TEACHING AVERAGE THROUGH IMPLEMENTING COGNITIVELY DEMANDING STATISTICAL TASKS

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The purpose of this study is a)to investigate how a mathematics teacher implements cognitively demanding statistical tasks(CDST) and b)how student learning occurs during the implementation of such tasks. The study is a case study with a mathematics teacher. Two 7th grade classes were observed during the lessons in which statistics were taught. The focus is to explain teaching practices about CDST in the context of teaching average. Therefore, 8 lessons and the student artifacts were analyzed. The unit of analysis is CDST; The Mathematical Task Framework(MTF) and Task Analysis Guide(TAG) were used to analyze CDST (Stein&Lane,1996). The results showed that when the teacher was provided with CDSTs, she implemented most of them in high-level that in turn provides high learning gains to the students.

INTRODUCTION

Statistics and Its Teaching

Statistics helps people to understand the world. Teaching statistical skills is an important part of any mathematics curriculum, contributing to students' making sense of the data through reading, analyzing and making inferences (Shaugnessy, 2007). Mathematics education tries to make students use mathematical processes in less abstract skills including problem solving and reasoning (NCTM, 2000). Having statistics in mathematics curriculum enables learners to develop problem solving skills through statistical processes (Shaugnessy, 2007).

Teaching Average

The idea of average is one of the tools in statistics that reflects a problem-solving skill. It can be defined as: to "summarize information about an entire data set" (Shaughnessy, 2007, p.968). Therefore, average helps students to understand summary statistics and other statistical tests using summary statistics.

School mathematics has too much emphasis on mathematical aspect of average concept (e.g. the formula of arithmetic mean) (Shaughnessy, 2007). In many Turkish curricula, before 2005, the average concepts including arithmetic mean, mode and median have been established as mathematical formulas; i.e. the computational aspects of the average concepts were the focus; there was no focus on statistical aspect of average as summarizing a data set given in a context (Toluk Uçar & Akdoğan, 2009), in other words they were not put into meaningful context, i.e. no examples were given as of what they can be used for. In Turkey the emphasis in curricula regarding statistics has been shifted towards statistical processes and statistical thinking. Although there are studies illustrating how students think or reason statistically (e.g. Toluk Uçar & Akdoğan, 2009), few studies investigate how new curricula are implemented.

Topic of This Study

Teachers are the ones taking curricular and pedagogical decisions based on the needs of students (Lombaerts, Engels, & Athanasou, 2007). Studies indicated that effective mathematics teaching will result in improvements in student learning (Schoenfeld, 2013).

Mathematical tasks are the units of instruction in mathematics. They provide a medium for students to solve problems, reason and understand mathematical concepts (Remillard, 2000). Stein and Lane (1996) categorized the tasks into two different process levels; high and low level. In high level tasks were doing mathematics tasks and procedures with connection tasks. Low demand tasks consisted of memorization tasks and procedures without connection tasks (PWC-out).

Teachers' daily teaching practices include the cycle of planning and enacting mathematical tasks (Stein & Lane, 1996). The task demanding high level thinking processes from students helps students to engage in doing mathematics. In particular, teachers need to choose tasks that help

students to have a productive struggle and a high level mathematical thinking and reasoning during the implementation. Boston and Smith (2009) found that teachers can plan high level mathematical tasks, but when they implement them in the class, they usually routinize them or cause the challenging aspects of the tasks to disappear. In this case, student learning from these tasks would not be as effective as intended (Stein & Lane, 1996).

The recent changes in Turkish school mathematics curriculum and curricular materials provide opportunities for teachers to use cognitively demanding tasks (Ubuz, Erbaş, Çetinkaya, & Özgeldi, 2010). Therefore, it is important to implement challenging or cognitively demanding mathematical tasks to improve student learning. However, when mathematical learning outcomes of Turkish students are examined in international studies like Programme for International Student Assessment (PISA), Turkish students fail to show high level performance compared to international average. The assessment tasks focus on the mathematical literacy that students use in real life context and need high cognitive demand. Although such tasks were provided (Ubuz et. al., 2010), it has not clear if teachers bring such tasks into the classrooms.

Shaughnessy (2007) states the importance of providing challenging tasks to support student learning while teaching statistics. This raises the question how the mathematics teachers implement CDSTs when the tasks brought to the classroom are cognitively demanding and how students benefit from such learning opportunities. This paper investigates (a) how a mathematics teacher implements CDST and (b) how student learning occurs during the implementation of such tasks. Teaching average is selected as the main topic to be investigated because statistics in nature need students to read and analyze real life situations.

METHOD

A qualitative case study design is adopted for the current study. Beril is a mathematics teacher with 5 year-experience in teaching and working at a public middle school. She taught mainly seventh graders. She was willing to participate in the study because she wanted to experience different ways of teaching statistics. She described her previous teaching practices related to statistics as giving too much focus on computational aspects of averages. Since the purpose is to observe how teachers use CDSTs, the researchers provided the tasks to the teacher. Before implementation, each task was introduced and possible implementations of them were discussed with teacher.

In order to explore the case, the researchers collected data through various sources including interviews, observation, and artifacts. For this paper, only observations and student artifacts were used. There were 8 lessons observed during which average concepts were taught for two classes between March 2017 and May 2017. However, the focus of this study, average concepts, were covered in only two out of four weeks. Furthermore, for this study, the unit of analysis is the mathematical tasks as defined above. There were seven tasks in total; five of the seven tasks were implemented in two classes, the other two only in one class and are therefore not subject to the current study. Charalambous (2010) developed a method to describe pathways referring the cognitive level of a mathematical task from the task as a curriculum material to the enactment of the task based on MTF. This study is only interested in the defined paths for high level tasks as curricular materials. Path A describes how high demand maintained during set-up and implementation phase. Path C describes how high demand decline in low during implementation phase. Path D describes how high demand decline in set-up phase.

Furthermore, to analyze student learning, a pretest and a posttest on statistical literacy were conducted to 30 students in each class (Yolcu, 2012). Stein and Lane (1996) explained that if the tasks were implemented in high level for three phases, the learning gains would be high; but if the high demand was declined while implementing tasks, the learning gains would be moderate. The tests included 12 questions in total. Only one question was on average in context analyzed for this study for both tests. The average questions were statements about the average of people dying in car accidents in a year and average number of newborns in a year. The students were asked what average means in the statements. The answers were coded based on the rubric developed by Yolcu (2012). These codes were blank/wrong answers, pre-statistical answers, descriptions via average concepts, statistical answers. The frequencies of each category for pretest and posttest were provided.

RESULTS

Beril set up and enacted 12 tasks. The first two tasks were related to finding and interpreting the arithmetic mean. The other tasks included using appropriate average concepts (arithmetic mean, median or mode) according to given data and the context. The Table 1 below showed the frequency of paths used by the teacher. Then, in the following paragraph, two illustrative episodes were provided.

Table 1: The paths Beril followed based on Charalambous (2010)

Path A		Path C		Path D	
n	%	n	%	n	%
8	66.6	0	0	4	33.3

In the first episode, Beril implemented a leveling task called Project Homework adapted to Turkish from Smith, Stein, Arbaugh, Brown, and Mossgrove (2004). In the task, students were asked to find the arithmetic mean without using a formula; instead they were supposed to use the leveling method. The task aims to teach the meaning of the arithmetic mean; to allow students to experience how data was summarized while finding the arithmetic mean. The question asked in the task was "Anita has four 20-point projects for science class. Her scores are shown below. What is her average score? Find the average scores by leveling off the stacks" given with the Figure 1 (p. 58). The teacher asked students to bring the bricks in the same level and asked students how to find the arithmetic mean. She set up the task as it was written in the question. However, in the enactment part, the students explained the procedure of how they shared the bricks. Then the she wanted them to calculate the arithmetic mean by the formula. She stated that they could use both methods to calculate the arithmetic mean. She did not relate the formula and the leveling method; expressed them in two different approaches. Therefore, the high level what declined into low level as PWC-out.
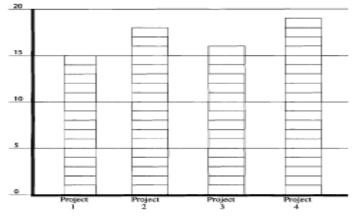

Fig1. Leveling task (Smith et. al., 2004, p.58)

In the second episode in which path A was followed, Beril implemented the Candy Bars task from Mathematics Assessment Project (2016). The task demands students to analyze the data; and analyze why the given statement related to data is wrong and to write one valid interpretation related to the data. The teacher asked students to analyze the data as the task demands. She presented the task in high level. She expected students to analyze the data. In the enactment phase, they shared why the given statement is wrong. Most students discovered that they could not reach any conclusion because the total number of candies does not mean anything when the number of people is not the same in the comparison groups. Students discovered that they needed to calculate an average. As the students were forming such explanations, the teacher asked why-questions and asked for explanations for their decisions.

The results related to students' learning gains showed that Class A showed improvement in

analyzing given statement related to average concept. For Class A, students made no statistical comments in the first semester; 80% of the answers were pre-statistical, and only 7,7% of the answers were the descriptions of averages. For the second semester in Class A, 14,2% of the answers were statistical, %28,5 of the answers were related to the descriptions of averages and 50 % of the answers were pre-statistical. In addition, Class B, for the two semesters, did not have statistical answers. However, while 16,6% of the answers were the descriptions of average in the first semester, the percentage slightly increased to 20,8%. Such improvement could be evidence that the paths Beril chose while implementing CDSTs contribute to learning gains moderately or in high levels.

DISCUSSION AND CONCLUSION

The study aimed to investigate the implementation of CDSTs and how student learning occurs while implementing CDST. The results showed that when the teacher was provided with CDSTs, she implemented most of them in high level that in turn provides high learning gains to the students. She maintained the high level helping students to explain and to discuss their thinking. However, in certain tasks, she reduced the demand of the task in the enactment phases. Research explains that there are certain reasons for teachers to lower the demand including time constraints, teachers' knowledge and beliefs, and students' demands for simplifying the task (e.g. Charalambous, 2010; Stein & Lane, 1996). In the current study, it seems the teacher lowered the high demand because she made too many explanations to the students to complete the task easily without frustration. In addition, the teacher had a lack of particular insight related to the meaning of average (e.g. leveling tasks) so she could not help students to relate the concepts.

The study contributes to the literature on teaching statistics meaningfully through giving both teachers and students opportunities to implement CDSTs. Especially in Turkish context, there are limited number of studies investigating what teachers do or should do in the classrooms while teaching statistics. Therefore, this study informs researchers and authorities on what in-service teachers need to learn for maximizing learning opportunities for students. As the next step, a professional development program for in-service mathematics teachers might be developed to improve classroom practices related to teaching statistics through implementing CDSTs.

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