# PROTO-BAYESIAN REASONING OF CHILDREN IN FOURTH

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*This paper is dedicated to the ecological rationality of certain information formats that facilitate proto-Bayesian reasoning by children. The formats of representation introduced are en-active, iconic and even symbolic and are based on so-called natural frequencies. Experiments are described that have been performed with fourth-graders recently at schools in Berlin and Ludwigsburg and which exhibit the success of these formats for fostering proto-Bayesian reasoning. Results on activities with a special website dedicated to conditional probabilities and Bayesian reasoning (www.eeps.com/riskicon) are also presented and discussed.*

## NUDGE VERSUS EDUCATION

Nudge versus Education is a relevant new dichotomy in the scientific scenario (Bond, 2009) tightly connected with the sustainable statistical empowerment of citizens in democracies. Nudge is a top-down way of inducing people to make the "right" choices and the "right" decisions where, as the nudge philosophy claims, the right decisions are those which will satisfy them in the end. Decision makers make choices in environments where many features, noticed and unnoticed, can influence their decisions. Most of these environments in which people make decisions are far from natural: they are created by other people. The persons who tailor these environments are, in the terminology of Nudge, *choice architects*. The goal of Nudge is to show how choice architecture can be used to help stir people towards better choices (as judged by themselves), an attitude that is called libertarian paternalism. The tools typically used by choice architects are: defaults, anticipating errors, understanding mappings, giving feedback, structuring complex choices, and creating incentives.

Education, as ascribed by Bond (2009), is the solution to (some of the) human biases and fallacies, which is proposed by Gigerenzer, Hertwig and their school of thought (Gigerenzer, Hertwig, & Pachur, 2011), and is perhaps most adequately described by the German term Bildung[1]. Bildung is a special kind of Education, which corresponds to an intellectual "formation" and empowering, namely a bottom- up process which empowers people to think for themselves (Selbstdenken) and make choices in the environments they encounter. Their Bildung equips them, for instance, with strategies for translating information provided by possibly any environment into formats they can naturally understand and deal with. Once they are confronted with adequate information formats, they can make choices and decisions, often by means of very simple heuristics. Empowerment (Bildung) is directly connected with Democracy[2] and the vision of competent citizens. As an example, people encounter information often in percentages or even in probabilities. These formats can be misinterpreted by the punter. Nevertheless those who have been empowered in the sense of Gigerenzer and Hertwig, will have heuristics for translating these formats into so called natural frequencies, which are akin to the mind and foster understanding.

Ecological rationality is an aspect of bounded rationality, which has to do with the environments the mind has to interact with. Nudge in its present form is seen as a consequence or reaction to the Heuristics & Biases (H&B) program (Kahneman, Slovic, & Tversky, 1982), or more specifically to aspects of this program, which describe the fallacies the mind is riddled with when dealing, for instance, with choices based on probabilistic assessments. In particular, Daniel Kahneman, one of the two main protagonists of H&B, has expressed only a very moderate optimism towards the possibility of "training" human intuitions for solving probabilistic tasks. In his recent book "Thinking fast and slow" (Kahneman, 2011), which is a wonderful, well written recount of the discoveries of H&B, Kahneman devotes one short chapter to Nudge as developed by Richard Thaler and Cass Sunstein, yet does not mention Education as a means to foster the "right" choices of humans. In fact Education is hardly mentioned in his book at all.

While the H&B program and its followers – Cass Sunstein and Richard Thaler among them –have been seen as pessimistic as far as the possibilities of empowering the human mind are

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concerned, Gigerenzer and Hertwig can been seen as *panglossian* thinkers, for whom the human mind is, if not *the best of all possible minds*, definitely one that can be empowered to succeed by means of its own choices and heuristics. Gigerenzer does acknowledge, for instance, that doctors, medical journalists or financial speculators, often fall into probabilistic traps. He reports, in fact, that in one experiment, he asked 160 gynecologists to interpret some basic probabilities about a woman's chances of having breast cancer, given that her mammography screening had come back positive and found out that just 21% gave the right answer. Gigerenzer sees the solution to this problem in an early empowering (Bildung) of children in probabilistic thinking by means of natural frequencies. His beliefs have influenced those of modern educators in the UK and in the German speaking countries with respect to the fostering of probabilistic competencies at an early age.

## RISK COMPENCY INSTEAD OF GEOMETRY IN PRIMARY SCHOOL

The concept of competency (Kompetenz) was first introduced by Wolfgang Klafki and later defined in precise terms by Franz Weinert in the nineties (Weinert, 2001). Competency has replaced concepts like skill and ability in the educational models across the world. Competencies can be measured in terms of output and are hierarchically structured in levels. The essential aspect of competencies is that once they are acquired they (should) remain in the rucksack of the mind: reading and comprehension and elementary number computations typically become acquired competencies. Gigerenzer has promoted the thesis that a first level of competency in dealing with aspects of risk should be achieved in Primary School. His motivations are corroborated by discoveries of Mathematics Educators which confirm that first level competencies *should* be acquired in Primary School. One crucial motivating example Gigerenzer has reported in his talks at Mathematics Education conferences, concerns a warning by the UK Committee on Safety of Medicines in 1995: The Committee publicly stated that the third-generation contraceptive pill increased the risk of dangerous blood clots by 100%. The warning was followed by an additional 13,000 abortions in the UK the next year, many of them by teenage girls. The fact that the increased risk amounted to just an extra 1 in 7,000 was lost on most people — and, crucially — was not passed on by the media. Gigerenzer has used this example to show how fundamental it is that school students learn to distinguish between relative and absolute risks early enough in their lives. In fact he has coined the term "risk competency", which is now understood as a bundle of first level competencies, like proportional thinking, dealing with relative/absolute risks, conditional frequencies and loss/benefit trade-offs, to be taught in Primary School (Martignon & Krauss, 2009; Spiegelhalter, 2011; Till, 2014). This has been a revolutionary position, because the traditional first level competencies acquired in Primary school had been long confined to Arithmetic and Geometry. Given that the number of hours dedicated to Mathematics in Primary school has remained unchanged, introducing this bundle of first level competencies is now happening at the cost of Geometry. Gigerenzer is famous in the Mathematics Education community for having said and written that Geometry is beautiful but useless when compared with a first level of risk competency. This extreme position has been heard by educators both in the English speaking countries and in the German speaking countries. Gigerenzer has been invited to express his beliefs on empowering children with a first level of risk competency in Primary School in keynote talks at conferences like ICOTS 8 (International Conference on Teaching Statistics) in Ljubljana, 2010, and at the meetings of the German Society for Teaching Probability and Statistics. Several Mathematics Educators have carried intervention studies based on ideas propagated by Gigerenzer proving that the use of analogue representations in Primary School, like tinker-cubes, can foster a first competency levels of probabilistic thinking (Martignon & Kurz-Milcke, 2006; Kurz-Milcke & Martignon, 2006; Martignon & Krauss 2009; Till, 2014). These studies, on turn, have influenced committees in the Ministries of Education in Germany - in Baden-Württemberg and Bayern for instance - that now assign portions of the primary school curricula in Mathematics to the basics of probabilistic thinking.

## NATURAL FREQUENCIES AND BAYES' THEOREM

Historically, the philosophical and mathematical complexity of probability, involving either notions of infinity and convergence in the classical approach, or notions of sets and measures in the modern formalization, had led Mathematics educators to think that the inception of probabilistic concepts should be confined to the last years of secondary school. In countries like Germany this late inception had, until 2003, seldom been implemented with real enthusiasm by teachers, although most did acknowledge that Stochastics is more useful than other branches of Mathematics for everyday life.

The first impact of Gigerenzer's ideas was in secondary school. Large intervention studies were carried out in Nordrhein-Westfalen, Germany, in which school students in ninth class were instructed in natural frequencies and Bayes' theorem. These studies included follow-up tests several months after instruction, which proved that students had become competent in translating probabilistic data into natural frequencies for solving problems, finding the correct solution and translating it back into probabilities (Wassner, Martignon, & Biehler, 2004). Several mathematical textbooks for ninth class in Germany now introduce natural frequencies in the chapters on conditional probability as a preparation for Bayesian reasoning. And even the textbook used for training future teachers in Germany has a section on natural frequencies, before introducing conditional probabilities and Bayes' Theorem (Büchter & Henn, 2007; 2011).

## CHILDREN'S PROBABILISTIC INTUITIONS

Traditional developmental theory, as initiated by Piaget and Inhelder in the second half of the last century, suggests that children do not become proficient at making probabilistic inferences until age 7 (Piaget & Inhelder, 1975). Nevertheless, more recent research has shown that very young children are capable of performing simple probability calculations when task demands are reduced (Zhu & Gigerenzer, 2006). In general, many of Piaget's postulates on children's logical and probabilistic intuitions have had to be revisited and modified. For instance in their book "The early Growth of Logic in the Child" Inhelder and Piaget (1964 [1959]) reported an experiment in which they showed 5- to 10-year-old children pictures, of which 16 were flowers and 8 of these 16 flowers were primroses. The children were asked several questions. One of them was "Are there more flowers or more primroses?" Only 47% of 5- to 7- years old children gave answers in accord with class inclusion − that is that reflected an understanding that the flowers were more numerous because they included the primroses as a subset. Among 8-years olds, however, a majority (82 percent) gave answers consistent with class inclusion. This kind of experiment, according to Inhelder and Piaget, indicated that children acquire logical intuitions even without instruction. Of course, this experiment has probabilistic connotations as well: flowers are more probable than primroses. It is important to note, in fact, that the step towards a proportional statement like "8 out of 16 flowers are primroses" is quite natural, once children learn to understand and formulate proportions. Here again, Piaget's pioneering results have been revised by recent experiments on children's early estimates of proportions for simple categorization.

The important discovery in this context has been (Martignon & Krauss, 2007; Multmeier, 2012) that the representation of information plays a fundamental role in children's and adults' understanding of proportions and probabilities. Multmeier investigated children in second and in fourth class and divided his samples out of second and fourth classes in two groups. Both groups had to solve six tasks like the following:

*In a small and faraway fairyland village live 10 inhabitants: 2 princesses and 8 mermaids. Of the 2 princesses, 1 wears a crown. Two of the 8 mermaids also wear a crown. If I tell you I met an inhabitant of the fairyland village and mention this inhabitant was wearing a crown, would you bet it was a princess?*

In one group children, second graders and fourth graders, had to solve the task based on the texts of the tasks only. In another group children, second graders and fourth graders, had to solve the tasks based on texts and iconic visualizations, like the following, which corresponds to the task above, on princesses and mermaids:

Figure 1: Iconic representation of a fairyland village

The following graph exhibits the results of this experiment:

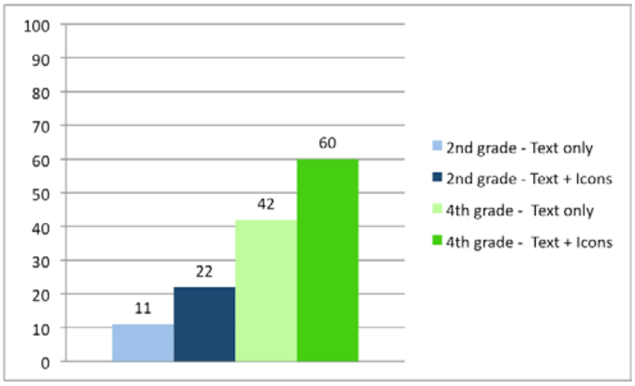

Figure 2: % correct of second graders (n = 91, 45 text only)
and fourth graders (n = 85, 42 text only)

The results show that children in fourth class have much stronger intuitions for proto-probabilistic thinking than children in second class and also show that iconic representations foster such intuitions in both classes. They also encourage instruction in fourth class on proto-Bayesian reasoning with the systematic use of such appealing iconic representations.

## A DYNAMIC, ECOLOGICALLY RATIONAL WEBPAGE FOR INSTRUCTING CHILDREN IN PRIMARY AND SECONDARY SCHOOL

Picking the idea of mermaids and princesses as an inviting environments for teaching first steps of conditional probability the authors devised a dynamic webpage that presents different environments allowing the user to vary parameters: www.eeps.com/riskicon .

In this webpage the user finds, for instance, environments like the following:

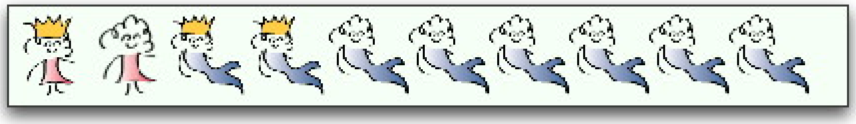

Figure 3. Icon Array to represent a subpopulation

In Figure 3 ten stylized fairy folk are represented: two princesses and 8 mermaids. The user has a menu of possibilities: he can vary the base rate of princesses among the fairy folk. She can vary the “sensitivity” of crown, i.e., the probability, that a princess wears a crown, and also the probability that a mermaid wears a crown. She can click on the “tree” button to obtain the “double tree” describing this situation, say of 20 fairy folk, 6 princesses, 4 with a crown and 14 mermaids, of which 6 wear a crown:

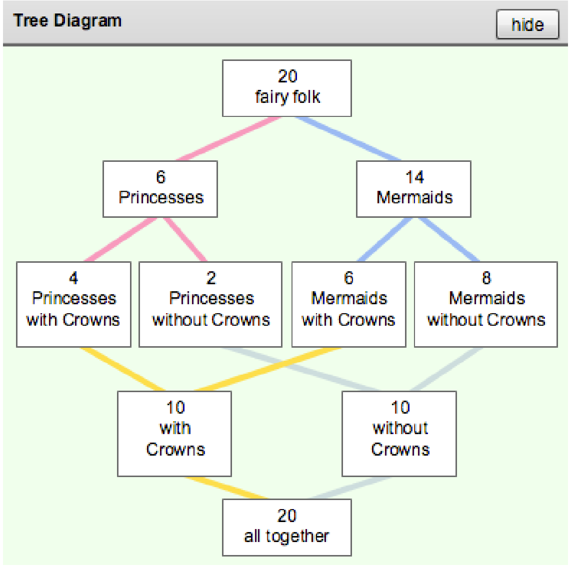

Figure 4: The double tree describing natural frequencies of a fairy folk environment

The double tree represents both directions, the direction from “princesses” to “crowns”, which can be called the causal direction (“being a princess may cause wearing a crown”), and the diagnostic direction, from “crowns” to “princesses” (“wearing a crown can be a sign that one is a princess”).

The double tree facilitates reasoning not just from cause to effect, but also in the inverse direction, from effect to cause, i.e., Bayesian reasoning with natural frequencies.

The dynamic webpage also presents an HIV testing scenario, designed for school students in ninth grade, where again the user can vary many parameters and visualize the corresponding double trees.

The dynamic webpage also presents materials that foster the understanding of relative and absolute risks.

We have started studies with future teachers using the dynamic webpage and find that it fosters understanding in many ways. In a group of 73 students of probability and statistics at the Ludwigsburg University of Education, 67 were able to answer questions on Bayes’ Theorem easily and correctly, when tested at the oral exams more than 8 months later.

Reasoning with natural frequencies and double trees can be seen as a form of proto-Bayesian reasoning, which constitutes a good heuristic for Bayesian Reasoning with probabilities. First elements of proto-Bayesian reasoning can be conveyed to children as early as in fourth class, not just with good iconic representations (mermaids and princesses, as in Figure 1) but also with concrete hands-on materials (Till, 2014).

## ENDNOTES

[1] Education corresponds, in its Etymology and meaning, to “Erziehung” and means "the result of conducing pupils along a certain path".

[2] The Max Planck Institute, where both Gerd Gigerenzer and Ralph Hertwig are directors, is an institute for Bildungsforschung, i.e., the research on Bildung typically devoted to questions of Empowerment for Democracy.

## REFERENCES

Bond, M. (2009). Decision-making: Risk school. *Nature, 461*(7268), 1189-1192.

Büchter, A., & Henn, W. (2007/2012). *Elementare Stochastik, Eine Einführung für das Lehramt.* Heildelberg: Springer Verlag.

Gigerenzer, G., Hertwig, R., & Pachur, T. (Eds.) (2011). *Heuristics: The foundations of adaptive behavior*. New York: Oxford University Press.

Inhelder, B., & Piaget, J. (1959/1964). *The growth of logical thinking from childhood to adolescence*. Basic Books.

Kahneman, D., Slovic, P., & Tversky, A. (1982) *Judgment under uncertainty: Heuristics and biases*. New York: Cambridge University Press.

Kahneman, D. (2011). *Thinking fast and slow*. New York: Farrar, Straus and Giroux.

Kurz-Milcke, E., & Martignon, L. (2006). Lebendige Urnen und ereignisreiche Bäume: Überlegungen und Versuche zu einer Didaktik der Stochastik in der Grundschule. In Meyer, J. (Hrsg), *Anregungen zum Stochastikunterricht*. Bd. 3. Hildesheim: Verlag Franzbecker.

Martignon, L., & Krauss, S. (2009). Hands on activities with fourth-graders: a tool box of heuristics for decision making and reckoning with risk. *International Electronic Journal for Mathematics Education*, *4*(3), 117-148.

Martignon, L., & Kurz-Milcke, E. (2006). Educating children in stochastic modeling: Games with stochastic urns and coloured tinker cubes. In A. Rossmann & B. Chance (Eds.), *Working cooperatively in statistics education. ICOTS 7 – Salvador de Bahia*. Retrieved June 22, 2009 from http://www.stat.auckland.ac.nz/~iase/publications/17/C443.pdf

Martignon, L., & Wassner, C. (2005). Schulung frühen stochastischen Denkens von Kindern. *Zeitschrift für Erziehungswissenschaft. 8*(2), 202-222.

Multmeier, J. (2012). *Representations facilitate Bayesia reasoning: Computational facilitation and ecological design revisited*. Berlin: Freie Universität Berlin (Dissertation).

Piaget, J., & Inhelder, B. (1975). *The origin of the idea of chance in children*. Routledge and Kegan Paul. Translation of original work (1951).

Spiegelhalter, D., Pearson, M., & Short, I. (2011). Visualizing uncertainty about the future. *Science 81*, 1333-1393.

Till, C. (2014). *Risk literacy: First steps in primary school*. Paper presented at ICOTS9, Session 8I.

Wassner, C., Martignon, L., & Biehler, R. (2004). Bayesianisches Denken in der Schule. *Unterrichtswissenschaft, 32*(1), 58-96.

Weinert, F. E. (2001). *Leistungsmessungen in Schulen*. Weinheim und Basel.