

TEACHERS AND STUDENTS: FROM AN INTUITIVE APPROACH TO A RATIONAL EVALUATION OF PROBABILITY

Maria Pia Perelli-D'Argenzio¹ and Silio Rigatti-Luchini²

University of Trieste, Italy¹

Department of Statistical Sciences, University of Padua, Italy²

carlodar@alice.it

The paper describes a research conducted in Italian schools and connected to the m@t.abel project for the improvement of teaching and learning mathematics. A group of selected teachers produced 56 didactic units on mathematical subjects. Other teachers involved in the experimentation, received a short in-person presentation and then worked with the units in a virtual environment available on line under the supervision of a tutor. The in-class experimentation was on 4 units and at least one had to be a statistical one. Teachers had to write a log book for each unit including a report with the reasons for their unit choice, a close-ended questionnaire on the experimentation and a final report with the results. By analyzing the context of the unit "Stem and leaf plots", one of those implemented in the classroom, we observed the teachers and students transition from a purely intuitive probability approach to a rational estimate.

INTRODUCTION

In Italy Statistics and Probability has been introduced into the official Mathematics curriculum of the lower secondary school since 1979, but without any emphasis on teachers' professional development. There is substantial evidence that both pre-service and practicing teachers have poor understanding and insufficient preparation to teach Statistical concepts (Begg & Pfannkuch, 2004). Teachers have mainly been trained in a traditional way with little, if any, specific training in Probability and Statistics. Many of the senior teachers have never studied the subject formally. Younger teachers may have taken an introductory Statistics course, which however does not prepare them adequately to teach in a way that develops students' intuition about data and uncertainty (Rossman, Medina & Chance, 2006). As a result, the majority of teachers tend to focus on the procedural aspects of Probability and Statistics rather than on the conceptual understanding (Nicholson & Darnton, 2003; Watson, 2001). Teachers who do not have a specific training in Statistics get their information directly from the school textbooks and may not detect the errors presented (Aureli & Ottaviani, 1992; Hawkins et al., 1992). Consequently, teachers are unable to recognise and respond to students' conceptions and misconceptions (Russell & Mokros, 1991). Therefore there is still a wide discrepancy between the curriculum as it should be followed and what actually is taught in the classroom (Mullis et al., 2008). Moreover in Italy there are some additional challenges regarding the educational programs such as the high average age of the secondary Italian teachers (the average age of our sample is 52), the low wage differentials (OECD, 2007) and a recruitment system which does not include specific training in teaching. Furthermore, Italian teachers declare, more frequently than in other countries, that they lack opportunities to get feedback on their work (OECD, 2009).

In the last few years Italian Maths teachers have strongly demanded proper training, particularly as far as Probability and Statistics is concerned (IARD, 2010; TALIS, 2009) and this has been offered to them directly inside schools. However, teachers seem not to appreciate these courses since, in the majority of cases, they are based on a traditional face to face teaching style for both disciplinary contents and educational approaches. Alternative training methods, such as e-learning and blended learning, remain very rare. Given the relatively poor performances of Italian students in international Maths assessments (i.e. IEA, TIMSS and OECD PISA), there has been a recent boost in initiatives to help teachers to improve and understand student' achievements. Starting in 2006, the Italian National M@t.abel Plan for the renovation and improvement of teaching and learning Mathematics, aimed at Maths teachers in the lower secondary school levels (6-8 grades) and the first two years of upper secondary school (9-10 grades).

Teacher training approaches vary widely and there is little empirical evidence on factors affecting both teaching practice and student achievement (Guskey, 2003a; 2003b; Fraser et al., 2007). Previous research on in-service teacher training stresses the importance of extended

duration, content-focus and peer collaboration. US cross-section data Garet et al. (2001) focus on content knowledge, opportunities for active learning and coherence with other learning activities as key features for successful training programs. Other researchers suggest that teaching practice and student performance are likely to improve when professional development is focused on academic content, based on teachers' collective participation and administered through long-term activities rather than one-day generic workshops (e.g., Kennedy, 1998; Ingvarson et al., 2005; Timperley, 2008).

The present study investigates the results of M@t.abel program for Statistics and Probability subjects, restricted in the lower secondary school and suggests some starting points of consideration.

BACKGROUND AND OVERVIEW OF M@t.abel PROGRAM

In Italy, only three projects regarding blended learning training programs have been introduced at a national level: 1) M@t.abel, for teaching Maths; 2) Poseidon, for teaching languages; 3) TIC, to acquire Information and Communications Technology skills.

The activities of M@t.abel provide a Mathematics teaching-learning based on three fundamental and interwoven aspects: 1) the content necessary to solve problems; 2) the situations and contexts in which the problems are posed, providing an incentive for students; 3) the processes that the students must use to solve the problems. The contents are related to four basic Standards that are included in the Mathematics curriculum of many countries around the world, as well as in OECD-PISA tests, albeit with different terms: 1) Numbers; 2) Geometry; 3) Relations and Functions; 4) Data and Forecasts (this last point corresponds to Statistics and Probability.).

Considering the existing evidence, M@t.abel seems a promising training program, since it has features characterized by success according to international literature. It concentrates both on teacher practice and on Math content, offering specific teaching materials and promoting group work in the classroom. It lasts the whole school year, and, finally, it requires teachers' active participation and a continuous interaction through an online virtual community.

The part of the M@t.abel program, evaluated in this study, aims at increasing lower secondary students' achievements in Statistics and Probability, providing teachers alternative approaches for presenting traditional contents. The main idea is that students, rather than learning abstract formulas and ideas, should be engaged in solving real life problems through Mathematical tools and concepts. The program is based on formal and on-line tutoring and it lasts a full school year. There is a repository of teaching materials facing different Math concepts by adopting a problem-solving perspective. The DU (didactic units) in the M@t.abel plan are subdivided into equal numbers among the four basic Standards (Numbers; Geometry; Relations and Functions; Data and Forecast). The participants in M@t.abel go through the following phases: 1) Initial in-person training; 2) Online training (virtual class); 3) Experimentation in the classroom; 4) Document production; 5) Final in-person meeting. At various stages the Tutor acts as a group moderator.

Teachers are required to use at least four of the teaching materials in their classrooms (one per basic Standard) and to discuss their experience with their tutor and their peers through a structured logbook. Moreover, the program encourages a virtual community of teachers, where they will have the opportunity to exchange views through on-line forums and discussion groups, and they could participate from their home.

EXPERIENCES ON PROBABILITY UNITS

DU in the basic Standard Data and Forecast (Statistics and Probability) comply with the necessary requirements related to the teaching of Statistics and Probability and they are inspired by some basic principles encoded at an international level (Garfield & Ben-Zvi, 2008). Schools units and individual teachers are enrolled in the program on a voluntary basis. The formal training takes place at selected schools with proper facilities for tutors. Teachers' meetings, although a substantial part of the training, are taking place at distance through an e-learning platform. Each teacher who participate this project had to fill a final logbook which needs to include: i) the description of the whole experience; ii) the relationship between the teacher and the training group; iii) the effect of the experimentation on teacher's professional development. In addition, a questionnaire was given

at the beginning and end to assess teachers’ perceptions of a possible improvement regarding their own teaching and the students’ learning as well. The basic Standard Data and Forecast included seven DU: four with a content based mainly on Statistics and three on Probability. In order to identify elements on the choices, strategies and meanings that teachers attribute to Statistics and Probability, the logbook, filled out by 206 teachers who participated the project M@t.abel, has been examined also using textual exploration techniques (Caputo & Pennisi, 2011). One of the main emerging issues through the analysis of the logbook, was the perception that the teaching of Statistics and Probability is not very well integrated in the other Maths basic Standards. Teachers’ unfamiliarity regarding these contents was also observed. In particular, it reveals teachers’ difficulty in choosing a DU that provides appropriate tools to deal with uncertainty and random choices.

Figure1 shows the distribution of selected DU on Data and Forecast. Three DU clearly linked to Probability (6,7%) (1. Many roads lead to the probability; 2. I want a daughter with red hair; 3. Experiments .. outcomes .. events), contrary to those related to Statistics which constitutes a better known and a more familiar subject to teachers. In fact, the teaching of Probability seems to be a challenging topic for the teachers who often address it in a purely formal manner mainly connected to games of chance. Consequently, they tend to use (sometimes mistakenly) the classic definition of Probability as a relation between favorable and possible cases forgetting, either because of neglect or ignorance that the possible cases have to be equally possible. In addition, they tend to restrict the use of Statistics to a mere technique for quantitative data handling, resulting in a procedural and trivial attitude towards Data and Forecast.

This leads to an ambiguous vision of the core Data and Forecast resulting at the same time to a marked technicality and an excessive trivialization. It seems that the small number of Probability DU that was chosen by the teachers, shows not so much a lack of interest for the subject, but rather a superficial knowledge of the content and a general uncertainty on how to handle any further analysis.

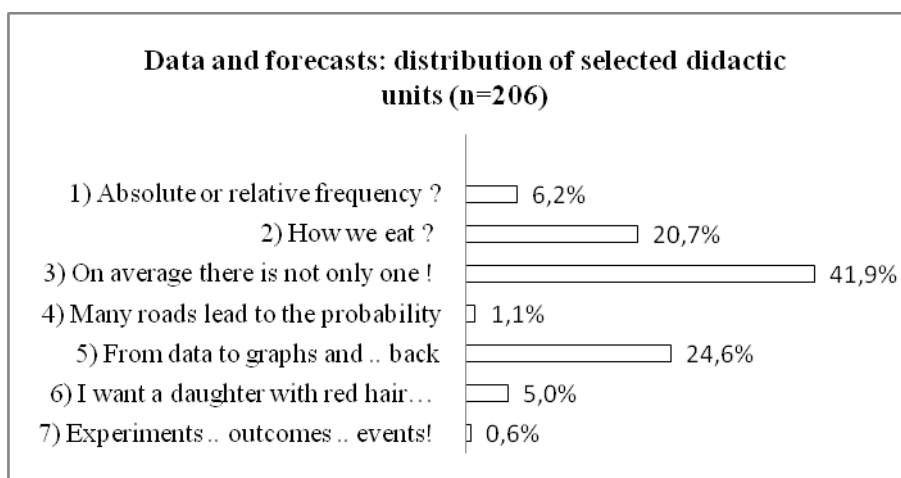


Fig. 1. Distribution of selected DU on Data and forecast (206 teachers)

Some other DU regarding Data and Forecast are also discussed here. Particularly, one of the DU (There are also trees in Statistics) was created in order to prevent the observed difficulties. The goal of this DU was the estimation of the probability of an event by collecting and elaborating empirical data relating to the same event. In addition, through this DU teachers were supposed to be encouraged to experiment with other DU based on Probability.

The main goal of the DU “There are also trees in Statistics” is the introduction of the stem and leaf graph representation regarding the date of birth of all students in the classroom. By looking at the graph, which is rather endearing, the next step to Probability comes spontaneously. The chosen branches represent the decades (tens). Before data collection, students are invited to estimate the number of leaves on each branch. Generally they come to the conclusion that the fourth ten is basically leafless as one can only find 30 or 31 day, while they cannot work out the

different values for the other decades. By observing the diagram obtained from the data collection the students are encouraged to make conjectures concerning the characteristics of the data under investigation.

In particular, the teacher urges students to notice that the length of the four decades is different and invite them to estimate them. Students will then be able to compare the frequency of births with the corresponding frequency in the days of the decades. They can reasonably guess that births are distributed along the four decades as the days of the year. If the collected data do not support such an assumption, they will rather extend the investigation on friends' and parents' dates of birth. By increasing the data volume we can assess more accurately the validity of our assumption. In that way, one may observe that the bigger the number of people surveyed about their date of births is, the more likely it is that the empirical distribution be very similar to the one hypothesized. Through this activity teachers and students shifted from simply collecting empirical data to representing them, make speculations based on information related to the different numbers of days in months, assign a probability value corresponding to the event "being born in the ... decade" and finally comparing data results with the initial assumptions.

Teachers were inspired by their experimentation on the DU stem and leaf and they eventually adopted a less formal approach to Probability. After this DU many of them also chose to try the DU "Many roads lead to Probability", always being supported by a tutor and a virtual class. The objective of this DU is to find ways to assign probability to elementary non symmetrical events in simple random situations. Through this DU, teachers suggest experimental activities related to toss symmetrical objects (coins) as well as asymmetric (thumbtacks). Using the outcomes of the experimentation, students can create two graphs related to the data distribution and then compare them in order to find the diversity in the frequencies of the possible events based on the symmetry or asymmetry of the objects used. Consequently, they reasonably come to assume that the coin is symmetric, since it is well balanced, and the probability value of the two outcomes (head or tail) can be set as $\frac{1}{2}$. On the other hand, a thumbtack is obviously asymmetric and not balanced; therefore the probability value of the two outcomes (pin up or down) can only be estimated in an experimental way.

All the teachers believe that they improved their professional skills in the subject. These two DU, suggest a frequentist approach for the estimation of probability, and this acted as a motive for teachers to use them in their class. The analysis of the logbooks shows that teachers appreciate very much this approach, and they also preferred this to the traditional approach for teaching probability, the determinant approach since then. Moreover, all the teachers admitted a positive effect of these DU on their professional development regarding the teaching of probability.

CONCLUSIONS

The data obtained from the M@t.abel project has been variously explored (Argentin et al., 2013) and the association between teachers' different teaching styles and students' learning have been considered in the evaluation of the project as well. Generally, despite teachers being the prime recipients of training, they are usually treated as a means to transfer knowledge to the students. The importance of these' results stems from two main considerations. First, noticeable shifts on the students' achievements are linked to a change on the teachers' practice (Desimone, 2009). Such shifts occur when teachers develop their knowledge of the subject (very important especially in Probability and Statistics) on the one hand, and develop awareness for the teaching of these certain contents on the other hand. The observed shifts regarding teachers' attitudes and practices show that there has been a change towards a more innovative way of leading the classroom. This change was particularly affected by teachers' collaboration through virtual teachers' class.

The second consideration is related to the fact that, once teachers have been trained they will be able to assist their students in a more effective way for the following years of their teaching. Therefore the analysis based on the effects of M@t.abel training should be considered potentially. However, the major result that emerges from the empirical evidence is the positive effect of the training on both content knowledge and related pedagogical knowledge (Garet et al., 2010; 2011). The utilization of the virtual class and the tutor's service seem to be the strengths of the project, especially for the contents of Statistics and Probability, as one can observe from the logbooks. The logbooks' analysis (Caputo & Pennisi, 2011) reveals that teachers consider that Data and Forecast

UD had a positive effect on students with difficulties in mathematics (81%), on bright students (91%), and on their personal teaching improvement regarding their previous teaching approach (36%).

Moreover, the logbooks reveal teachers' appreciation for the effectiveness of the DU tasks regarding both their content and the teaching approach used. Teachers have noted that the activities in the Probability DU has been useful to them and helped them to change their initial approach to this subject. Particularly, some teachers mentioned that they learnt how to read critically the Italian textbooks which are often full of mistakes: i.e. in a textbook, two possible results of tossing a drawing pin were given as equiprobable and so they were taught to students (Perelli D'Argenzio, 1996).

Logbooks' textual analysis were used to explore the teachers' choices, strategies and meanings about the usefulness of probability compared to other concepts (Caputo & Pennisi, 2011). Headwords more associated with Probability are: "game", which is the initial reference model when probability is taught; "chance" which confirms the confusion between probability and uncertainty; "statistics" seen as inevitable link with the variability of the phenomena.

The textual analysis also reveals that the word probability is more frequently used by younger teachers (<10 years teaching experience). This could imply that they are more open-minded towards the context of uncertainty. The effect on students' achievements has also been studied through the analysis of the logbooks, the questionnaires given at the beginning and end, and teachers' focus and comments in the international forum M@t.abel (Cosenza, 11-13 Oct. 2010).

This study's conclusions are aimed to contribute on teaching approaches regarding a new and stimulating subject, on the workshops methodology and on the group work. The introduction of new DU in the probability area such as "There are also trees in Statistics", in the second phase of the program, could reinforce the adoption of investigative approaches by the teachers when they teach Probability" and will surely improve the positive effects of M@t.abel.

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