

A CASE STUDY OF AN ELEMENTARY SCHOOL STUDENT'S UNDERSTANDING OF STOCHASTIC PROGNoses

Judith Stanja

University of Duisburg-Essen, Germany
judith.stanja@uni-due.de

Stochastic prognoses are assumed to be a key concept for elementary school stochastics. They may be characterized by the following structural components: focus, evaluation and justification. A qualitative research project with third graders (age 8-9) was conducted in the frame of epistemological interaction research (Steinbring, 2009). The aim of the study was to learn about elementary school students understanding of stochastic prognoses. The study encompassed pre-interviews, a series of lessons and post-interviews. The contribution aims at analyzing and comparing one student's understanding in the pre- and post-interview. The data used for analysis are transcribed episodes from the pre- and post-interview. The analysis and comparison gives insights in the students developing understanding of stochastic prognoses.

INTRODUCTION

Studies on the way students deal with the task to make prognoses for future events are common in stochastics education research. A prognosis might be understood as a justified (reflexive) statement about some future event that contains a focus (what the prognosis is about) and an evaluation (rationalized probabilities or degrees of belief and by means of expressing possible deviations taking variability into account). Stanja and Steinbring (2014) argued that stochastic prognoses may be a key concept for primary school stochastics that is orientated at the particular nature of stochastics as applied science (for details on the epistemological nature of stochastics see for instance Batanero, Henry, & Parzysz, 2005; Heitele, 1975). Heitele (1975) expressed some doubts whether the “separation of reality and model⁵⁷, or even of consciousness about it” might be addressed at all levels of education (compare Heitele, 1975, p. 201). For primary school students, the question is still not answered.

A qualitative research project was conducted in the frame of epistemological interaction research (see Steinbring, 2009). The relations to and distinctions from research on probabilistic thinking and informal inferential reasoning are discussed in Stanja (2013). The project's aim was to use the frame of epistemological interaction research to describe and study the range of primary school students' perspectives on and understanding of stochastic prognoses. This means that the interpretation and coordination of semiotic means in interactive situations is of particular interest. The design of the study (that took place in grade 3, age 8-9) is based on the following considerations. Due to the complexity of the concept of stochastic prognoses and the necessity of appropriate semiotic means to express one's ideas, children were not expected to have a highly sophisticated understanding when dealing with the task to give stochastic prognoses for the first time. In order to learn about more elaborated perspectives an intervention was developed that gave children the opportunity to get to know some means (i.e. lists, diagrams, language) and to develop their understanding further. So, children were interviewed in a 1-1 situation prior to the intervention of twelve lessons (each of 45 minutes) and afterward. The lessons included: an introduction to random experiments, the provision of elementary tools to formulate and study stochastic prognoses for random experiments with spinners (lists, diagrams, technical terms for evaluations for frequencies); the formulation, evaluation and justification of statements about future outcomes of random experiments; comparisons of the outcomes of various spinners; the reconstruction of spinners from available data; and general discussions about prognoses.

The aim of this contribution is to describe and characterize two ways of interpretation and coordination of given means in the interviews. The next section provides more information about the design of the interviews and about the data analysis. Then, some short episodes from a pre and post interview of one child (Nelli) are presented (that therefore also allow to speculate about some developments in one student's understanding).

INTERVIEWS AND DATA ANALYSIS

The half structured interviews were about giving and justifying stochastic prognoses for an experiment with a spinner as shown in Figure 1. An overview of the course of the interview as well as other materials used can be found here: https://www.dropbox.com/s/ldpauks016ewefh/ICOTS9_interviewmaterials_finalversion.pdf. The materials encompass templates for lists and elementary diagrams (to record absolute frequencies) for the purpose of giving a prognosis and to record outcomes of experiments, filled lists as well as filled elementary diagrams, cards with given justifications and cards with statements concerning the possibility to give prognoses.

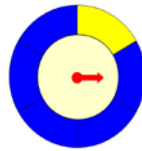


Figure 1. Spinner used in the interviews

In order to deal with the complexity of stochastic prognoses it has been argued that primary school students need appropriate semiotic means to study and understand stochastic prognoses (see Stanja and Steinbring, 2014). The lists and diagrams are understood as a communicative tool that offers the possibility to record possible or actual outcomes that a child might refer to by verbal or gestural means for further explanations. The cards with given justifications and statements about the possibility to make prognoses are taken from interviews and classroom discussions in a pilot study. They serve manifold purposes. In cases where children have language problems, these cards (introduced as ideas from other children) may provide language tools for a child to articulate its own ideas. In every case, the cards provoke additional statements from the child and therefore form a possibility to gain more insights in a child's understanding. A third purposes concerns the quality of the interpretative reconstructions. Since the cards are introduced after a child has stated its own ideas, they can serve to validate the previously made reconstructions. For the analysis it is not primarily of interest which cards are chosen but what explanations for (non) choices are given by the child.

The data used for the case study are transcribed episodes of videotaped interviews. The analysis is done in two steps. First an epistemologically oriented analysis will be carried out using the epistemological triangle (see Steinbring, 2009). Steinbring (2009) who studied knowledge construction in mathematical classrooms, states that “[...] there is no mathematical knowledge independent from interactions and communications. Thus the interpretative reconstruction could not and should not be undertaken with the idea of an objectively existing, correct mathematical knowledge [of the child] in the background.” (p.35). This also holds for interview situations as interactive situations. Moreover, the frame of epistemological interaction research meets the need for non-normative perspectives on students' thinking and understanding (see for example Pratt, 2000). During the first step of analysis, one has to bear in mind that the interaction between interviewer and child also determines what may be reconstructed or to put it with Hallden, Haglund and Strömdahl (2007): “[...] to make valid propositions about an individual's conceptual structure, we have to take cognitions, as well as discursive practices, into account.” (p.26). For the analysis not single statements but sequences are considered that perform a unit of meaning and both the child's as well as the interviewers statements are taken into account. The analysis draws on verbal utterances and gestures that are understood as an integrative system for the person speaking and listening. They are understood not only as a tool for communication but also as a representational system that seems to be particularly interesting for early stochastic learning (see for example Tatsis & Kafoussi, 2008). A second step of analysis then tries to relate the reconstructions to an analytical framework based on the structural definition of stochastic prognoses (focus, evaluation, reference) and by means of comparison of reconstructions from several episodes tries to make statements about the students understanding of stochastic prognoses.

Nelli's Understanding of Stochastic Prognoses

For space reasons a full analysis can not be presented here. Instead some snippets from the interviews are chosen to illustrate the different ways of understanding that could be reconstructed (see Figure 2). The full episodes can be found in the additional material. By using episodes from one child's pre and post-interview some interesting changes may be observed concerning the usage and interpretation of the given semiotic means and in the implicit understanding of the status of prognoses.

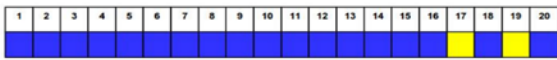
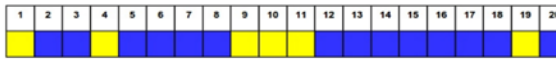

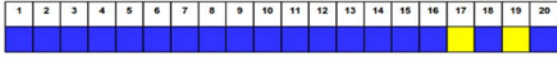
Pre Interview	Post Interview
<p>Outcome of the first experiment</p> 	<p>Outcome of the first experiment</p> 
<p>I Mh, can you explain, why the outcome looks like that?</p> <p>N Mh, because the blue here is also just much more often on there (<i>moves the finger round the spinner several times</i>) and, as I said, one just has to have <u>luck</u> and (.), I actually would not have imagined now, that it will come out like that at the seventeen and nineteen. I would have rather thought that it will be blue, but, well, I would have, well, until the sixteen always blue and this alternating simply, that is just like that, because, eh, the yellow is simply only once on it here (<i>moves the pointer to the yellow field</i>) so, and then one has to have that so, so, so or so (<i>moves the pointer by bits always pointing at yellow</i>) and that is quite rare. One rather has that at the blue (<i>moves the pointer at a blue field and then taps several times on it</i>). That is why it is also until the sixteen blue and then only the seventeen and the nineteen yellow.</p>	<p>I Ehm, can you explain why the outcome looks like that now?</p> <p>N Mm (<i>shakes her head</i>). Well, yes, eh, it simply is only one times yellow on the spinner but with so many I would have rather thought now that there would have to be two fields yellow (<i>points at one blue field and the yellow field of the spinner, laughs, looks at the list</i>). [...] But there is still simply more blue.</p>
<p>Comparison with her initial idea</p>  	<p>N has explained before that she refers to the spinner and available data to give a prognosis</p> <p>I What, eh, does it help you, when you look at the outcome or# the spinner?</p> <p>N # It can become something like that [...] because, if one has done that now, if that has come out, than it also can just come out something like that, because one can not always spin the same outcome with the same spinner. That is actually very, very unlikely.</p>
<p>N Well, I actually had everything wrong (.), except for here then, eh, the twenty B, the sixteen B, the fourteen, the thirteen, the twelve, the nine, the eight the seven, the four and the one (<i>points to the according fields</i>). [...] There I had only these numbers right, because I believed that yellow could also come successively, maybe. [...] I have just planned too less blue.</p>	<p>Justification for non-choice of the „guess-card“</p> <p>N I think that this one is actually nonsense, because if one already, if one has spun now (<i>makes a spinning gesture</i>) and already has an outcome, then one can look at that (<i>looks at the spinner</i>) and if one looks at the spinner (points to the spinner), then it would still be nonsense, if one just writes in there, in the conjecture, somehow (<i>shakes her head</i>) that, eh, there comes twenty times yellow and zero times blue. [...] That would actually be nonsense then, because that is almost not possible.</p>
<p>Predictability</p> <p>N Not exactly. In my case there are always two possibilities now. (<i>takes the „wish“-card and looks at it for 7 sec.</i>) Mmm (<i>shakes her head</i>), if one wishes for the outcome, then it will not come out. (<i>takes the „approximately“-card and looks at it for 9 sec.</i>) Approximately, I can say something about the outcome. (<i>takes the „nothing“-card and looks at it</i>)(...) But I can also not say nothing about the outcome. [...] because I have already done the two experiments (<i>points to the lists and diagrams</i>) and then I have those two in my mind. [...] Before the experiment I can not say exactly, well, before the experiment I can, which outcome will come , but this I also can not do. Before the experiment I can approximately say what will come out there. [...] That there are three yellow and seventeen blue or that there are eighteen blue and two yellow. [...] But it could also be that there are nineteen and one yellow. [...] Mm, I, it could maybe be also, if one has a lot of luck, that one has four yellow and sixteen blue.</p>	<p>Predictability</p> <p>N For this spinner (<i>points at the spinner</i>) I can say about the outcome, for example, that absolutely more blue comes out [...] then yellow, probably and then also that, eh, yellow comes less and that for this spinner it comes five or six times.</p>

Figure 2. Snippets of the interview data

In the pre interview the behavior of the *concrete* spinner stands in the foreground but Nelli also creates a relation from the structure of the spinner via the pointer positions to frequencies of blue and yellow outcomes. We can observe how she begins to build a new *system of symbols/signs* that she uses in the post interview as a basis to formulate prognoses. While she conducts the experiments with the interviewer she observes possible frequencies for yellow and blue and in consequence considers them possible for future experiments. At first, she restricts the possibilities to the observed ones so far. But then she goes beyond these and adopts a *pluralistic perspective* with four possible pairs of frequencies (1:19, 2:18, 3:17, 4:16) where (4:16) can only occur when one “has a lot of luck”. In this extension of her perspective there is a change from an exclusive usage of the lists and diagrams as protocols for experiments with the concrete spinner to a tool for thinking. In the post interview we can see how she uses the lists and diagrams not only as a representation for actual outcomes but as a semiotic mean to speak about expected relations of frequencies of yellow and blue. Also, the spinner is used in a different way. It does not stand any longer only for itself but is used as an *ideal* object. The given snippets in figure 2 illustrate how Nelli coordinates the ideal spinner and lists/diagrams – for instance when she expresses that an outcome is unexpected for her referring to the structure of the spinner and relating a different spinner to the actual outcome. Her symbol system of spinner and lists/diagrams is not only regulated by a (preliminary) notion of proportions what becomes clear when she states that the relation of more-blue-than-yellow of the spinner *still* holds for the actual outcome. According to Nelli, actual outcomes may be used as an orientation with the restriction that the outcomes of experiments are not always the same. In fact, she evaluates this with “very, very unlikely”. In several episodes of the post interview we can observe that Nelli expresses the universality of her perspective (for example the usage of “one”, „something like that“, „the same outcome“, „the same spinner“). Interestingly, we can also find hints for how important the new symbol system is to Nelli when she contrasts it with guessing (“nonsense”). Guessing would for instance give a bad prognosis like “twenty times yellow and zero times blue”. She evaluates this with “almost not possible” thus expressing the possibility and the low probability. As in the pre interview, the relation of the frequencies is addressed in the post interview. Though Nelli is convinced that this relation will show in the outcomes, she does not postulate it as absolute when she evaluates it with “probably”. Moreover, she articulates a range for the expected frequencies of yellow. From pre to post interview, a change of the status of prognoses can be reconstructed. While in the pre interview Nelli's statements concern what will actually happen and resemble guesses that may be right or wrong, in the post interviews, she seems to be aware of the status of these statements as theoretical in nature. Coming back to Heiteles doubts, we can see in Nelli's case that the distinction of the real spinner and experiment and the theoretical statements about it might be captured at least implicitly by a primary school child.

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