LEARNING AND TEACHING PROBABILITY IN THE 21ST CENTURY

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Contemporary technology affords considerable enhancements to the pedagogy of probability. Traditional approaches focused on mathematical presentation of theory, supported by theoretical and numerical exercises. Utilising current electronic media in the form of e-books, available to students on mobile devices such as the iPad, allows probability to be presented and learned in a more active mode, with seamless integration of text, images, video, animations, presentations and self-assessment exercises. Such a format promotes deep learning of the underlying theory and an essential connection to practical application of the ideas. While this is beneficial for all students (and teachers), it is especially so for those who will be future teachers of mathematics and statistics. They not only get access to contemporary learning materials in probability, but they are also exposed to models of teaching from which they can develop their own pedagogical repertoire.

INTRODUCTION

Probability has long been a 'difficult' subject to learn and teach. It is replete with intuitive challenges and even paradoxes, it utilises a language and terminology that is demanding even for those with some statistical background, and it moves very quickly from simple calculation into the realms of complicated mathematics. Little wonder that students at school and university so often struggle with the material, and that some school teachers avoid the topic whenever possible, maybe delaying it until the last week before holidays. Traditional approaches to teaching and learning probability focused on mathematical presentation of the underlying theory, supported by theoretical and numerical exercises. The predominant tools for such learning consisted of written materials such as textbooks and lecture notes, mainly mathematical investigations sometimes supported by calculators. At the school level, this reflects the position of probability as part of the mathematics syllabus. At the university level, probability was traditionally used as a first topic for introductory statistics courses, and then re-visited as an advanced subject for specialist statistics major students – particularly those with a stronger background in mathematics.

Approaches to teaching courses in statistics have changed profoundly over the previous decades: conceptually in terms of the inclusion of material such as exploratory data analysis and graphical techniques, practically in terms of utilising computer packages in the investigation and analysis of data, and pedagogically in terms of incorporating awareness of students' conceptions about key ideas such as chance and variability. This is true not only for university courses, but increasingly also for school courses. In terms of probability, however, there have been more limited changes in approach, and such changes have been concentrated at the school level – most commonly at primary school – rather than at the level of university studies.

Currently, students and teachers of probability can access an increasing range of technology in order to approach the subject of probability in more varied ways. Contemporary tools such as interactive e-books and mobile devices allow probability to be learned and taught in a more active mode that incorporates a variety of media – written, visual and audio – and can promote greater engagement with the material of probability and a deep approach to its learning. While this is beneficial for all students, it is particularly important for those who are planning to become future teachers of mathematics and statistics, giving them access to learning materials and pedagogical techniques that can provide a strong basis for their future professional work in helping the next generations of students approach the learning of probability in a more positive way.

BACKGROUND

Every generation of teachers of probability makes use of the technology available at that time. In the late 20th century, computer packages and televised materials began to be utilised in probability (and statistics) teaching and learning. Particular features of video – its ability to bring reality into the classroom, to show visual and graphical details, and to communicate on the affective as well as intellectual plane – were highlighted by statistics educators (e.g., Moore, 1997;

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Petocz, 1998). However, the range and power of contemporary technology are substantially greater and changing at an increasing pace. While e-learning (electronic learning) is now a standard component of most university courses, the term m-learning (mobile learning) has recently appeared in the literature; it harnesses the power of mobile technologies such as the latest telephones and tablet computers ('post-PC devices', Murphy 2011) that have the potential to make fundamental changes to the process of learning and teaching. In particular, m-learning encourages constructivist and collaborative approaches to learning, and flexible and adaptive approaches to teaching.

Although the iPad is relatively new (the first version was released in 2010), it has already been the subject of studies investigating its use in higher education. Kinash et al. (2013) described an Australian study evaluating the impact of such mobile devices on learning; feedback from students was positive and optimistic, although most did not believe that their learning had improved. Hall and Smith (2011) described an iPad initiative in a U.S. graduate management program; though they could not demonstrate improved learning outcomes, student convenience and flexibility were enhanced. To date, m-learning has rarely featured in discussions of statistics education (but see Lee, 2009), though there are more general investigations of the role of technology in learning probability and statistics (e.g., Chance et al., 2007; Garfield & Ben-Zvi, 2007; Mills & Raju, 2011). Chance et al. point out that "it takes time and thought to effectively incorporate new technologies [into teaching]" (p.15) and that "technology should be chosen to facilitate student interaction and accessibility" (p.17). Garfield and Ben-Zvi state that "the computer can also be used to improve students' understanding of probability by allowing them to explore and represent statistical models" (p.388).

Consideration of the role of new technology in learning probability and statistics should go beyond discussion of the technology itself, and rather concentrate on how the technology can help the learning process and how it can allow changes in pedagogy that may lead to enhanced learning. A pedagogical focus on techniques of probability will push students towards narrower conceptions of the subject; a focus on various probability models and their uses will allow a broader conception of probability; and an incorporation of the role of probability in professional and even personal life will encourage students towards the broadest conception of the subject (Petocz & Reid, 2010). In the following section, we will give some examples of how this can be carried out.

SOME EXAMPLES OF LEARNING FOSTERED BY CURRENT TECHNOLOGY

In disciplines such as probability that are 'difficult' to learn and teach, educators have to choose their teaching strategy carefully. Possible consequences of less-than-ideal approaches can include not only unsatisfactory student outcomes, but also disengagement, increased surface and strategic learning, waste of students' resources, and difficulties in the learning activities in general. In this section we present examples of how contemporary technology can be used as a tool to effectively support students' learning. Compared to their older peers, students of the m-learning generation are used to different ways and times to get information, different 'languages', and different stimuli. The examples that we discuss are varied, but all of them try to introduce students to a technical discipline by shifting the focus from the technicalities to the concepts, presented in an engaging and easily understandable format.

The use of technology is important in this context. Although learning can take place without technology, students live immersed in a world of smart phones, tablets and PCs, and not using technology would require an adjustment and an increased fatigue for students. It is important to underline the idea that technology should never be used for its own sake, but rather, it should be organically integrated into the curriculum. Ideally, it should disappear. The important aspects to address are generally related to students' learning and experiences, but technology, if used correctly, can play an important role, facilitating learning and making it deeper, improving engagement and the accessibility to course resources, favouring active participation and dialogue with other students and the teacher. We illustrate this next.

One student of *Mathematical Background for Biostatistics*, a distance course offered at Macquarie University, wrote (in 2011): *"I really enjoy the video lectures. I think that as humans we learn from sensory stimuli and having a lecture that stimulates the ears as well as our eyes in my view reaffirms the learning process."*

A problem with technical disciplines is that the content is often presented through the written word. Of course, traditional books can be replaced by modern ones that propose approaches pedagogically more valid, and books are not the only possibilities, as students can access websites, blogs and discussion forums, to name a few. But human beings learn from a variety of sensory stimuli, and books may not be the most effective, nor the easiest way to present the content of a technical course to a non-specialist audience – and participants in this course include students with scientific rather than statistical background.

A video can carry the same information as a book, but if carefully produced it can talk to the audience in a more effective and efficient way. Taking into consideration the short attention span typical of some learners in the current generation (Bunce, Flens & Neiles, 2010), the use of short 'spot-on' video clips, from a few minutes to half an hour in length, has many advantages. It allows the educator to modularise the course so that the content becomes easier to navigate and integrate. It lets the student engage with the course content on his/her own terms. And it allows the educator to expose more effectively the structure of the material, helping the students to understand the linkage between the curriculum, the assessment and the learning outcomes.

Mathematical Background for Biostatistics is a course offered in distance mode to students who are mature aged and have not studied mathematics since high school. The pedagogical challenge in this case is to build the mathematical knowledge required to continue their studies (the course is part of a *Master of Biostatistics*). Assuming almost no previous knowledge, the course ends up with the matrix formulation of least squares and the notion of spectral decomposition. With no face-to-face sessions, videos (e.g., http://youtu.be/G9Agns4IXnM) have dramatically and positively changed the learning experience of students as shown in their formal evaluations.

If systematically implemented in a course, the approach also introduces a new paradigm: information can be better presented using media other than the written word. A possible corollary is that students will develop new expectations when they raise questions (e.g. through the discussion forum). Educators are able and expected to reply in ways that are more effective than traditional emails or forum messages, maybe with a video clip focused on a theoretical topic or a specific exercise. (e.g., see the video produced to help students of *Introduction to Probability* with a particular question at http://youtu.be/0QNP3AUO9YY). This has the power to transform the learning experience of students, allowing the educator to incrementally increase the course data base with high-quality materials.

A course built with this approach can be easily designed as a distance or a 'flipped' learning experience (e.g. Ronchetti, 2010). Here, technology is present, but it does not require any additional effort by students. They continue using the tools they use in their real life, instead of changing them in favour of traditional and often cryptic ones. From the educator's point of view, there are several tools and platforms that allow them to create videos easily. Tablets, among others, are an easy and cost-effective way to record narrated videos in which documents, or simply whiteboards, are annotated directly on the screen. This has been possible for a long time using PCs, but the importance of finding a quick and easy tool to produce video materials makes the difference between actually recording them or giving up and falling back on traditional approaches.

Longer video lectures are not the only product that can be offered to students to enhance their learning experience. Interest in a particular course, module or topic can be enhanced using introductory video clips not dissimilar to movies. A trailer is a short film (1-2 minutes), usually with catchy background music, that highlights the important points of what is offered (a movie, a course, etc.), with a quick sequence of 'scenes' (e.g., see the trailer for students of *Mathematical Background for Biostatistics* at http://youtu.be/8s3CVvcC2lQ highlighting how videos will be used in the unit). In a minute, the educator can achieve two goals: increasing students' interest and highlighting some key concepts. Both are relevant to the learning experience.

Another tool that can be very effective for students is to accompany written learning material with animations. Animations can be ideally placed between spot-on video clips and trailers. They are usually short (1–2 minutes), without narration and with background music like trailers, but are more similar to video lectures as they only present a single concept or idea. Sometimes a concept does not need a voice-over explanation to be clear; in those cases an animation is an excellent alternative to a video lecture (see, for example, the explanation of 'boxplot' at http://youtu.be/hoOh8hqVdE0).

Other resources such as image galleries, interactive images and automated assessment can be included in the curriculum to offer students a simple yet deep learning experience. The risk is that with this multitude of resources the overall experience becomes too complex. One way to avoid this is to gather all the material in a tool flexible enough to contain the different file formats and yet able to offer an integrated and organic experience. This has been the goal of e-books from their conception. Today the two main formats available are *epub3* and *Apple iBooks*. The latter has the advantage of being a mature platform that offers an easy and free software to create e-books, *iBooks Author*. We are in the process of using it to produce an e-book for an introductory course offered at Macquarie University (*Gambling, Sport and Medicine*). The effort and the time required to create the e-book have been limited, but the result, compared with the traditional book in PDF format, will provide a richer learning experience for students.

The examples offered so far have the principal effect of smoothing the obstacles in the learning experience, making it simpler and deeper, facilitating access to the course materials, establishing new expectations on teachers' answers and favouring a dialogic approach. Increased engagement and participation can be side effects, even if they are not immediate goals.

When disengagement becomes a problem, one possible solution is the 'gamification' of the unit – the incorporation of gaming dynamics as an incentive to changing learning behaviour (Kapp, 2012). The authors have trialled an activity developed at Macquarie University called *The Reading Game* (Parker, Manuguerra & Scheaffer, 2013). In this online game, students are required to write and post questions weekly, and to answer other students' questions regularly. They gain points when they answer correctly and when their questions are positively flagged by their peers. The game aims to involve students, allowing them to review their preparation (answering questions) and deepen their understanding (writing questions). To make the game even more attractive, several reward tactics are possible, from weekly small-value prizes for the best players to the possibility of basing part of the final assessment on the best questions of the game.

STUDENT REACTIONS TO THE USE OF VIDEO

The systematic use of contemporary approaches and learning materials in the authors' courses has had the consequence of an abundant spontaneous and formal feedback from the students; this is particularly the case for the use of video clips – an innovation that we have been using for several semesters. In this section we present a selection of their comments, organised under various themes.

Many students comment that being introduced to a new topic with a narrated video has the effect of making the content and the book more approachable:

The video lectures are just brilliant. When you said you'd be making some tutorial vids, I was thrilled. The videos form a great introductory base on which more detailed math can be learned. It's a very, very nice change from all the terrible math teachers and lectures I've had in the past.

The video lectures are fantastic. Maths doesn't come easy to me so having a video lecture to explain the important concepts before I tackle the text book is a great help.

Students find the videos and video clips particularly useful where a textbook fails:

I have also really enjoyed the video lectures and found them really beneficial. At times I've struggled to understand the concepts from reading the text alone; the video lecture has enabled me to comprehend these concepts and greatly reduce my frustration and enhance my learning. Thanks for posting these. I really appreciate the effort, it's made this subject much more enjoyable.

The video lectures are a godsend. Watching them before tackling the textbook makes the topic much less daunting. Also, watching someone go through the steps methodically allows me to make the vital connections that the textbook never explicitly states. Please keep them coming!

In particular, students commonly have problems understanding the mathematical steps involved in proofs. Carefully-constructed video materials can help in this regard:

[Video]-lectures are excellent, especially when there is a difficult problem which requires a detailed answer. They are also helpful when the book is not clear in its explanations. Please keep them coming.

I think the video lectures are great too! I think it's particularly helpful that you write each step as you explain it. It makes it so much easier to follow compared to a pre-written powerpoint slide where all the steps are shown together.

Accessibility is another problem that can be solved by short video clips:

I absolutely love the video lectures. They are convenient in that I can access them at a time that's convenient for me and I can access them more than once. Being able to see you write and draw as you are talking is a big bonus because it adds immediate meaning and context to what is being explained.

In terms of communication, dialogue with the teacher becomes easier and more fruitful:

I think that as distance students, this method of bringing you to us, so to speak, makes learning difficult concepts much easier. A great example of this was the video you posted in response to forum questions about inequalities. It allows us and you to be interactive in our learning.

Distance students have particular needs and short videos fit well into their workload:

Love the video lectures! This is my third attempt at studying statistics via distance ed [education], and the only time I haven't felt overwhelmed by it all and withdrawn. Recordings of lectures delivered on campus are often not helpful for external students and have left me feeling lost and disconnected from my studies. These short videos keep my attention and I have had many "ah ha" moments watching them. I wish all external units could be just like this!

I really enjoy the video lectures as they give distance learning a personal touch and help explain concepts. I also feel like I am in a class rather than just learning over the net.

Maybe the most obvious benefit of contemporary video-based material is in the increased engagement that can result:

I have been saying to my friends that I feel 'included' in the course and that I feel I actually 'belong' in the course, which is a revelation for me. … It's hard to be engaged when you are studying at a distance. Those videos are great for engagement with the material.

Contemporary students expect that current technology (including video) will be utilised in the curriculum, especially when there are real needs to be addressed, as in the case of students with a non-English background:

As an foreign student I need the visual integration of difficult and abstract concepts, and obviously the video lectures are the only way to get this help without having a real life lecture. On the other hand, the fees that I have paid deserves that new technologies will be applied but not only in your unit of study, in all the units of study!!

Students appreciate the ability to engage with the course content using a variety of technological forms and in a way they can understand it. What matters most is giving all students, face-to-face or distance, young or mature, full-time or working, the possibility of enjoying their studies. This is not always easy to do, especially in technical courses in probability and mathematics, but it is nonetheless possible.

I am really enjoying your videos and instructions ... You make studying maths almost enjoyable (within limits, of course!).

CONCLUSION

In this paper we have illustrated several ways in which contemporary technology can be utilised in helping students engage with and learn about probability. We have discussed the use of short video lectures focused on specific points and video clips created in response to students' questions or to elucidate some area of difficulty. These materials can be prepared easily using current mobile devices, and can be accessed by students in the same way. Students' reactions to these video materials seem consistently positive. We have also talked about extensions of these approaches to creating animations and interactive images, and to 'packaging' the materials and the associated assessment in the technology of an e-book. To this we add the utilisation of course management technology in the form of gamification to enhance our students' learning experience. These are aspects that we are currently implementing; they seem to be positively received by our students, though we have not yet had the opportunity of carrying out formal evaluation of them.

When inappropriate technology is imposed on students or forced into the curriculum the outcomes are usually negative (Spitzer, 1987). But when current and familiar technology is

integrated into the course offering, students appreciate the resulting flexibility in their 'personal learning environment' (Dabbagh & Kitsanis, 2012). Their comments, some of which were shown in the previous section, talk about an increased engagement with the course materials leading to a deeper and more satisfying learning experience. This is particularly important in a technical topic area such as probability, and more generally in the mathematical aspects of statistics. Probability has a long history of being difficult to learn and teach, and technological solutions can help to remove obstacles to learning and produce an enhanced learning experience for all students.

For those of our students who are planning to become teachers, there is a double benefit in such an approach. First, they will learn the concepts of probability at a deeper level and will be less likely to avoid it in their future teaching. Secondly, they will be exposed to a pedagogic model that focuses on providing a technology-rich environment for student learning, resulting in a greater level of engagement with the learning process. They can use this as a model for their own teaching in the future, helping to break the cycle of avoidance and formalization that has been a characteristic of probability pedagogy for some time.

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