

## A GLIMPSE OF TWO YEAR COLLEGE INSTRUCTORS' UNDERSTANDING OF VARIATION IN HISTOGRAMS

Monica Dabos

College of the Canyons, Santa Clarita, CA, USA

[Monica.dabos@canyons.edu](mailto:Monica.dabos@canyons.edu)

*Statistics education researchers are urging teachers of statistics to help students develop a more sophisticated understanding of variation, since variation is the core of statistics. However, little research has been done into the conceptions of variation held by instructors of statistics at the post-secondary level. This exploratory study was designed to map the conceptions of variation held by two-year college mathematics instructors. A total of 52 instructors from 33 different California community colleges responded to a survey designed to reveal instructors' conception of variation. The results indicate that a tendency to focus only on the center of the distribution seemed to prevail and very few instructors gave explanations integrating different aspects of the distribution. The majority focused on the center or on the range, and instructors' lack of consideration for context was also notorious. This study has opened the gate and laid the groundwork for understanding conceptions of variation held by two-year college instructors.*

### INTRODUCTION

The reality of introductory statistics courses at two-year colleges in the United States exposes the challenges that teaching and learning statistics at this level face. According to the Conference Board of the Mathematical Sciences (CBMS) report in 2010, there were 137,000 students taking introductory statistics courses at two-year colleges in the United States, yet only 2% of full time and 2% of part time instructors have a degree in statistics. While the degree may not reflect the capacity of an instructor to teach statistics effectively, its lack does reflect a paucity of representation of statistics experts in the field of teaching and learning at the college level. Instructors face additional complications because introductory statistics courses have been labeled the most challenging course in the undergraduate curriculum for students (Delucchi, 2007). Also, Yilmaz (1996) states "statistics is a difficult subject for non-specialists, not just from the viewpoint of the student but from the teacher as well" (p. 2).

Empirical studies have extensively, but not exhaustively, investigated students' difficulties in statistics at all levels of the education system (Batanero, Godino, Vallecillos, Green, & Holmes, 2000; Meletiou-Mavrotheris & Lee, 2005; Stevens & Palocsay 2012). Students' difficulties have been identified as the result of researchers' endeavors to map out students' struggles. In almost every case, the researchers at the end of their investigation provided a guide for those teaching statistics. The results of these studies on students' difficulties not only provide practical applications to those teaching the subject, but they also demonstrate the central role that instructors play in minimizing students' difficulties and in fostering students' understanding. Some researchers suggest "teachers have the same difficulties with statistics concepts as the students they teach" (Shaughnessy, 2007, p. 1,000). If the goal is to minimize students' misunderstanding and maximize students' statistical literacy, then there is an urgent need to seriously investigate the statistical knowledge of those teaching introductory statistics courses at two-year colleges.

### METHODOLOGY

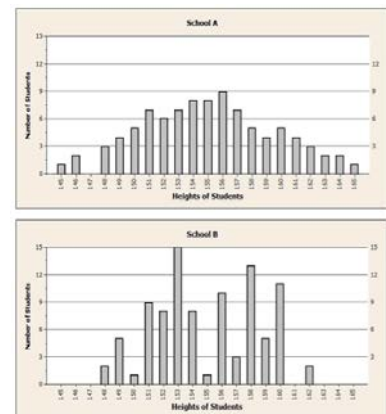
This paper provides a glimpse of two-year college instructors' understanding of histograms. The results presented here are part of a larger study that involved ten survey questions and six interview questions. From the set of 16 questions only the four questions dealing with histograms will be presented here. A total of 52 professors from 33 different California colleges participated in the study; 23 of them had never taught statistics (M) and 29 had taught statistics for several years (MT). Seven of the latter held a statistics degree (ST). Since the justifications for their choices reveal more about instructors' thinking than a correct or incorrect answer does, instructors' justifications will be the focus of this discussion. The study consisted of two instruments - a written survey answered by all instructors and an interview that was answered by only 12 instructors, seven of whom had statistics teaching experience. Of the four questions discussed here, two were from

the survey (Question A.4 and Question A.10), and two from the interview (Question B.3 and Question B.4).

Question A.4 prompted instructors to determine which histogram showed more variability in students' heights. Question A.10 was also designed to assess instructors' ability to identify histograms with greater or smaller variability. Question B.3 (interview) asked instructors to recognize the best display of the proportion of hits from baseball players so that the shape, the center and the spread of the data could be easily described (this question is part of the CAOS assessment). Question B.4 was designed to investigate instructors' reactions to students' misconceptions with histograms. These four questions asked instructors to justify their answers either by writing (in the survey) or by verbalizing their thinking (interview).

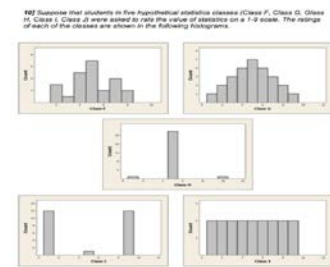
RESULTS

Question A.4 prompted instructors to determine which school had more variability in students' height and to explain their reasoning. The results show that even when giving the correct response, instructors did not always use all aspects of the distributions to substantiate those answers; the majority used only one aspect of a distribution, namely the range. The responses to the first part of the question were coded as being correct if instructors identified School A as having more variability and incorrect if they identified School B as having more variability. The results from this part of question A.4 seem very encouraging since 77% (n=40) of the instructors were able to correctly identify that School A had greater variability, while 19% (n=10) incorrectly identified School B as having more variability or decided that both were equal. However, while many instructors chose the correct graph (School A), their justifications revealed limitations and misunderstandings. For example: "School A. Greater range/ though comparing other measures might give different answers"; "The range is greater in A. I could be wrong since I didn't calculate the standard deviation"; "Graph A has slight increases or decreases going from each height to the next. Graph B has larger jumps going from each height to the next."



The most common reasons for choosing the wrong graph (School B) were given by instructors who decided that school B had more variability because it was not normally distributed; for example: "Pure bell vs. bi-modal", "Because School A is more normal in distribution, with less variation". Some of the instructors' reasoning was more difficult to comprehend. For example, a few instructors wrote that their decision was based on the spread of the data from the center, which is the right conception of variation, but they used this understanding to choose the wrong graph (School B). For example, ST22 chose School B as having more variability, and wrote, "top graph shows more concentration around the mean." This instructor seemed to ignore the frequency of the values because there were more observations in School B in the center range (150 to 160) than in School A and there were more observations outside that range in School A than in School B. His/her judgment is therefore unsubstantiated.

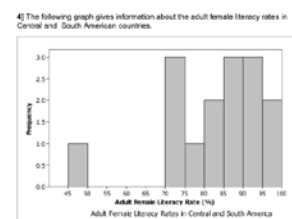
Question A.10 was also designed to assess instructors' ability to identify histograms, which had greater or smaller variability, and to find out what aspects of the distribution they paid attention to for such identification. Several instructors recognized Class G as having greater variability, but their justifications focused mostly on the range of the distribution and showed a limited consideration for variation. The answer was coded as being correct if the instructor identified Class G as having greater variability and incorrect if they identified Class F as having greater variability or stated that both had the same variability. About 48% (n=25) of instructors were able to correctly identify Class G as having greater variability, while 44% (n=23) said that Class F had greater variability, 2% (n=1) said that they were equal, and 6% (n=3) did not respond. It is interesting to note that even though the question is similar to Question A.4, a smaller



percentage of instructors identified the correct graph in this question. This suggests that the limitations found in instructors' previous justifications are now becoming apparent in their inability to recognize the graph with more variability.

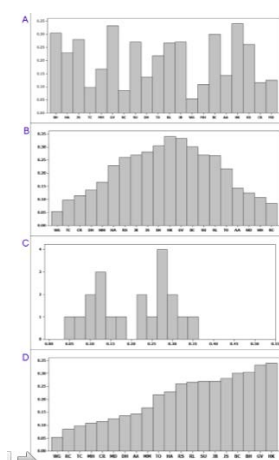
Looking at the justifications given by instructors who correctly identified Class G (n=25) as having more variability reveals that only a few instructors (n=6) used several aspects of the distribution, indicating that while the majority was able to recognize the appropriate graph, they gave very little consideration for all aspects of the distribution. Instructors who indicated erroneously that Class F had more variability (n=23) included justifications that showed misconceptions. *"F because there are more values out to each extreme in the x distribution"; "F has more variability because there are larger jumps", "F because it was not normally distributed".*

Question B.4 (interview) was designed to investigate instructors' reactions to students' misconceptions with histograms. In order to understand students' misconceptions, instructors had to first demonstrate consideration for context and realize that the histogram was portraying, in the x-axis, the female literacy rate of South American countries, and realize that the y-axis showed the number of countries with a particular range of literacy rate. Instructors' responses were considered correct if instructors recognized that the student was wrong, and also identified the student misconception, namely that the student was counting the numbers of bars instead of adding the frequencies to find out how many countries were represented in the data.



Most instructors were able to recognize students' errors and correctly identify the reasons for students' misconceptions. However, it was surprising to find instructors who had difficulty with this question who teach statistics (ST17 and MT10). *"So I see they add the frequency 1 plus 3 plus 1 plus 2 that is probably more than seven ah? wrong ah..... students.... 70 in the middle and they move the decimal. I don't know why can have 70 countries but that is my best of what student where thinking. Yes, I think that is right....Oh it is Central and South America. I don't think you have 70 countries there I bet there is only 20 countries total so good intuition student have"(MT10); "I will say wrong. Student is thinking he or she just added the frequency [pointing to the y-axis and counting]. No does it not it.... [Reading the question]... Conclusion, umm.... I don't know why somebody would have chosen seven, I can't tell why..." (ST17). While difficulties with this question from instructors who do not teach statistics may be less surprising, they are still noteworthy. "y-axis is telling me the frequency [pointing to x-axis] what is this telling me? This is the age? No, 45 to 50... Make sense the age. Okay, adult literacy rate between 45 and 50. These are the ages. No, this is literacy rate. Don't worry about age. So they are two percent literate age. Well this cannot be the age because there is no info about age. Adult so can be... This frequency... How frequent, like a probability. This is percentage. The student is wrong, in Latin and South America there are more than seven countries"(M14).*

Question B.3 aimed to identify if instructors could recognize the best display of the proportion of hits from baseball players so that the shape, the center and the spread of the data could be easily described. This question presented a challenge for instructors. It is possible that the difficulty derived from the fact that several of the graphs presented looked like histograms (A, B and D), as they had bars with something in the x-axis and something in the y-axis, but since the x-axis is not a quantitative measure in any of the three graphs (A, B and D), they are not histograms. Instructors should have recognized that histogram C was the only graph that would address the goal of the problem as well as be the only way to represent the data at hand. However, half of the participants (n=6) did not recognize that graph C was the only histogram. Moreover, looking deeper into the justifications even instructors with statistics backgrounds seemed to struggle with this question. For example, "I



*"I think this one is the best [pointing at B]. It sort of ranks them from umm well... no actually I take that back... I don't like that one... I change my mind. Well, I... I like it better ranking from lowest to highest... I think that's interesting, although I like that one too... they're both interesting. But I'd*

say I'd pick probably D. And why? Yeah, yeah am I supposed to say why? Okay... alright... I think it's the... it just it just puts them in order from lowest to highest so I think so one way to represent it. So it's a more organized way. Here they're kind of although... I find that, I find that one interesting but that one's useful too, so... I like this one too [B]... alright let's go to the next page" (ST12).

## DISCUSSION

Overall, these questions unveiled some interesting aspects of the conception of variation of two-year college instructors when dealing with histograms. First, the identification of the appropriate graph with more variability does not guarantee the appropriate reasoning. Second, while some instructors seemed to grasp the idea of variability as a measure of spread about the center, a few still used this knowledge inappropriately by identifying the wrong graph. Third, it seems that there is an incorrect association of seeing the normality shape of a distribution as an indication of low variability. This means that they based their judgment solely on the shape of the distribution, ignoring the range, the spread from the center and the frequency of each value. This led them to the wrong conclusions. Also the results indicate that getting the correct response did not necessarily indicate the appropriate understanding, as shown by the justification. It can be deduced that those who identified the graph correctly tended to focus solely on the range of the distribution and those who identified the graph incorrectly tended to focus on the frequency of the extreme values. Paying attention to the range and to the frequency of the extreme values are both good considerations to determine the amount of variability in a graph. Using them in isolation, however, led instructors to wrong conclusions. Additionally, the context was a commonly overlooked characteristic.

This study was exploratory in nature and while there could be faultfinding in the methods and procedures used, instructors' direct quotes utilized throughout the study should give at least a glimpse of the state of statistical understanding of two-year college mathematics instructors. This study can serve as a springboard for further in-depth studies where the inclusion of interactive interviews, using both clarifying and prompting, would improve the depth of the knowledge exposed in this study. Group interviews could also be beneficial since not everyone seems to hold the same opinion and the interactions and discussion with several instructors with different viewpoints could be very beneficial to understanding the depth of instructors' thinking. Whatever path is taken to investigate mathematics instructors at two-year colleges, it would be significant progress to further understand this overlooked population. Researchers need to seriously consider this population if the aim is to serve students and the community at large.

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