objects that represent subject matter based on a *structural* or *functional analogy* between the subject matter and the model (cf. Schnotz & Bannert, 1999). The analogy of mental models refers always to a specific point of view of the represented situation. Analogical processing and reasoning by using analogies can become especially meaningful when making predictions (Forbus & Gentner, 1997).

Mental models are differing with regard to their granularity (cf. Schnotz, 1994): Whereas mental models of experts are standing out with regard to a structural and functional analogy to the modeled stochastic situation, the mental models of novices are often characterized by surface properties of the situation's representation given in the task. Thus, the question which elements are constituting the mental model is depending on previous knowledge and level of expertise in drawing inferences on the one hand (cf. Seel, 1991), and on the specific problem and its contextualization on the other hand (cf. Dutke, 1994).

Mental models are of a dynamic nature. They are not fixed knowledge entities being only recalled but situational constructions being processable with regard to the specific demands of the situation which have to be cognitively mastered (cf. Baguley & Paine, 1999; Forbus & Gentner, 1997). By "running" (de Kleer & Brown, 1983, p. 156) a mental model on base of previous knowledge inferences can be drawn. However, such inferences do not necessarily need to make use of formal rules of inference (cf. Johnson-Laird, 1083, p. 131); they are part of an individual sense making process within a situation. Thus, these inferences are not necessarily correct in an objective sense but they can be useful. The runability of mental models allows for deriving answers via mental simulation of systems by anticipating possible results of stochastic processes given, for example, by throwing dice or deterministic processes given, for example, by going through the components of an electric circuit. De Kleer and Brown (1983, p. 155) are talking of "qualitative simulations". These qualitative simulations are not only internal depictions or mental movies of an external process. They require sense-making about the system or process that should be simulated, its constituting components and their (causal) relationships. Mental simulations do not result in quantitatively exact conclusions. By contrast, mental simulation could result in qualitative ideas about the expected outcomes of such system simulations (De Kleer & Brown, 1983).

STATISTICAL THINKING IN TERMS OF MENTAL MODEL THEORY

The mental models' characteristics of analogy concerning structure and function are useful to describe in more detail what can be expected when students have to cope with situations of uncertainty like in the M&M task mentioned above. We shortly exemplify this by meanings of this task and illustrate it by short statements of students we have interviewed (see Eichler & Vogel, 2012):

- *Structure:* The initial problem situation makes a request not only for a superficial viewing at the shape of the color distribution in the jars but also for considering available information about the production of the M&M-jars (e.g. average number of candies in one bag, information about randomness impacting the filling process, assumption that the candy bags are filled up independently from each other). Furthermore, concrete memories about M&M-candy bags as well as general beliefs concerning deterministic or stochastic ways of thinking will influence the cognition of the problem situation.
- *Function*: These beliefs also have an impact on the way mental model is being applied to the M&M problem situation by qualitative simulations or deriving answers by making decisions or predictions. Especially when asking for possible future outcomes like the color distribution of the next two candy bags this becomes important for purposes of diagnoses of different kinds of statistically thinking: Because there is no exact answer derivable by calculation but only estimations of more or less probable outcomes we can get information about the students' preferences concerning arguing mainly deterministically, stochastically or some hybrid kind of both.

In the M&M task mentioned above the difference between the color distribution of the problem situation and the resulting situation leads to the assumption that this student is aware of the randomness affecting the bag filling process and that she knows something about the phenomenon of stabilization of relative frequencies, the so called "empirical law of large number". Apparently, she accepts influence of randomness and applies this knowledge by thinking the stabilization phenomena backwards and deducing qualitatively a higher variability in the smaller sample. The following statement of an interviewed student seems to be affected similarly by a more stochastic influenced mental model of the situation:

> "In each candy bag, there will be variable numbers of each color.''

On contrary, the following student we have asked seems to prefer a more deterministic model:

> "The M&Ms will be mixed in the candy jars at random. Thus, anywhere, there must be the same number of M&Ms.''

It is even possible that one person constructs multiple mental models by sense of affected by a mixture of deterministic and stochastic ways of thinking (which have not necessarily to be consistent) and apply them in different ways:

> "Concerning the 20 candy bags, they [the colors] are approximately equally distributed. I have estimated [for the next candy bag] a little bit more blue, but a little bit less yellow. In the second candy bag, I have put less orange, because I think the candy bags will rarely be filled equally, in general."

In summary, the kind of the situations' perception and processing and, in consequence, the process of building a mental model and operating on it is highly individual. The analogy concerning structure and function allows for qualitative simulations as well as for qualitative reasoning. Following Forbus and Gentner (1997) qualitative reasoning is an essential part of mental model reasoning. Properties of such reasoning are (ibid., p. 2):

- "*Handling incomplete and inexact data*: Qualitative information is easily extracted via perception, and […] more likely to be easily remembered than precise details.
- *Support for simple inferences.* Simple, everyday "obvious" inferences can be carried out easily. […]
- *Representation of inexact knowledge.* Qualitative representations […] provide a vocabulary for expressing partial knowledge about causal theories and mathematical relationships, and methods to assemble this partial knowledge on demand for reasoning.
- *Representation of ambiguity.* In […] prediction tasks we can imagine several distinct outcomes. Qualitative simulations capture this ambiguity."

It seems to be evident, that especially in situations of decision making under uncertainty deriving from missing information needed for exact calculations and solutions, these properties mentioned by Forbus and Gentner (1997) become meaningful.

ASPECTS OF REPRESENTATIONS OF STATISTICAL PROBLEM SITUATIONS

According to the information processing model of Schnotz and Bannert (1999) mental models are constructed according to a task and its requirements within a situation representing the structure or the function of the modeled object in an analogous way. There is much empirical evidence that the kind of presenting statistical problem situations is critical for the kind of adequate mental model building and problem solving, e.g. Hoffrage and Gigerenzer (1998) showed that people arguing with natural frequencies instead of using probabilities succeeded in solving tasks of conditional probabilities in a significant better way. A crucial result of our previous studies was that the more *abstract* the situation's representation was, the more the students struggled with understanding it (Eichler & Vogel, 2011). We use the term *abstract* in two ways:

- Either if a situation's representation is described by text only: In this case according to the information processing model of Schnotz and Bannert (1999) the learners have firstly to build an internal text surface representation by sub-semantic processing the text and subsequently an internal propositional representation by semantic processing before they can construct a mental model being analogues to the statistical problem situation.
- Or if a situation's representation is described by drawings of the situation, e.g. using pictograms, symbols or a dot plots representing a data distribution: According to the distinction of different kinds of illustrations referred by Vogel (2006) such pictorial representations are at the highest level of abstraction. Thus, according to Schnotz and Bannert (1999) the learner's thematic selection of the internal visual perception of situation's drawing has to be complemented by decoding graphical elements and semantic processing of symbol-combinations being involved.

In both cases, the information process includes more steps and requires more cognitive capabilities. Because focusing on young students without schooling in realistic statistical situations we try to represent the problem situation as easily as possible to understand (i.e. in sense of mental modeling). According to principles of multimedia learning in terms of Mayer (2001) and due to impacts of cognitive load theory of Chandler and Sweller (1991) we represent statistical problem situations in form of text-picture-combinations of the lowest abstraction level (mostly real pictures, s. above) and as short as possible text-passages formulated in an elementary style of speech which explain the problem situations illustrated by the pictures (for details cf. Eichler & Vogel, 2013). Thus, we want to support coherence in learning from multiple representations (Seufert, 2003) which is especially important with regard to mental model construction of novices such as those we were interested in our research into statistics education.

For purposes of reconstructing underlying mental models on which students operate when dealing with statistical problem situations, the analogous characteristic becomes important: According to the information processing model of Schnotz and Bannert (1999), the elements of a mental model may be changed, enriched or modified during the persistent mutual processes of internalizing and externalizing, when a student is working on representations of a statistical task but they do not disappear at all. Thus, information about statistical mental models, and hence statistical thinking and statistical knowledge, could be made available by analyzing the tasks (content, representation), the learner's specific situation (experience and pre-knowledge with regard to statistical content, statistical methods and statistical context) and the learners' outcomes (e. g. videotaped explanations incl. gestures, drawings, written responses) after having worked on the tasks. Not the statistical mental models themselves are claimed to be captured, but the researchers' reconstruction of these models based on learning outcomes. In our studies we were able to empirically back up our hypotheses about factors the influence mental models by deriving a model of complexity levels which allows for characterizing different kinds of statistical problem situations (Eichler & Vogel, 2012).

CONCLUSION

Based on previous works of our research into young students' mental modeling of uncertainty (cf. Eichler & Vogel, 2012) in this paper, we discussed central concepts of a theory of mental models with regard to its applicability to describing and analyzing processes of students' learning statistics. We embedded theoretical considerations within the field of learning statistics by making references to the seminal work of Wild and Pfannkuch (1999) concerning statistical thinking. Analogues characteristics of mental models play an important role with regard to information processing and thus, with regard to adaptive representations of statistical problem situations. Also, the analogy concerning structure and function and, based on this, qualitative simulating and reasoning are important aspects of mental model theory which especially allow for reflecting on situations of learning statistics in an appropriate way: handling incomplete and inexact data, supporting for simple inferences and capturing representations of inexact knowledge, qualitative representations which make causal knowledge explicit as well as representations of ambiguity – these all are aspects which become relevant in learning to draw inferences and to make well-founded decisions under uncertainty. Thus, in our opinion, the mental model approach could complement those important and high-yield works of e.g. Ben-Zvi et al. (2012), Makar and Rubin (2009) about informal statistical inference from a theoretical point of view.

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