IS DATA A QUANTITATIVE THING?
AN ANALYSIS OF THE CONCEPT OF THE MODE IN TEXTBOOKS FOR GRADE 4-6.

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This paper presents analyses of seven popular Swedish textbooks for grades 4-6 examined for their presentation of statistics, focusing on average mode. Since textbooks in mathematics have great impact on both teaching and content in Swedish classrooms, it is of interest to examine both definitions and tasks. Tasks and definitions were analysed to identify quantitative or qualitative context. Tasks were also analysed whether they evoke procedural or conceptual knowledge. Findings suggest an excess of quantitative data in the context of the tasks and definitions, which may be a consequence of the ambiguity of vocabulary used. Findings also show an overwhelming focus on procedural knowledge and, in some cases, doubtful use of levels of measure in the tasks. These findings have implications for future understanding.

INTRODUCTION

Textbooks play an important role in teaching and learning, as instructional design of textbooks might influence the students' understanding of concepts (Bryant et al., 2008). This is true for Sweden as well: textbooks have a large impact on mathematics education in Sweden (Boesen et al., 2014). Teachers trust the textbook to cover what the students should learn and are synonymous with what the teacher teaches (Johansson, 2006). Furthermore, it is common in Sweden that students work individually, approximately 60 % of the lessons, mainly with tasks from the textbook (Boesen et al., 2014; Johansson, 2006). Johansson (2006) found that teachers even in the public parts of the lesson used definitions, background and procedures for discussion from the textbook. Often templets on proposed algorithms are given to the tasks (Lithner, 2017) and an international study involving textbooks from twelve countries show that 79% of the tasks are of the type that apply a given procedure (Jäder et al., 2015). Due to these results, a textbook analysis of Swedish textbooks is performed concerning mode. Groth & Bergner (2006) highlighted the lack of research on the learning of averages, particularly focus on mode. Thus, mode is the choice for this paper to ascertain a better understanding of how it might be presented to students through their textbooks. Mode can be presented through both qualitative and quantitative data, evoke both procedural and conceptual knowledge, and provide clear and meaningful definitions to work with. The paper will address the following research question: What knowledge, procedural or conceptual, and quantitative or qualitative context, do textbooks in years 4-6 afford Swedish students on the concept of mode?

THEORY

The tasks in this study have been analysed if the solution require procedural or conceptual knowledge. Procedural knowledge consist of the formal language, or of algorithms and rules (Hiebert & Lefevre, 1986), and many tasks implies students to apply a given procedure or recalled algorithm (Jäder, Lithner, & Sidenvall, 2015; Lithner, 2017). The same goes for averages where the rule/algorithm is close to the definition (Groth & Bergner, 2006), especially for mode, since mode is less complex to calculate compared to the procedures for mean and median (Groth & Bergner, 2006). Conceptual knowledge is rich in relationships, and often described as a web of knowledge linked through relationships (Hiebert & Lefevre, 1986), and the conceptual challenge in a task concerns what conceptual knowledge is in play (Lithner, 2017). For averages that could be an understanding that all three averages are "used to find central and/or typical values of data sets" or to "recognize that in some instances one measure is more suitable than another" (Groth & Bergner, 2006, p. 39). When solving a task the conceptual knowledge "determines how the advanced mathematical properties (such as in representations and connections) of the task need to be understood in order to construct a solution" (Lithner, 2017). For the concept of mode, the mathematical properties are: treats both qualitative and quantitative data; does not have to be unique or exist; and is not affected by extreme values. Below, is a description of two different webs of knowledge concerning mathematical properties of averages: Firstly, there is a risk to apply an algorithm without taking the context into

In M. A. Sorto, A. White, & L. Guyot (Eds.), *Looking back, looking forward. Proceedings of the Tenth International Conference on Teaching Statistics (ICOTS10, July, 2018), Kyoto, Japan.* Voorburg, The Netherlands: International Statistical Institute. iase-web.org

account (Jacobbe & Carvalho, 2011). It is only in a quantitative context that all three averages are available, why it is important to be able to formalize the distinction between the different levels of measurement (Groth & Bergner, 2006). Qualitative data can be either nominal or ordinal, and both have their issues. When students try to find averages for nominal data, there sometimes is a willing to order that could lead students to do mistakes, for instance ordering nominal categories as they use different systems, for instance idiosyncratic rules, trying to determine a median (Groth & Bergner, 2013). Ordinal data have on the other hand a natural order, as for size of shoes, 38 is larger than 37 because of a hierarchic order. What makes this scale ordinal is that the scale is not equidistant. For this reasons the only possible average for qualitative data is mode. Sometimes median is applied on ordinal data but it could be problematic. If we have the answers poor, fair, good and excellent, the median of fair and good cannot equal fair and a half, not even one assigns integers to represent fair and good (Kuzon Jr, Urbanchek & McCabe, 1996). Secondly, there is an issue that different averages represent different facts of a data set and one needs to know under what conditions different averages are suitable (Groth & Bergner, 2006). Using a formula or a definition is only a step in the development of an average towards how it can be representative in a data set (Watson & Fitzallen, 2010). The property that mode is not affected of extreme values, is an important knowledge when contrasting mode to other averages (Groth & Bergner, 2006). Another issue is that students sometimes confuse the variable value with the frequency (Watson, 2014). This leads to a numerical answer of the mode instead of answering with the category. For instance that the student answer that the mode is 8 instead of blue. Also the representations could affect the cognitive level in a task. Often tasks present data in a row of numbers, in a table or in a graph. This could cause difficulties finding the mode, or other averages, when the students are to interpret new graphical representations of data (Leavy & O'Loughlin, 2006). Furthermore, use of words might be a hurdle to the student if the concepts are lexical ambiguous (Richardson, Dunn & Hutchins, 2013), especially wording of a definition is important as this could limit the knowledge for the student (Perrett, 2012). One example is the Swedish concept typvärde (mode) consisting of two common words typ (typical) and värde (value). The ambiguity might disturb the student´s conceptualisation since värde strongly denotes something quantitative. For mode that means that information of no/one/more than one mode and that it applies for both qualitative and quantitative data is important in a definition. Choice of textbook is in this sense vital for afforded knowledge, as there are differences among definitions and tasks, even between different editions of the same textbook (Perrett, 2012).

METHODS

The empirical data is analysed deductively through a thematic analysis (cf. Braun & Clarke, 2006). This explicit analyst-driven (top-down) approach is drawn from the research question and specific analytical focus on the tasks related to mode. Each task was coded through levels of measurement and cognitive demands, whereas, the definitions were coded for specific vocabulary and levels of measurement in given examples. Seven commonly used Swedish textbook series, for years 4-6, were analysed in this study. The purposive selection of textbooks is based on popularity, in that analysing the most popular textbooks, offers the field insights into what Swedish students are offered as example tasks for learning mode. All text, graphs and tables, exercises and tasks on statistics were analysed in all books. The textbooks contained different exercises and problems with varying demanding on students, e.g. apply a given procedure, compare different averages or interpret a diagram. The smallest division in these activities were considered a task (Jones & Jacobbe, 2014). Type 1 shows a task that out of this definition equals one task; e.g.: The result of ten rolls of the dice was: 3, 1, 5, 3, 6, 1, 4, 3, 2, and 3. Calculate the mode. Type 2 equals two tasks; e.g. Calculate the mode for the ages: a) 7, 5, 5, 4, 8 and 5, b) 3, 4, 4, 2 and 4. In type 3, there are several questions, but only one on mode; e.g. A die is rolled five times with the result: 3, 1, 5, 1, and 6. Find out the a) mode b) median c) mean. Finally, type 4 tasks on mode that do not explicitly ask for mode; e.g. A table and a bar graph is showing the frequency of different colours as follows: Blue, 3, White, 4 and Green, 2. Which colour was the most popular? The tasks were then coded for: levels of measure: qualitative (Ql) or quantitative (Qn) data, and whether they covered: procedural (P) or conceptual (C) knowledge, which provide four codes QlP, QlC, QnP or QnC. Tasks coded as procedural knowledge are close to definition or algorithm, tasks coded as conceptual knowledge deal with mathematical properties as described. Some tasks were open in their character and could give answers on more than one code for example, sometimes there was no mode in a set of data, but students were required to explain why. Fisher´s exact test was made to test the null hypothesis that there is no difference between the distributions of the different types of tasks. If $P < 0.05$ we reject the null hypothesis. To ensure high reliability one other researcher coded a sample of every eighth task followed by changes concerning a few tasks with ordinal data. At first, these tasks were coded as qualitative, but since the tasks were treated as quantitative by the textbook, an agreement was made that this was the impression given to the student, and these tasks were recoded as quantitative. One example is a task on shoe sizes, asking the student to calculate both mean, median and mode. Finally, definitions were analysed out of use of words, for instance value or number, amount of modes and if exemplified with quantitative or qualitative values.

RESULTS

The result show that approximately 75% of the tasks are on quantitative data and in total 81.9% of the tasks are considered procedural. The result also shows that of the tasks that attend to conceptual knowledge (18.1%), 12 % are on qualitative data (QlC), corresponding to approximately 2.2% of all tasks. The proportion of the qualitative tasks that are conceptual is approximately 8.8%, the corresponding figure for quantitative data is 21.2%. Of the 62 QlP-tasks 46 are of type 4, tasks that do not ask explicitly for mode but for example for the most popular or the most frequent.

Table 1. Comparison between different types of tasks.

	Procedural	Conceptual	Fishers exact test
Quantitative	164	44	P = 0.0283
Qualitative	62	6	

The Fisher´s exact test show a two-tailed P value < 0.05, why we reject the null hypothesis. The result is statistically significant, indicating that there are difference between the distributions of the tasks. See Table 1 above. The analyse of definitions on mode show that six textbooks provide definitions describing mode as the value that is most usual, occur most times or is the most of. One textbook defines it as the most frequent number. No definition illuminate that there might be none or more than one mode. In six of the textbooks, examples adjacent to definitions give only numerical examples with one mode. One textbook add in the definition that values do not need to be quantitative and give qualitative examples with more than one mode.

DISCUSSION AND IMPLICATIONS

The aim of the study was to investigate definitions and tasks of the concept of mode in Swedish textbooks. Consistent with the findings from Jäder et al. (2015) the textbooks have an excess of procedural tasks. These tasks, as well as the conceptual tasks are predominantly quantitative. The fact that textbooks have significant influence on the students' understanding of concepts (Bryant et al., 2008) and the frequent use of textbooks in Sweden (cf. Boesen et al., 2014)), indicates a risk, considering the results, that students conceptualisation of mode will have a strong focus on procedural knowledge and quantitative data. The tasks considered as conceptual discussed the representativeness of averages in a dataset, the first kind of the mathematical properties. There were also a few examples of the third kind comparing datasets with or without extreme values. The second mathematical property, more than one mode or no mode at all, is only present in one of the textbooks. Depending on the choice of textbook, students might not meet tasks on QlC at all. In this study, 5 out of 6 QlC-tasks were in the same textbook series. The initial findings also illustrate a misuse of levels of data, and as teachers trust the textbook according to Johansson (2006), this could lead to misconceptions, for example: where the textbook treats ordinal data as quantitative. In addition, the use of definitions affect students learning depending on the formulation in the textbook. In Swedish, the average mode is called typical value (typvärde), and value (värde) might signal numbers, hence could be confusing. Most of the textbooks use the word value in the definition, but only one is explicit with the meaning of the word. One textbook defines mode as the most frequent number. Just as Richardson et al. (2013) suggest, the use of problematic language in explanations and, or definitions might imply implications of possible lexical ambiguity. If teachers, as Johansson (2006) findings show, use definitions and background for discussion from the textbook, students might get an incomplete concept image. Use of a textbook that defines mode as the most frequent number might limit the students´ conceptualisation of mode. Other evidence strengthens this image to the argument through the use of a higher degree of quantitative examples. The students´ thereby might not be given enough experiences on mode with qualitative data. The fact that definitions express there is one mode, gives an incomplete picture, since there could be more than one mode, or even none. These findings are in line with Perrett (2012), who state that misconceptions can arise from the formulation of a definition. When textbooks present averages, qualitative data and graphs are often already worked with. It is in these circumstances most of the OlP-tasks were presented, though not explicitly asking for mode. There were only a few exceptions where the mode was specifically asked for. Most of these tasks connected the highest bar with for instance the most popular. Later on, the textbooks introduce mean, median and mode in quantitative contexts. This may suggest why mode is asked for in quantitative contexts. Another issue is the misuse of ordinal data as quantitative, something that needs further examination.

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