INTERPRETING VARIATION OF DATA IN RISK-CONTEXT BY MIDDLE SCHOOL STUDENTS

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The aim of this research is to explore students' reasoning concerning variation when they compare groups and have to interpret dispersion like/as a risk. In particular, we analyze in this paper the responses to one problem of a questionnaire administered to 80 ninth-grade students. The problem consists of choosing between two groups of data by comparing them; each one consisting of losses and winnings coming from a hypothetical game. The results show the difficulty students had in interpreting variation in a risk context. Although they identify the data group with more variation, it is not enough for interpreting the variation in terms of risk and making a rational decision. The psychological categories of risk-seeking and risk-aversion are used to explain the behavior of students who choose one game or another when they identify correctly the risk in each game. As a conclusion, it is suggested that more risk context situations should be studied.

INTRODUCTION

Many authors have emphasized the importance of statistical variability (Moore, 1990; Watson, Kelly, Callingham & Shaughnessy, 2003; Franklin, 1999; Wild & Pfannkuch, 1999). The concept of variability or variation is related to a number of fundamental statistical ideas: representation, mean, distribution, and inference. Garfield and Ben-Zvi (2008) noted that "understanding the ideas of spread or variability of data is a key component in understanding the concept of distribution, and is essential for making statistical inferences" (p. 203). In the Mexican education program, Statistics in secondary school is mainly reduced to graph contents and measures of central tendency. Only at the end of third grade (ages 14-15) is the study of *dispersion measures* mentioned; it is suggested to address both range and mean deviation (SEP, 2011).

On the other hand, it is very difficult to analyze the understanding of dispersion isolated from the notion of center (i.e., mean); for example, Konold and Pollatsek (2004) use the signal-noise metaphor to explain the relationship between center and dispersion; Garfield and Ben-Zvi (2008, p. 203) state that "it is impossible to consider variability without also considering center, as both ideas are needed to find meaning in analyzing data."

Although dispersion is a fundamental characteristic of a data set, its use in making inferences or decisions, concerning its interpretation, is not entirely clear in elemental school tasks. Take for instance one of the tasks studied by Watson (1999). In a protocol on comparing data sets, Watson (1999) suggested the following question: Look at the scores of all students in each class, and then decide, did the two classes score equally well, or did one of the classes score better? The students compared two graphs of scores from a mathematics exam. The scores had the same mean (5), but a different range (A = 2 and B = 4).

Does considering a measure of variation, range for example, help in making a decision in favor of one class or the other? Some answers may tend towards group B because there is one with better score; however, we may object that there is also one with the worst. Another criterion would be choosing group A since it is more homogeneous. More or less variation is not a clear criterion to make a choice; an additional criterion is necessary. Other option could be considering only the arithmetic mean without including variability; so which is the best solution? In this report, we propose a task in which a game must be chosen. In this game, variability may play an important role, and we see how sensitive students are to it.

FRAMEWORK

The interpretation of dispersion depends on the situation which data come from. One situation is in terms of risk: When the uncertainty present in a process implies any threat to the goodness of a result, it is called a risk. These situations appear when there are potential and unwanted results that, as a consequence, lead to losses or damages.

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Defining risk means to specify both the valuable and unwanted results in a way that reflects the value attributed to them. Analyses of risk situations offer information for decision-making. The theory of decision making under situations of risk has two aspects. On one hand, it defines abstract rules for what people should do; and, on the other, it studies what people really do when facing the risk. "If people do not follow the rules, then either they need help or the rules need revision" (Fischhoff & Kadvany, 2011, p. 65). Descriptive studies about decision making have identified two attitudes in the subjects facing risk situations: risk seeking and risk aversion (Tversky & Kahneman, 1991). A situation may be approached from a conservative or an adventurous point of view. So, when facing two different game situations in which the average winnings are the same, a conservative person will choose the game with the smallest stakes while an adventurous person would choose the game with the highest ones.

METHODOLOGY

This is an exploratory study on students´ spontaneous perception and interpretation of variation and mean in risk situations. The context corresponds to bets in gambling.

Eighty students divided in four groups of third grade of secondary school (9th grade, ages 14-15) in a private institution in Mexico City participated in the experiment. Two teaching activities were designed and implemented. The students also solved two problems administered before and after the teaching activities. In this paper we report the results of one of the problems (Figure 1) dealing with comparing two data sets. The problem has two sections: one is concerned with decision making when the student elects the game that would be most advisable to play; the other, with the perception of variation. We should note that the mean is the same in both sets while the dispersion is different.

Teaching activities between the pre and post problems revolved around two problems in the context of measurement and games in the context of risk. The problems dealt with comparing data sets, and the students considered mean, range and mean deviation. Doing the activities helped the students learn terms like dispersion, range and average. However, this understanding did not affect the interpretation of the variability. The influence of the activities in the students´ performance was only manifested in the increase of responses based on the sum of the data; that may be a result of having worked with the average.

In a fair, the attendees are invited to participate in one of two games, but not in both. In order to know which game to play, John observes, takes note and sorts the results of two 10 people samples. The losses (-) or cash prizes (+) obtained by the 20 people are shown in the following lists:

Game 1:

15	-21	-4	50	-2	11	13	-25	16	-4

Game 2:

120	-120	60	-24	-21	133	-81	96	-132	18

a) If you had the possibility of playing only one of the two games, which one would you choose? Why?

b) b) In which of the two games is there more dispersion of data? Why?

Figure 1. Decision making and the perception of variation in a risk situation

Teaching activities were carried out in two 50-minute sessions. The authors expected improvement in the students' performance in the posttest.

RESULTS AND DISCUSSION

Most of the answers to question *a)* can be classified in broad terms in four levels. In the most elementary level the answers came from focusing attention on the maxima or the minima. When they are focused on the maxima, the subjects choose Game 2 "because you win more". When their attention is focused on the minima, they prefer Game 1 "because you lose less". Thirty two answers were classified in the first case and 6 in the second (Table 1). However, there are 15 subjects who preferred Game 1 because it has more possibilities of winning, they do not give any more information. In this first level, the students do not choose a representative value such as the mean or the median, to reduce the variability of each set. Nevertheless, they picked a “bad” representative, such as the maximum or minimum.

Table 1. *Reasons (pretest and posttest questionnaires)*•

	Level 1				Level 2		Level 3				Level 4			
	Win More		Lose less		Lose-Win the same		Wrong sum		Right sum		Aversion		Seeking	
	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
Game 1	15	13	6	12	1	1	2	3			3	2		
Game 2	32	27			1	2	2	2					6	3
Any					5				1	9				

* Some answers are not included in the Table 1 because they were unclear or involved students´ experiences in other games.

In the second level the responses classified in “win-lose the same” make reference only at the signs relations without consider the magnitude; they observe that in both games five people lost and five people won. For example, “because 5 persons won and 5 lost, and in game 2 is the same”. In a third level are the answers that are based on the sum of the data of each set. In one answer (the correct one) the global winnings were calculated to be equal to 49, while four committed mistakes. One only subject confirmed that winnings are the same for both games; he answered: “Any of the two; at the end, you get the same result when you add up the prizes and subtract the losses, and in both cases that result is 49.” Finally, in a fourth level, are the answers that derive from comparing the opposite ends of both sets and which are reflected in expressions such as “in game 2 you win more, but you also lose more” (risk seeking), or “in game 1 you win less, but lose less” (risk aversion). In one case (Figure 2), the student shows signs that let us know he added up the winnings and proved they were equal: He states in his answer that “In the second one, if you win, it is more money, but if you lose, you lose more.”

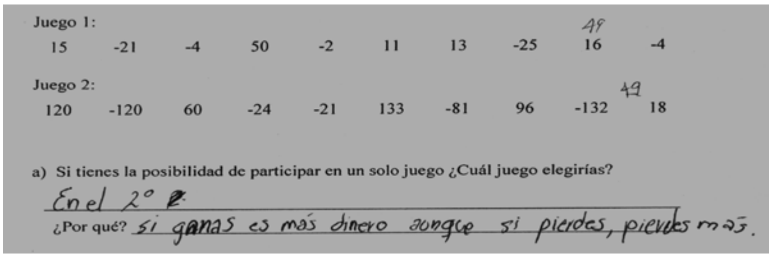

Figure 2. “In the second one, if you win, it is more money, but if you lose, you lose more”

The answers to the posttest are not too different from the pretest´s, although there is a slight improvement. We should stress that in 9 answers the students chose “Any” of the games based on the calculation of winnings in each game (Table 1). There is an increase in answers that prefer game 1 because “you lose less.” Finally, there is a slight reduction in the number of answers that show a preference for risk (aversion or seeking). In this task (of comparing data in the context of risk) it is shown how range interpretation is not derived from calculating, but it implies a perception of the uncertainty present in the data, a fact reflected in the formulation “you win more money, but when you lose, you lose more money”, which means that the game with the widest range is the riskiest. We can also see how the sole consideration of the sum of winnings (that prefigures the mean) is not enough for the students to reach the conclusion that it does not matter which game they choose since they somehow neglected the risk in both games, that is, the variability in the data. Even if it was expected that, in order to answer the question correctly, a good option was calculating the mean, no one did that. In this task, the students did not see the arithmetic mean as a resource to reduce the variability of every data set. Probably because in the teaching sessions they worked with the arithmetic mean, the number of students who summed up the winnings increased from 1 to 9, although they did not manage to obtain and compare the arithmetic means. In contrast, there was a decrease in the fourth level answers in which there were risk considerations. Probably due to the fact that they were influenced by the equality in the global winnings, some students hesitated when answering those questions in which they preferred one of the games taking in account their aversion or seeking to risk.

CONCLUSIONS

The context of risk offers an opportunity for students to find a meaning of variability through range. A first step to make this possible is the use of arithmetic mean (or median) to reduce variability of data finding a representative value for each set for a first comparison. A second step consists of considering variation interpreting it in terms of higher or lower risk. In this case, there is not a normative answer to the question and it depends on the resolutor´s preferences concerning risk; this relativity of the answer introduces a subjective element in the final selection.

Most of the students identify the data set with highest dispersion because they generally associate dispersion with the magnitude or separation between the numbers. However, knowing that a set has more or less variability does not seem to help them to make a decision on which set to choose. So, the difficulty lies in interpreting the variation, and not only identify where it´s more or less.

In conclusion, the problem turns out to be complex because it requires the consideration and interpretation of the mean which is difficult in itself. Then, we must analyze variability in terms of risk, this does not lead to a unique selection but depends on the resolutor´s preference regarding risk. The administration and discussion in the classroom about tasks such as the one shown here may contribute to the understanding and use of statistical variability.

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