HIGH SCHOOL, TINKERPLOTS, FATHOM… AND INDEPENDENCE AND ANOVA. HOW COME?

Hugo Hernández Trevethan
Colegio de Ciencias y Humanidades, Plantel Vallejo. Av. 100 metros S/N, esq. Fortuna, Col Magdalena de las Salinas, Del Gustavo A. Madero, C. P. 07760, Ciudad de México
animal_estocastico@hotmail.com

Formal concepts on Inference are fundamental on teaching Statistics since Inference is the summit of the statistic process, but at the same time those concepts are difficult to learn. A possible solution is aiming formal concepts from informal inference, supported by the use of software with contextualized data. A course was given to senior high school students to develop the basic ideas about what ANOVA and independence proofs are and how to interpret their outcomes. Some of the students discriminated when to use different tests and to give an interpretation to their outcomes. Since those subjects are not taught at high school, it seems possible to develop higher concepts on Statistics at that level by going from informal to formal inference using software.

INTRODUCTION

On recent times statistic education has focused on the use of technology with real and contextualized data, informal inference and statistics literacy –assumed this as knowing how to construct, manipulate, interpret and critically evaluate statistical information-.

The importance of using computer when teaching Statistics has been pointed widely. It's common nowadays to have Statistics classes with a computer projected on a screen or in a laboratory, students may own a powerful calculator or a portable computer, and even on-line courses are available; since technology has changed the way statisticians work, it has also changed the way Statistics is taught (Chance et. al. 2007).

Speaking about teaching concepts using technology, Konold (2002) highlights computer's power to illuminate key concepts through simulations and multiple-linked representations which eventually allows focusing on the process of data analysis, exploring questions of interest, searching for and interpreting patterns and trends and communicating findings.

About the informal inference, Rodríguez (2012), quoting Rossman (2007) and Ben-Zvi (2006), defines it as the capacity of going beyond data at hand, trying to explain or justify those observed data through reasoning which does not use any formal method, technique or calculation, and to point at the necessity of the evidence revealed by data in order to argue under this kind of reasoning. In addition, and in order to differentiate, Watson (2008) quotes Moore (1991) when defining statistical inference as "formal methods for drawing conclusions from data taking into account the effects of randomization and other chance variation". Besides, Garfield & Ben-Zvi (2008) divide statistic inference in two great subjects, estimation and test of hypothesis, and two questions towards inference, which are the generalization from samples and comparison and determination of a cause.

How deep can statistics education may go from this points depends on scholar program, which clearly varies among schools and grades. In the Mexican case, there's one high school system –Colegio de Ciencias y Humanidades (CCH), depending on the National Autonomous University of Mexico- which has being teaching statistics up to estimation and test of hypothesis both for one mean and one proportion to senior students, and nowadays the use of technology, informal and formal inference and statistics literacy are present in its syllabus (Ávila et. al., 2005). The subjects of formal inference consider the basic concepts of statistical inference, their applied nature and the subsequent interpretation, which raises the question about if it's possible for high school pupils to get involved in some other themes of inference, particularly ANOVA and test of independence at the level of understanding when those test are to be used, what do they measure and how to interpret their outcomes obtained from some statistical software, avoiding all the algorithms involved on each proof.

For this study, a group of high school senior students were taught along a regular course on Statistics at the CCH up to the topics of estimation and test of hypothesis using two software altogether, TinkerPlots (University of Massachusetts, 2014) and Fathom (KCP Technologies, Key

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Curriculum Press, 2007). At the end of the course, three subjects out of the scholar program were discussed: Test of hypothesis for two means, ANOVA and test for independence. Then two test were applied; one in which the students had to discriminate when to use those test for different situations, and a second one in which they had to interpret an outcome for each of the test within a context.

PROBABILISTIC IDEAS AND DIDACTICAL CONSIDERATIONS

As stated, some subjects were chosen for taking the students further on the scholar program, under the idea of going from informal to formal inference using software. The topics and the teaching methodology were not randomly picked.

Two fundamental stochastic ideas

Batanero (2004) highlights as fundamental stochastic ideas both conditional probability and independence, among others. Speaking about independence, idea closely related to conditional probability, the same author states "Even when the mathematic model of independence, expressed by the rule of product, is easy to understand on daily life, a lot of people, including scientist, are not able to apply this idea consistently in practical situations. […] The fact that people picks just one lottery number and gamble to it every year or that they try year after year to invent new methods for discovering "patterns" for the roulette or some other games of chance are other examples of the intuitive difficulty of the idea of independence" (Batanero, 2004). About conditional probability, the same author points that this concept "formalizes the change in our degree of belief whit new information"; in addition "it's not always assigned in absence of information, but when we have knowledge about the involved events, which we may use to improve our assignation", and "the idea of conditional probability will be frequently used in inference and other statics methods, for example in the study of association of variables. Its correct comprehension will prepare the student to get into all of them" (Batanero, 2004).

With all this in mind, we can assume both the importance of learning these concepts and their difficulty within their comprehension.

Independence and conditional probability at high school

One point to consider is if the concepts of independence, conditional probability and test of hypothesis fit into high school level. There are scholar programs in Mexico and abroad as well in which those topics are openly established. Just two of them are to be mentioned. The first one corresponds to the CCH, as stated earlier; in its program some educative porpoises, defining those concepts, may be found (Ávila et. al., 2005). The second case lies on the Common Core State Standards Initiative (2015), which point at understanding independence and conditional probability and use them to interpret data, understanding and evaluating random processes underlying statistical experiments, and making inferences and justifying conclusions from sample surveys, experiments, and observational studies.

This two examples are nor exhaustive, but allow to think that the ideas of independence, conditional probability and test of hypothesis suits to the high school level.

Some didactical considerations

Teaching informal and formal inference has been discussed by several researchers. Zieffler et. al. (2008), quoting different authors, note the difficulties that people experience to make inferences about uncertain outcomes, in particular about comprehending and interpreting proofs of significations, p-values and concepts related to inference; at the same time, and given both the importance of reasoning in inference and the difficulties that students experiment with it, they point at some attempts to expose pupils to situations in which they may use informal methods to make statistics inferences.

Rubin, Hammerman & Konold (2006) state that most of statistics educators agree that inference is the main objective of the statistical reasoning but at the same time is one of the most difficult ideas for the pupils to understand; they also note that in the traditional approach, inference is presented as a quantitative problem, usually finding the probability of getting the observed outcome assuming the null hypothesis as true, instead of focusing on certain key properties in the statistical process.

To Ben-Zvi (2006) informal inference is closely related to argumentative activities since obtaining conclusions from data implies to sustain the argument on data analysis, as for Watson (2000) judging statistical statements within social context is fundamental for the statistical literacy, while this author proposes a hierarchy to categorize the abilities needed to correctly interpret statistical information, which its highest level states a critical attitude that may apply more sophisticated concepts to argue against postures that may not have the proper statistical fundaments.

Zieffler et. al. (2008) suggest challenging students whit tasks that imply to make judgments or predictions about a population, based on samples, but without formal procedures; to recover, use and integrate previous formal and informal knowledge; and to establish arguments based on evidences for their judgments, postures and predictions about population, based on samples. One of those possible tasks is judging which of two models is more likely to be true.

THE PROCESS

Whit all that has been previously exposed in mind, a full course in Statistics was given to a group of 46 senior high school students at the CCH, following the official scholar program (Ávila et. al., 2005). The course was based on didactical activities, designed under the theory of didactical situations stated by Brousseau (2002), and using in each one of them both TinkerPlots (University of Massachusetts, 2014) and Fathom (KCP Technologies, Key Curriculum Press, 2007) software *combined*. The main idea at combining both tools was precisely going from informal to formal inference, while the concepts and some operative tasks were discussed along the classes. After the whole scholar program was exhausted (including the ideas of test of hypothesis in general and not just for one mean or one proportion, and the p-value), the group worked with the topics of test of hypothesis for two means, ANOVA and test for independence under the same scheme used along the course. Then the class was faced at two instruments, allowing the students to work in groups of 2-4 students (with one student working alone); in the first instrument four situations were presented, and they were asked to state the proper statistical hypothesis –arguing their answers-, being the four situations tests for one mean, comparison between two means, ANOVA and test for independence; in the second instrument three Fathom outcomes were presented, one for each test among two means comparison, ANOVA and test for independence, and the students were asked once again to state the statistical hypothesis involved and to give the interpretation of the outcomes within the correspondent context, once again arguing their answers; at the second instrument they were also asked to define what the p-value is using their own words.

The results were classified using SOLO taxonomy (Biggs, 1982).

RESULTS

15 groups worked with the instrument. Using the SOLO taxonomy, the results showed groups at the prestructural, unistructural, multistructural and relational levels. Most of the groups were classified within the second and third levels of the taxonomy. Previous experiences working just the topics of the scholar program and using just Fathom has showed more or less the same results but going no further than test of hypothesis for one mean and one proportion. It's clear that the structure given to the course allowed going further on the scholar program, covering important subjects on formal inference from the perspective of statistics literacy, but it's also clear that a significant part of the class showed a poor level of understanding of the mentioned topics, even when it wasn't intended for the students to assimilate the underlying mathematical processes of the proofs nor the formal demonstrations proper of the topics´ theories. One possible reason may be that the time the students had to assimilate and maturate the concepts of the new topics was insufficient.

CONCLUSION

The goals for the study were a) to find out if it's possible for senior high school students to understand some ideas of the formal inference -particularly those involved with ANOVA and test for independence- when taught from informal inference to formal and using TinkerPlots and Fathom combined, and b) to find out if by the end of the experiment the students were able to differentiate when to use ANOVA and independence tests and what for, and how to interpret their outcomes within a context. The results show that some of the pupils actually understood the concepts and were capable to apply them in a contextualized situation, justifying their statements, even when the mentioned topics are not traditionally included in the high school curricula. Since a significant part of the students were not able to do so, a possible future study is trying the same approach but devoting more time to the extra topics in order to give the pupils the opportunity of maturing the involved ideas.

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