INDICATORS OF DIDACTICAL SUITABILITY TO EVALUATE THE STATISTICAL TRAINING OF FUTURE CHILEAN MATHEMATICS TEACHERS

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In the last three decades, a large number of countries have followed the international trend of incorporating statistics and probabilities throughout the complete school trajectory, leaving the responsibility for their teaching to mathematics teachers, who according to various investigations are not prepared for teach statistics or confront the difficulties of their students. Faced with this problem, following a qualitative methodology through a Content Analysis, in this work we present the process of construction of a Guide for the Evaluation of the Didactic Suitability of Educational Statistics Instruction processes for mathematics teachers. This guide is organized within the framework of Didactic Suitability Theory, in order to systematize the standards identified in search of improvements in the process of initial training of those teachers.

THE PROBLEM

Actually, statistic have been valued by various educational, political and social agents as a basic component to perform effectively in the information society. For this reason, the general tendency adopted by many countries has been to include it into the educational trajectory. The subject is introduced from an early age, through basic concepts of the discipline, until the end of compulsory education with aspects of statistical inference (Batanero & Borovnic, 2016). In this way, the success of the new curricular guidelines depends strongly on the training of those responsible for its implementation, that is, the teachers. They are the ones who ultimately perform the role of adapting and interpreting the new requirements according to the characteristics and conditions of their students. However, Batanero, Burrill & Reading (2011) state that many teachers consider that they are not well prepared to teach statistics or to confront the difficulties of their students.

In this sense, Godino, Batanero, Rivas & Arteaga (2013) approached the complexity of the design of mathematical and didactic training plans for future teachers. In that work, we can observe the evolution of the "Assessment Guide of the Didactic Suitability of Mathematical Instruction Processes" (AGDS-MI) by Godino (2013), towards the called " Assessment Guide of the Didactic Suitability of Educational Mathematics Instruction processes" (AGDS-EMI). This new instrument aims to be a tool for reflection on the instructional processes of mathematical-educational knowledge of training teachers.

Therefore, taking the theoretical structure that allowed the realization of the AGDS-EMI as the conceptual framework, the problem of assess the plans of educational statistic to future teachers is approached by means of the design of a "Assessment Guide of Didactic Suitability of Educational Statistics Instruction processes" (AGDS-ESI).

FRAMEWORK

The theoretical foundations that support this study are framed in the Onto-Semiotic Approach (OSA: Godino, Batanero & Font, 2007). The OSA is presented as an inclusive, open and dynamic theoretical system that, since its inception, has aspired to include in it the necessary and sufficient theoretical/methodological notions to investigate the teaching and learning processes of mathematics and statistics (Godino, 2012). Specifically, we take the Didactic Suitability Theory (Godino, 2013) from the OSA to confront the problem of design and assessment of training plans in educational statistic of future teachers.

The *notion of didactic suitability* is understood as a systemic criterion of relevance of an instructional process, which mainly values the convergence between the personal meanings achieved by students and the institutional meanings intended or implemented (Godino, Bencomo, Font & Wilhelmi, 2007). In this way, the suitability of an instruction process

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(programmed/implemented) is defined as the coherent and systemic articulation of the following six facets (Godino, Ortiz, Roa & Wilhelmi, 2011):

- *Epistemic suitability facet* refers to the representativeness degree of institutional significances, which are implemented (or intended), with respect to a reference meaning.
- *Cognitive suitability facet* expresses the degree to which the intended/implemented significances are in the potential development zone of the students, as well as the proximity of the personal significances achieved according to the intended/implemented significances.
- *Affective Suitability facet* degree of implication (interest, motivation…) of the students in the study process. Affective suitability is related to factors that depend on the institution and factors that depend basically on the student and their previous school history as well.
- *Interactional suitability facet.* A teaching-learning process will have greater suitability from the interactional point of view if the configurations and didactic trajectories allow to identify potential semiotic conflicts, and to resolve the conflicts that occur during the instruction process.
- *Mediational suitability facet* degree of availability and adequacy of the necessary material and temporary resources for the development of the teaching-learning process.
- *Ecological Suitability facet* degree to which the process of study adjusts to the educational project of the center, the school and the society and to the conditions of the environment in which it is developed.

Therefore, because the components of the facets of didactic suitability are not directly observable, it is necessary to infer them from empirical indicators, understood as a "*heuristic that considers the restrictions related to the context and the corresponding facet*" (Beltrán-Pellicer & Godino, 2017, p. 95). In this way, with the aim of build a tool that allows to evaluate the suitability of teacher educational statistics training, it is necessary to distinguish two focus from which it is possible to analyze this training process. A first focus, which allows to characterize the work of the future teacher according to the institutional learning expectations of its students, is considered according to the OSA as part of the Epistemic Facet of the GVID-IDE instrument. This focus is related to institutional knowledge about the teaching and learning of statistics, which considers various educational-statistical topics involved in the teacher's professional work with respect to future students, such as the statistical, cognitive, affective, interactional, mediational and ecological content. The second focus is contemplated in the Cognitive, Affective, Interactional, Mediational and Ecological Facets that involve the trainer with the trainee teachers. In this work, due to the maximum allowed extension, they will only present the results with respect to Epistemic Facet.

METHODOLOGY

This research is classified within the qualitative approach and it is considered exploratory descriptive type (Hernández, Fernández & Baptista, 2014). The technique used to collect information is the Content Analysis (Andréu, 2011) wich consider that the usefulness of the message fragment to be analyzed is the presence or absence of a content feature. In the context of our research, the aforementioned analysis is used to identify, classify, compare and infer didactic suitability standards. The inferred didactic suitability norms o rules consider two curricular documents, one of international consensus called 'Statistical education of teachers' (SET, 2015) and another specific for the Chilean reality called 'Guiding Standards for pedagogy careers in secondary education' (MINEDUC & CPEIP, 2012).

The first document corresponds to the Statistical Education report for Teachers (SET), developed by the American Statistical Association (ASA). This papes describes the content and conceptual understanding that teachers need to know to help their students develop statistical reasoning skills. The second one is intended to provide guidance to the training institutions of Chilean teachers, with respect to knowledge and skills that a future teacher from levels 7 to 12,

must demonstrate to teach statistics in the six grades that considers this level of schooling. Based on these norms suitability indicators, classified according to their six facets of interest, are inferred with respect to the teaching and learning processes of statistics for future teachers of mathematics, process that is detailed below.

At the beggining, fragments of text are identified and classified in Units of Analysis (UA), according to the different facets of suitability. Subsequently, these UA were subjected to assessment by expert judgment to guarantee the validity of content included in each category and to ensure its correct classification. This procedure involved two academicians with a wide path of investigation in the field of mathematics and statistics education, as well as a complete mastery of the theoretical framework and followed methodology. Thereby, the results of this assessment allowed a new revision of the proposed classification, relocating a small number of UA that produced discrepancy.

Then it was necessary to compare and reduce the UA included from the previous stage to avoid repetitions. The process carried out was able to identify and label the UAs that may be contained in another UA or that not provide new information. They were labeled with the expression "contained in" accompanied by the letter (a, b, c...) of the corresponding UA, to leave represented its content in a single final UA. In Table 1, the explained procedure is exemplified, where the letters do not follow a sequential order since their original classification is maintained and only those that are contained in others are shown.

Table 1. Example of the process of comparison and reduction of the UA

COMPONENTS	UNITS OF ANALYSIS
Situations-Problems	d. “Statistical issues must be developed through meaningful experiences with the process of solving statistical problems” (SET, 2015, p. 22)
	e. “The statistical contents must be developed through significant experiences with the process of solving statistical problems "(SET, 2015, p.30) *contained in" d "*
	g. " The development of the statistical thought must begin with a problem that could be solved through data" (SET, 2015, p.45) *contained in "d"*
	i. “Be familiar with the historical development of the solution of problems that originated the calculation of probabilities " (MINEDUC & CPEIP, 2012, p. 126)
	j. “Using strategies of resolution of emblematic problems in probabilities (…) it solves related problems " (MINEDUC and CPEIP, 2012, p. 126) *contained in "i"*
	k. “Solve problems of probabilities that involve uniform distribution "(MINEDUC & CPEIP, 2012, p.121) *contained in" i "*

Finally, after comparing and reducing the different UAs, we proceed to infer "Didactic Suitability" indicators from the included fragments, considering that: (1) Two or more UA can produce a single indicator, and (2) One UA itself can produce one or more indicators. Thus we obtain a first proposal of indicators to assess statistical training processes for future mathematics teachers’, which is exemplified in Table 2. It is important to mention that with the intention of proposing a guide that can be used to evaluate both training actions as well as plans, the indicators have been written in a global sense by replacing expressions such as "solve" by "promote" or "include", since the first one could only be applied to a study process implemented and not in the design stage.

Table 2. Example of the inference process of suitability indicators

UNITS OF ANALYSIS	INDICATOR
a. "The problem-solving framework emphasizes the omnipresent variability of data and recognizes its role within	• Significant problems of the

each component" (SET, 2015, p. 21) b. "Statistical issues must be developed through meaningful experiences with the process of solving statistical problems" (SET, 2015, p. 22) c. "To model valuable problems of the real world" (SET, 2015, p. 31) d. "Emphasize several components of the process of solving statistical problems: formulate questions, collect data, analyze data and interpret results" (SET, 2015, p. 43) e. "Be familiar with the historical development of the solution of problems that originated the calculation of probabilities" (MINEDUC & CPEIP, 2012, p. 126)	real world are included and they could be developed by means of the process of resolution of statistical problems (PPDAC), emphasizing the omnipresent variability. (UA a, b, c, d) • The use of historical problems that originated the calculation of probabilities (UA e) is promoted

RESULTS

After the process described previously, we have obtained a first version of the GVID-IDE that considers the six corresponding facets, along with possible interactions between them. However, due to the maximum length allowed for this work, we will present in Table 3 part of the indicators that make up the Epistemic Facet, specifically those referring to the statistical, cognitive and affective contents that intervene in the teacher's professional work with respect to their future students.

Table 3 Indicators of Didactic Suitability of the GVDSPI-DS

COMPONENT	STATISTICAL CONTENT
Situations-Problems	• Significant problems of the real world that could be developed by means of the process of resolution of statistical problems (PPDAC) are included, emphasizing the omnipresent variability. • The use of historical problems that originated the calculation of probabilities is promoted.
Languages	• Different representations are included to explore, summarize and describe patterns of variability in categorical and quantitative univariate data (tables, graphs, numerical summaries). • Various representations are proposed to describe association patterns of two variables, either categorical (two-way tables) or quantitative (scatterplot). • The use of two-way tables, tree diagrams and other representations for the teaching of probabilities is proposed. • The use of written or oral language is promoted to communicate clearly and accurately with the statistical language.
Rules: Definitions, propositions and procedures	• Work is promoted with statistical models that describe the variability of the data (data = structure + variability). • Transforming the research questions into statistical questions is proposed (which can be solved by means of data in the PPDAC cycle). • Identifying and distinguishing the different types of variables and data when approaching a statistical question. • The concepts, procedures and fundamental properties of descriptive statistics are included. • The main concepts, procedures and properties of probabilities and random variables are included. • Primordial concepts and procedures are promoted to model the association between variables through regression analysis. • The main concepts, procedures, techniques and properties of parametric statistical inference are promoted.

	• It includes differentiating situations where an observational studies or a comparative experiment is necessary.
Arguments	• The construction of viable, clear and precise arguments with the statistical language is promoted. • It includes evaluating the plausibility of alternative conclusions. • The distinction between the correct statistical reasoning and the defective one is promoted.
Relations	• Establishing connections between the design of the study and the interpretation of results is promoted. • Interpreting the results of a statistical study with aspects such as the bias and the scope of the inferences is promoted. • Relating the distribution of the data and its variability with biased and unbiased models is promoted. • It includes connecting the simulation with inferential procedures. • It promotes the assessment of context and variability throughout the process of statistical problem solving. • It promotes giving value to the role of data and the need to produce them considering the variability.
COMPONENT	COGNITIVE CONTENT
Previous knowledge	•Working from informal reasoning to introduce the understanding of topics of greater difficulty for students is proposed. •Developing not only the knowledge of the disciplinary content, but also its progression and connection between the different school levels is promoted.
Curricular adaptations according to individual differences	•It includes identifying learning styles, special educational needs and specific talents of students. •The relevant use of the differentiated evaluation is promoted. •The inclusion of all students with their respective social, sexual, ethnic, physical appearance and academic development is promoted.
Learning	•Being familiar with the conceptions, difficulties and common mistakes of students when learning statistics, probabilities and inference is promoted. •Focus on the students of the school system, its characteristics and modes of learning are encouraged. •The use of teaching and evaluation strategies to develop learning is promoted.
COMPONENT	AFFECTIVE CONTENT
Interests and needs	• The interests and motivations of students are considered in the proposal of problems and statistical tasks. • Tasks and activities that recognize the importance of discipline in society are promoted. • Real situations are analyzed to identify ways of thinking, feeling and acting of students.
Attitudes	• A positive attitude towards the resolution of statistical problems is promoted. • Attitudes such as perseverance and intellectual and moral growth of students are promoted.
Emotions	• The physical and emotional safety of the students is promoted.

CONCLUSION

As a closure, it is important to note that the indicators presented correspond to the partial results of the epistemic facet of the construction process of the GVID-IDE. These results have to be compared with the specific literature where this type of instruments have been designed (for example, Godino et al., 2013; Godino, 2013; Rivas, 2014) and have to be checked with specific research on the training of secondary school teachers (or equivalent levels) in the statistical and probabilities axis. At the end of this stage, we will propose an instrument that allows us to evaluate teacher training processes in the statistical and probability subjects and its didactics, which is expected to be a valuable input for both teacher trainers and those responsible for designing or evaluating training plans for future teachers.

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