A SURVEY OF TEACHERS' SELF-REPORTED PRACTICES OF PROBABILITY TEACHING IN PRIMARY AND SECONDARY SCHOOL LEVELS IN QUÉBEC

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In Québec (Canada), the teaching of probability is mandated across the primary and secondary grades. However, probability teaching poses numerous challenges related to its conceptual complexity and its impact on the development of probabilistic thinking. To investigate this issue, we conducted a survey of teachers' self-reported practices of probability teaching at the primary and secondary school levels in Québec. Based on responses to an online questionnaire, we draw a portrait of probability teaching practices among a sample of 626 teachers. Our results include descriptive statistics of respondents' perceptions about probability, the place occupied by its teaching in their classes and the resources used in their teaching. Additionally, we use inferential statistics to explore the existence of significant differences among variables in the sample.

CONTEXT OF THE STUDY

Probability teaching and learning have attracted the interest of numerous researchers for several years (Martin & Thibault, 2016). Indeed, probability holds a prominent place in society, given that probabilistic thinking is useful to many professions and that random events affect individuals on an everyday basis (Batanero *et al.*, 2014). In Québec, probability instruction is prescribed throughout primary school (6–12 years old) and secondary school (12–17 years old), as the province's curriculum show. This said, the inherent conceptual complexity of probability gives rise to numerous instructional challenges, with resulting impacts on probability learning (Stohl, 2005). Moreover, studies suggest that teachers come up against difficulties when teaching this topic (e.g. Borovcnik & Kapadia, 2010). Some authors (e.g. Watson, 2001) have also noted teachers' poor mastery of probabilistic content. Furthermore, studies have pointed out teachers' lack of training in order to be able to teach this subject topic (e.g. Batanero, 2014). In Québec, recent academic studies have investigated probability teaching at the primary level (e.g. Martin, 2014) and at the secondary level (e.g. Thibault, 2011). These studies have essentially drawn on qualitative approaches. As a result, they have helped shed light on specific elements of probability teaching and learning in some contexts, particularly by targeting one education level at a time. The intent here is to compare education levels. The research objective is to paint a broader and more reliable picture of probability teaching in both primary and secondary school across Québec.

CONCEPTUAL FRAMEWORK

In this study, which aims to describe self-reported practices of probability teaching in Québec, practices are defined as "purposeful individual actions of a teacher in the course of the preactive, interactive and postactive phases of working with students" (Deaudelin *et al.*, 2005, p. 83, our translation). Self-reported practices are defined as instances in which teachers themselves supply information on their practices in the context of questionnaires (Marcel *et al.*, 2002). This definition stands in contrast to observed teaching practices (e.g. observations in the classroom).

To situate more specifically the concept of self-reported practice of probability teaching from a mathematics education standpoint, we draw on the concept of teachers' *Probability Content Knowledge* (PCK), in reference to Gomez-Torres *et al.* (2016). This concept is a more specific version of the *Mathematical Knowledge for Teaching* (MKT) set forth by Ball *et al.* (2008) based on the concept of *Pedagogical Content Knowledge* developed by Shulman (1986). Ball *et al.* (2008) have sought to distinguish between different types of MKT, including *Common Content Knowledge* (CCK) and *Specialized Content Knowledge* (SCK). Gomez-Torres *et al.* (2016) consider that CCK "includes basic skills and general knowledge about the topics that are to be taught to students" (p. 198). In this study, we link the concept of CCK to participants' degree of mathematical proficiency with the probability content to be taught at school. SCK is "the mathematical knowledge and skill unique to teaching [and] not typically needed for purposes other than teaching" (Ball *et al.*,

2008, p. 400). In the view of Gomez-Torres *et al.* (2016), "this type of knowledge supports the teacher in representing mathematical knowledge, providing explanations, and understanding students' solutions to problems" (p. 198). In our study, we link the concept of SCK to participants' instructional confidence in teaching probability. Finally, in drawing up our portrait of the teachers' PCK, we also take into account their declared use of probabilistic approaches (Batanero, 2014) and of manipulatives (Thibault & Martin, 2016) to teach probability.

METHODOLOGY

Procedure. We designed an online questionnaire with 25 questions in order to collect the teachers' self-reported teaching practices. The questionnaire was submitted to four reviewers to validate its content and the wording of the questions, thus enabling us to make a few adjustments. Subsequently, 626 teachers answered on a voluntary basis over the course of spring 2017.

Participants. Among the respondents, 238 were primary teachers and 388 secondary teachers. The sample's characteristics are distributed in a manner that reflects the Québec context at large, for example with respect to the respondents' gender, level of teaching experience, and education levels taught, as shown in Table 1.

Table 1
Number of respondents by gender, teaching experience and education levels taught

| | Gender | | Level of teaching experience | | Education levels | | |
	Male	Female	Under 15 years	Over 15 years	1st cycle	2nd cycle	3rd cycle
Primary	19	218	142	95	100	128	145
Secondary	155	232	191	197	272	302	-
Total	174	450	333	292	372	430	145

Measures. Two independent variables were incorporated into the model: mathematical topic and education level. The study's dependent variables were degree of mathematical proficiency, degree of instructional confidence, use of probabilistic approaches, and frequency of using manipulatives. For the first variable, the question was, "From a mathematics standpoint, what is your degree of proficiency with these mathematical topics?" and the respondents were to rate themselves for each mathematical topic on a four-level Likert scale (very unproficient [1] to very proficient [4]). For the second variable, the question was, "From an instructional standpoint, what is your degree of confidence with teaching these mathematical topics?" and the respondents were to rate themselves for each mathematical topic on a four-level Likert scale (very unconfident [1] to very confident [4] on an instructional standpoint). For the third variable, the question was, "Which probabilistic approach or approaches do you use in your probability teaching?" and the respondents were asked to indicate (yes or no) whether they used each probabilistic approach. For the fourth variable, the question was, "How often do you use manipulatives in your probability teaching?" and the respondents rated their frequency on a four-level Likert scale (never [1] to very often [4]). Martin and Thibault (2017) provide a more detailed description of the study results.

Data analysis. The data was processed for compatibility with IBM SPSS Statistics 24 software. To verify the links between the various variables under study, univariate and repeated-measure variance analyses (ANOVA) were conducted for each variable, independently, in light of the absence of data for certain scales. In most cases, when the question referred to algebra, primary level teachers did not necessarily feel that this topic concerned them. The results are presented for simple effect (comparison between topics) and interaction effect (topic according to education level). Gender was used as a co-variable. In addition, correlational analyses were performed to verify links between two of the dependent variables, namely degree of mathematical proficiency (CCK) and degree of instructional confidence (SCK). The Pearson coefficient was used, as suggested by Field (2013).

STUDY RESULTS
Degree of mathematical proficiency and degree of instructional confidence

First, we wanted to determine the probability-related perceptions of the teachers participating in our study. To be able to accurately score the respondents' CCK through their degree of

mathematical proficiency with probability, we sought to compare these perceptions with those for the other mathematical topics taught at the primary and secondary levels. Generally speaking, the teachers in the sample report a high degree of mathematical proficiency, with relatively high average scores (Table 2). Based on these scores, probability emerges as the teachers' weakest topic of mathematical proficiency (M=3.32). At the primary level, the lowest level of mathematical proficiency is observed for algebra (M=2.57), but this is not surprising since primary teachers are not trained to teach this topic because it is not taught at this level.

All of the pair comparisons between probability and the other topics are significant at $p<0.05$, whether without (Topic) or with interaction effect (Topic*Level), controlling for the respondents' gender. In order, the topics in which the teachers report the highest mathematical proficiency are arithmetic, measurement and geometry. These topics reveal the greatest differences (η^2 between 0.22 and 0.32) compared to probability, regardless of education level. The differences between probability and the two other topics are very slight. It is also worth mentioning that, even if no difference can be seen between degree of mathematical proficiency with probability at the primary and secondary levels (F=0.019, p=0.89), the comparative analyses do point to differences for all the other topics. The teachers report a lower degree of mathematical proficiency with probability compared to all other topics, at both the primary and secondary levels, with the exception of statistics for which this difference is observed at the secondary level only.

Table 2
Mean (M), standard deviations (s) and ANOVA tests for degree of mathematical proficiency and instructional confidence in teaching according to the various subject topics

	Education level			Topic		Topic * Level	
	Primary M(*s*)	Secondary M(*s*)	Total M(*s*)	F	η^2	F	η^2
Degree of mathematical proficiency							
Probability	3.27 (0.77)	3.35 (0.71)	3.32 (0.73)				
Arithmetic	3.68 (0.58)	3.88 (0.37)	3.80 (0.47)	288.75*	0.32	4.62*	0.01
Measurement	3.54 (0.62)	3.85 (0.40)	3.73 (0.52)	187.18*	0.23	17.08*	0.03
Algebra	2.57 (0.94)	3.85 (0.41)	3.38 (0.90)	8.37*	0.01	301.16*	0.33
Geometry	3.54 (0.61)	3.82 (0.44)	3.71 (0.53)	170.70*	0.22	13.57*	0.02
Statistics	3.28 (0.76)	3.56 (0.60)	3.45 (0.68)	21.71*	0.03	19.96*	0.03
Degree of instructional confidence							
Probability	3.21 (0.79)	3.27 (0.73)	3.25 (0.76)				
Arithmetic	3.61 (0.59)	3.78 (0.44)	3.72 (0.51)	255.29*	0.29	4.08*	0.01
Measurement	3.48 (0.63)	3.77 (0.46)	3.66 (0.55)	189.39*	0.23	14.74*	0.02
Algebra	2.42 (0.93)	3.77 (0.48)	3.29 (0.94)	14.39*	0.02	325.67*	0.35
Geometry	3.49 (0.61)	3.77 (0.48)	3.66 (0.55)	192.17*	0.24	15.42*	0.02
Statistics	3.22 (0.77)	3.49 (0.62)	3.39 (0.69)	29.85*	0.05	20.18*	0.03

Note. Likert scale from 1 to 4. * $p<0.05$
The F test compares each subject topic with the topic of probability, which is the subject of our study.

We also asked the respondents about their degree of instructional confidence in probability in order to ascertain their SCK. The degree of instructional confidence reported by the teachers in the sample (for all topics) is relatively high, with the lowest average score being for probability, at 3.25 (Table 2). Primary teachers appear to feel much less confident in teaching algebra (M=2.42), but here again, this is not surprising since primary teachers are not trained to teach this topic.

All of the pair comparisons between probability and the other topics are significant at $p<0.05$, whether without (Topic) or with interaction effect (Topic*Level), controlling for the respondents' gender. In order, the topics in which the teachers feel most confident are arithmetic, geometry and

measurement. These topics reveal the greatest differences (η^2 between 0.23 and 0.29) compared to probability, irrespective of education level. The differences between probability and the two other topics are very slight. Although no difference can be noted between degree of instructional confidence with probability at the primary and secondary levels (F=0.017, *p*=0.9), the comparative analyses do point to differences for all the other topics. The teachers report a weaker degree of instructional confidence with probability compared to other topics at both the primary and secondary levels, with the exception of statistics for which this difference is observed at the secondary level only.

Since the results are similar for the teachers' levels of mathematical proficiency and their instructional confidence, we conducted correlational analyses between these variables. Table 3 reveals the existence of a strong relationship between degree of mathematical proficiency and instructional confidence for a given subject topic (r between 0.69 and 0.90), which in large part explains the closeness of these results. These results suggest that respondent's CCK and SCK may be interrelated for a given mathematical topic. Another interpretation of these results would be that respondents were unable to distinguish mathematical proficiency from instructional confidence.

Table 3
Correlation between degree of mathematical proficiency and instructional confidence

Degree of mathematical proficiency	Degree of instructional confidence (Pearson correlation)					
	Arithmetic	Measurement	Algebra	Geometry	Probability	Statistics
Arithmetic	0.72	0.59	0.44	0.61	0.39	0.45
Measurement	0.56	0.69	0.48	0.58	0.38	0.49
Algebra	0.37	0.40	0.90	0.41	0.22	0.35
Geometry	0.58	0.58	0.45	0.70	0.37	0.45
Probability	0.36	0.38	0.24	0.37	0.81	0.61
Statistics	0.43	0.50	0.36	0.45	0.56	0.80

Note. All correlations are significant at *p*<0.05.

Use of probabilistic approaches in probability teaching

Regarding the place of probability teaching, we set out to determine which probabilistic approaches the teachers used to teach probability. Without giving in-depth definitions of these approaches, the questionnaire stated that the theoretical approach is based on calculation; the frequentist approach, on experimentation; and the subjective approach, on personal judgment. The teachers' use of probabilistic approaches varies according to education level (Table 4). Indeed, the respondents who teach at the primary level state that they use the frequentist approach more than the theoretical approach, whereas the teachers at the secondary level report the opposite.

Table 4
Use of various probabilistic approaches according to education level

	Education level		
	Primary (n=237)	Secondary (n=388)	Total (n=625)
Theoretical approach	115 (48.5%)	353 (91.0%)	468 (74.9%)
Frequentist approach	174 (73.4%)	257 (66.2%)	431 (69.0%)
Subjective approach	81 (34.2%)	108 (27.8%)	189 (30.2%)
I don't know these approaches	31 (13.1%)	11 (2.8%)	42 (6.7%)

Note. Chi-square=140.99, p<0.001.
Respondents could declare more than one approach.

A chi-square test ($\chi2$=140.99, *p*<0.001, dof=1) indicates the existence of a significant relationship between education level and use of the theoretical approach. It appears that the secondary teachers use the theoretical approach more than the primary teachers. However, chi-square tests do not indicate a significant relationship between education level and use of the frequentist approach ($\chi2$=3.54, *p*=0.060, dof=1) or the subjective approach ($\chi2$=2.81, *p*=0.094, dof=1). Moreover, the

subjective approach is reported as the least used by all respondents in order to teach probability. This observation is unsurprising given that this approach is not part of the curriculum for primary or the first cycle of secondary school but appears only in the second cycle of secondary school.

Frequency of using manipulatives in probability teaching

Finally, we wanted to find out how frequently various resources are used to teach probability, with special focus on manipulatives in this text. Most of the teachers in the sample stated that they used manipulatives to teach probability, but how often they did so varied depending on the education levels they taught. The primary teachers reported using manipulatives more often than the secondary teachers (2.78 and 2.11 respectively; F=100.16, $p<0.001$, $\eta^2=0.14$). Hence, a significant difference can be observed, with a relatively high effect size for the frequency of using manipulatives according to education level. Indeed, the primary teachers stated they had more frequently used manipulatives to teach probability than the secondary teachers. This result may be connected to the use of probabilistic approaches. As noted earlier, the primary teachers reported more frequently using the frequentist approach, and the secondary teachers, the theoretical approach. We posit that manipulatives are mostly employed in cases where probability teaching is based on the frequentist approach, in which students are required to experiment and run trials. It seems possible that the primary teachers state that they have more frequently used manipulatives to teach probability than secondary teachers because their teaching more often entails experiments (frequentist approach).

CONCLUDING REMARKS

The results presented in this text reveal a portrait of the respondents' PCK that shows common points between teachers at the primary and secondary levels. For example, it uncovers that, of all mathematical topics, probability is the one for which they report the lowest degree of mathematical proficiency (CCK) and the lowest degree of instructional confidence (SCK). Watson (2001) has pointed out the low level of competence and confidence among primary and secondary teachers when it comes to probability teaching. Our results suggest that the participants in our study have a lower sense of competence in teaching probability than other mathematical topics. However, pre- and in-service teacher training offer few avenues for teachers' professional development in terms of probability teaching in Québec. In our view, significant work remains to be done to improve these two complementary paths of training for probability teaching.

In keeping with some of Watson's results (2001), the portrait of the teachers' PCK at the primary and secondary levels in our study also shows points of divergence, particularly in the use of probabilistic approaches and the frequency of using manipulatives to teach probability. Is it possible that ministry prescriptions (for example the Québec curriculum and mandatory exams) influence the use of certain probabilistic approaches and the frequency with which manipulatives are used? It appears essential, in our view, to examine these ministry prescriptions and their effects on probability teaching in schools. In addition, is it possible that the use of probabilistic approaches and of manipulatives to teach probability result from teachers' use of other instructional resources (e.g. textbooks or online exercises)—a situation that might rigidly dictate the nature and characteristics of the probability learning offered to primary and secondary students? It appears crucial to undertake a comparative analysis of the probabilistic tasks and content available in the instructional resources generally used to teach probability at the primary and secondary levels.

This said, consistent with Batanero (2014), we believe it is necessary to place renewed value on the use of the frequentist approach and of manipulatives at the secondary level, as these avenues are especially promising for exploring the conceptual particularities of probabilistic thinking such as variability, uncertainty, non-determinism, etc. Moreover, it is important to stimulate reflection on the connection between probabilistic approaches (especially frequentist and theoretical), as Martin and Theis (2016) have pointed out, both at primary or secondary level.

Finally, one limitation of this research is related to the fact that our sample is not a random sample but rather consists only of volunteers. This prevents us from generalizing our results to all teachers in Québec. In addition, given the nature of the questions on the questionnaire, the results we obtained do not provide insight into the whys and hows of the respondents' self-reported probability teaching practices. It would thus be helpful to ask teachers about the reasons behind their actions and choices, as well as how they put their choices into action. Accordingly, in the second phase of the

study, we will seek to describe and understand specific cases of self-reported probability teaching practices at the primary and secondary level in Québec, drawing on interviews with teachers who represent exemplar profiles, based on data collected here. This will help enrich the portrait of Québec primary and secondary teachers' PCK which we have begun to paint here.

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