ODDS THAT DON'T ADD UP

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This paper examines a common probability misconception that students have. Although students are generally able to correctly calculate the probability of two independent events occurring they are often not able to use this calculation in a meaningful way. This paper will contextualise this probability misconception by using examples from television game shows. Extracts from the game shows will demonstrate how contestants routinely overestimate the probability of combined events. The decisions made by the contestants reduce their probability of success by a significant margin.

BACKGROUND

In a previous paper, 'Odds that don't add up' (Fletcher M, 1994), I commented on a student's misconception in probability. In general if P(A) > ½ and P(B) > ½ then most students make decisions that are consistent with the belief that P(A&B) > ½.

Forty eighteen year old trainee teachers were shown two regular six faced dice that had four of their faces coloured red and two faces coloured white. The students were told that the two dice would be thrown and the colours of the upper faces noted. The students were asked:

i. What is the most likely outcome?
ii. Is the likelihood of this outcome greater than, equal to or less than ½?

In response to i) thirty six students stated that two red faces was the most likely outcome. Four students stated that one red face and one white face was as equally likely as two red faces.

In response to ii) the same thirty six students stated that the probability of two red faces was greater than ½. The other four students correctly stated that the probability of two red faces and the probability of one red face and one white face was the same and equal to 4/9.

The group of thirty six students was asked why this probability was greater than a half. The most common argument was 'the first dice is likely to show red and the second dice is likely to show red and so it is likely that both dice show red'. All the thirty six students considered this to be a sound argument.

This misconception has an interesting consequence. In the semi-final of any knockout competition a participant, be it an individual or a team, has to win two games in order to win the competition. Consider the case where the participants are A, B, C and D where, from past performance, A is better than B is better than C is better than D. Suppose that A is drawn against B and C is drawn against D in the semi-final. The probability that A will beat B is greater than ½ and the probability that C will beat D is greater than ½. Most people believe that the probability of A winning the competition is greater than ½. In fact if A is vastly superior to B the true probability may well be greater than ½. What is interesting, however, is that most people believe that A always has a greater than 50:50 chance of winning the competition. Their logic is that A will probably win the semi-final game and will probably win the final and hence will probably be the eventual winner. This is evidenced by the fact that the odds offered by bookmakers on A winning the competition are particularly ungenerous. Note that the bookmakers merely reflect the belief of the punters and the odds constructed by the bookmakers are in the ratio of the amounts bet by the public. It is usually the case that the odds offered on the favourite, at the semi-final, stage reflect the punters' belief that the favourite has a greater than 50-50 chance of winning the competition. Consequently the odds offered on the outsider at the semi-final stage are generous. i.e. the odds do not reflect the probability of success. As a result anyone betting on the outsider at the semi-final stage of a knock-out competition would have an expected positive return on their stake. Examination of the English FA Cup soccer competition over the past forty years shows that betting on the outsider at the semi-final stage would have been a profitable exercise.

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Robert Quinn, of the University of Nevada at Reno, has written an interesting article 'Odds that don't add up re-visited' (1996). This article considers in more detail the odds offered by bookmakers and uses the context of the USA Superbowl competition.

IMPORTANCE FOR PROSPECTIVE TEACHERS

This paper will consider:

i. further examples of how this misconception is evidenced
ii. how teachers can use these examples to help overcome pupils' misconceptions

The Colour of Money

The 'Colour of Money' was a television game show that aired in the United Kingdom in 2009.

Contestants are faced with twenty different coloured cash machines each containing a different amount of money. The machines contain either £1000, £2000, £3000, …….. £19000 or £20000 but the contestant does not know how much is in each machine. The task facing the contestant is to visit ten cash machines and attempt to withdraw a total of at least £50000. If a contestant attempts to obtain more from a machine than it contains no money is forthcoming. For a contestant who attempts to withdraw N thousand pounds, where N is an integer between 1 and 20 inclusive, the probability of success is (21 – N)/20. The expected amount the contestant receives is N(21 – N)/20. This is maximised if N = 10 or 11. Attempting to withdraw £10000 or £11000 maximises the expected amount received.

For a more detailed analysis of the game see 'The Colour of Money' (Fletcher, 2009).

In practice the vast majority of contestants opted to withdraw sums of money between £6000 and £9000. The probability of being able to withdraw an amount less than £11000 from a machine is greater than ½. The probability of being able to make successive withdrawals of this size is, however, not large. Consider, for example, the strategy, adopted by one contestant, to withdraw £6000 on six successive attempts. The probability that each attempt is successful is 15/20 x 14/19 x 13/18 x 12/17 x 11/16 x 10/15 = 0.129.

Some contestants attempted to withdraw £5000 on their first attempt. Presumably with the intention, if they were successful, of carrying on withdrawing £5000 for every one of their attempts.

The probability of successfully withdrawing £50000 using this strategy is 16/20 x 15/19 x …… x 7/11 which is approximately 4%. Considering that a good strategy has a probability of success of greater than 50% this is a particularly poor way to play the game. It is worth commenting, however, that the compere of the game show often pointed out to contestants that, on average, they needed £5000 for each withdrawal. This could partly explain why contestants opted to withdraw such small sums of money.

Students who play this game and opt to withdraw £10000 will quickly discover that this is a good strategy. There is a version online that students can play.

Play Your Cards Right

'Play Your Cards Right' is a popular television game show that aired in the United Kingdom and other countries between 1980 and 1999. In the USA it was known as 'Million Dollar Card Sharks'. The game is played with 52 playing cards. Contestants have to guess if the next card is higher or lower than the card that is showing. If the card has the same value the contestant loses. Two teams compete against each other. In one particular game Team 1 had a Jack showing and four cards not yet turned over. Team 2 had an eight and two cards not yet turned over.

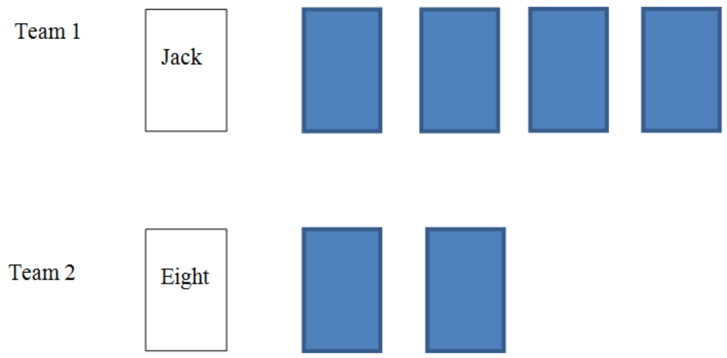

Figure 1. A game of “Play your cards right”

Team 1 was given the choice either to turn over four cards correctly from the Jack or force Team 2 to turn over two cards correctly from the eight. Their choice was to Play or Pass.

The better strategy, by far, is to pass. In fact the probability of Team 1 winning if they pass is approximately 2/3 whereas the probability of winning if they play is approximately ¼. (See Teaching Statistics (Fletcher 1995) and (Hunt 1996) for an explanation of how the probabilities are calculated.) In the TV show the contestants opted to play and lost.

When presented with this scenario the vast majority of my students opt to play.

The most popular reason given by my students is that the chance of guessing correctly from the Jack is high and, unless one uncovers an eight, the chance of guessing correctly thereafter is always greater than a half.

A moment’s thought, however, shows that Team 2 has a less than 50-50 chance of winning. The first card is an eight and the chance of guessing the next card correctly is less than ½. The chance of Team 2 making two correct guesses, therefore, is less than ½.

Students can find recordings of programmes on the internet. It is particularly interesting to view shows where contestants are given the opportunity to play or pass. Invariably contestants play and, for the most part, are unsuccessful.

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