

SUITABILITY CRITERIA FOR TEACHERS' EDUCATION PROGRAMS IN MATHEMATICS AND STATISTIC EDUCATION

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The design and evaluation of prospective statistics teachers' education programs require developing reference models for the desirable characteristic of these programs. It is necessary to consider the knowledge required to organize and manage statistics teaching processes, as well as the various aspects involved in the application of that knowledge by the teachers. The aim of this report is to identify components and indicators of didactical suitability for statistics teachers' education processes (following the model suggested in the onto-semiotic approach to mathematics education). Consequently, we propose a model of didactic-statistics knowledge and some criteria for developing this knowledge in prospective teachers.

INTRODUCTION

The synthesis of research on teaching and learning a specific content such as, for example, data analysis in primary education, is not an easy task for the teacher who has to design, implement and evaluate his/her lessons on that topic. Although the teacher could know the pedagogical-didactical principles guiding educational design, their application to specific issues involves too many degrees of freedom, which makes insecure and overly complex the teacher's action. Consequently, it would be helpful for the teacher to possess strategies or guides that facilitate instructional design, in an operative way, and support his/her systematic reflection on a specific content teaching and learning. The development of such guides, which should be based on educational research, is one objective of the *Theory of Didactical Suitability* (Godino, Batanero, Roa & Wilhelmi, 2008; Godino, 2011); its development and application to statistics teachers' training plans is addressed in this paper.

In the next section we describe the aim of this research and the theoretical framework in which we particularly address the notion of *didactical suitability*. Then, we describe the facets and components we consider necessary to take into account when assessing the didactical suitability of a teachers' training process. Based on the system of suitability indicators for mathematical instruction processes we identify the epistemic suitability indicators for a formative process directed to mathematics teachers. The indicators for the remaining facets (cognitive, affective, interactional, resources and ecological) and the application of the model described to a particular formative program are developed in Godino, Batanero, Rivas & Arteaga (2013). The paper concludes with some final reflections and implications. Since mathematics teachers usually are responsible for the teaching of statistics in primary and secondary educational levels, the indicators of didactical suitability for formative processes in mathematics and mathematics education are also applicable to the particular case of statistics and statistics education.

RESEARCH PROBLEM AND THEORETICAL FRAMEWORK

In this research we face the educational design problem from the perspective provided by the "onto-semiotic approach" to mathematical knowledge and instruction theoretical framework (OSA) (Godino, Batanero & Font, 2007; Font, Godino & Gallardo, 2013). We consider that the OSA, and, in particular, the notion of didactical suitability can provide original and significant elements for a theory of instructional design appropriate to guide the teaching and learning of mathematics and other curricular areas. Specifically, in this paper we address the problem of designing formative plans in statistical education for mathematics teachers; therefore, this work will be interesting to prospective mathematics teachers' educators.

Godino (2011) suggested a "Guide for assessing the didactical suitability of mathematics instruction processes" (GADS-MI), which includes a set of indicators or criteria of didactical suitability for teaching and learning mathematics based on the OSA assumptions and other theoretical frameworks. This guide summarizes mathematical and didactical principles in

agreement with the pedagogical and didactical assumptions of different approaches and theories currently accepted by the mathematics educators community.

The notion of didactical suitability, its dimensions, criteria, and an operational breakdown of this notion were introduced in the OSA as a tool that allows the passage from a descriptive-explicative to a normative didactic; that is, a didactic that is aimed towards the effective intervention in the classroom. The didactical suitability of an instructional process is defined as the consistent and systematic articulation of the following six components (Godino, Batanero & Font, 2007):

1. Epistemic suitability (extent to which the statistical content is representative of the curricular content for a specific teaching level and the extent to which its inclusion in the teaching is justified).
2. Cognitive suitability (whether the content is adequate to the students' previous knowledge and the extent to which the instructional goals can be achieved).
3. Resource suitability (sound use of technical tools, resources and time).
4. Affective suitability (whether the teaching/learning process takes into account the students' motivations, attitudes, affects and beliefs).
5. Interactional suitability (whether the interactions between the teacher and the students and among the students themselves favour the overcoming of learning difficulties).
6. Ecological suitability (degree to which the teaching/learning process is adapted to the social environment; possibility of establishing interdisciplinary connections).

One aim of this paper is to adapt and apply the aforementioned guide to assess formative plans to train mathematics and statistics teachers. This requires a previous reflection on what knowledge, understandings and competences a teacher should master to design and implement suitable curricular proposals, as well as on the most appropriate "ways" to build such knowledge, understandings and skills. Therefore, we need to adopt a model of "teacher knowledge" categories. For example, the "Pedagogical Content Knowledge" (PCK) (Shulman, 1987); the "Mathematical Knowledge for Teaching" (MKT) (Hill, Ball & Schilling, 2008), or the "Mathematics and Didactic Teachers' Knowledge" (MDTK) (Godino, Ortiz, Roa & Wilhelmi, 2011) models. Once a model is adopted, the design of formative process should ensure that teachers acquire the mathematical and didactic knowledge required, i.e., knowledge, understanding and competences that the prospective teacher requires in the particular educational level.

FACETS AND COMPONENTS OF A GUIDE FOR ASSESSING TEACHERS STATISTICAL EDUCATION TRAINING PROCESSES

The (GADS-MI) guide described in Godino (2011) is aimed to analyze mathematics instruction processes performed at any educational level. Therefore, it can be applied in teacher mathematics contents training courses, only in case these contents are addressed separately from the related specialized didactic knowledge. The guide is useful for mathematics teachers who want to design and evaluate the teaching and learning of specific mathematical contents, considering the different facets, components and indicators.

To achieve suitable instructional mathematics processes, teachers should know and understand this guide and acquire competence in its implementation, as its indicators "condense" the didactical and mathematical knowledge developed and agreed by the mathematics educators community (Godino, Font, Wilhelmi & Castro, 2009). Consequently, formative processes in statistics education should be focused, not on statistics alone, but in the didactical-mathematical principles embodied in the mentioned guide, its theoretical foundation and practical application. Such theoretical foundation is based on the contributions from various disciplines (statistics, epistemology, psychology, pedagogy, semiotics, etc) that statistical education "aims" to articulate in order to orientate the teaching and learning of specific contents.

Furthermore, the design, implementation and evaluation of teacher training processes should be oriented by a specific guide (GADS-DMI), which includes the components detailed in Table 1. The epistemic facet of these instructional processes is the didactic-mathematics/statistics content to teach, which can also be understood as the institutional learning expectations for

statistics education. But the formative processes of teachers must also consider the other facets involved in an instructional process: cognitive, affective, interactional, resources and ecological facets (Godino, Batanero & Font, 2007; Godino, 2011).

In the next section we describe some indicators for the epistemic suitability facet of teacher training processes. Because of the reduced space it is not possible to develop in this paper the indicators corresponding for the cognitive, affective, interactional, resources and ecological facets of these processes. We refer the reader to Godino, Batanero, Rivas & Arteaga (2013) where such indicators are presented as well as the application of this model to the analysis of a formative plan for mathematics teacher training in a Chilean University.

Table 1. GADS-DMI Components

EPISTEMIC FACET Didactical and Mathematical content (from the institutional point of view)	OTHER FACETS INVOLVED IN MATHEMATICS EDUCATION TRAINING
<i>Mathematical content:</i> Problems, languages, concepts, procedures, propositions, arguments, connections	COGNITIVE FACET: Teachers' learning of didactical and mathematical content
<i>Cognitive content:</i> Students' previous knowledge and learning of the specific content, curricular adjustments	AFFECTIVE FACET: Teachers' beliefs, values, interests, attitudes, emotions towards the learning of didactical and mathematical content
<i>Affective content:</i> Interests, attitudes, emotions towards the mathematical content learnt by students	INTERACTIONAL FACET: Ways of interaction and discourse in the teacher training process
<i>Interactional content:</i> Ways of interaction and discourse in the teaching and learning of mathematics process	RESOURCE FACET: Use of technological resources in the teacher training process
<i>Resource content:</i> Use of technological resources in the teaching and learning mathematics process	ECOLOGICAL FACET: Curriculum, didactical innovation in teacher training, interdisciplinary connections
<i>Ecological content:</i> Curriculum, didactical innovation, socio- professional adaptation, interdisciplinary connections	

INDICATORS OF EPISTEMIC SUITABILITY IN A FORMATIVE PROCESS IN MATHEMATICS EDUCATION

A training program for teachers should support the development of knowledge, understanding and professional competences that enable the implementation of mathematical/statistical instructional processes where the components and indicators of epistemic facet of GADS-MI (given by Godino, 2011 and presented in Table 2) are reached.

Consequently, the teacher training program should include the means to reach the objective that prospective teacher shares the epistemological assumptions underlying the components and indicators of epistemic suitability of teaching mathematics. Some indicators of the epistemic suitability for teacher training programs are the inclusion of situations and activities that make the teacher:

- a) To accept the importance of problem-situations in the construction of mathematical knowledge, i.e., lead he/she assumes a socio-anthropological view of mathematics. Equally, the situation must help teachers understand problem solving as a mean to give sense to

- mathematics/statistics contents, and also a basic competence (that involves exercising and application).
- b) To select and adapt mathematical problems/tasks in order to make mathematical knowledge meaningful, by taking into account the epistemological obstacles which has led to the progressive construction of knowledge throughout history.
 - c) To recognize the central role of mathematical languages (representations), their types, transformations and conversions during the construction and communication of mathematical knowledge.
 - d) To understand mathematics as an interconnected system of rules (concepts, procedures and properties), and to accept the diversity of meanings for each mathematical content, both formal and informal.
 - e) To recognize the central role of argumentation in the construction of mathematical knowledge, and the diversity of means of demonstration.

Table 2. Components and indicators of epistemic suitability

COMPONENTS:	INDICATORS:
<i>Situations-problems</i>	<ul style="list-style-type: none"> - A representative and articulated sample of situations is presented; including contextualization, exercising and application situations - Statistical problems with multiple sources and types of data are included¹ - Problem generating situations (problematization) are proposed
<i>Languages</i>	<ul style="list-style-type: none"> - Using different modes of statistical expression (verbal, graphic, symbolic ...), translations and conversions between them. - Use of appropriate language for the students - Situations of statistical expression and interpretation are proposed
<i>Rules</i> (Definitions, propositions, procedures)	<ul style="list-style-type: none"> - Definitions and procedures are clear and accurate, and are adapted to the educational level - Statements and fundamental procedures of the topic for the educational level given are presented (data collection, comparison of distributions, inference techniques, ...) - Situations where students have to generate or negotiate definitions, propositions or procedures are proposed
<i>Arguments</i>	<ul style="list-style-type: none"> - Explanations, verifications and demonstrations are appropriate to the educational level treated. - Situations where students have to argue are promoted
<i>Relations</i>	<ul style="list-style-type: none"> - Statistical objects (problems, definitions, propositions, etc.) are related and connected between them. - The different partial meanings of the intended statistical objects are identified and articulated

The objective is that teachers reflectively assumes a way to understand mathematics from a plural point of view, that articulates the functional (internal and external applications), with the linguistic, structural and argumentative components. The teachers' training process should include as a central objective the study and discussion of an *educational epistemology* of mathematics. This is a "specialized content knowledge" that serve, first, to the epistemology of mathematics (with cultural/anthropological roots), but also to a deeper knowledge of the content that is not limited to problem solving and knowledge of mathematical structures, but to the educational use of problem solving as a learning tool. Likewise, the mastering of a rich variety of meanings for mathematical objects and their articulation is procured (languages, structures, and arguments)

Epistemic suitability in the teacher training process is achieved when the educator plans, organizes the instructional process in such a way that it makes the teacher know, understand and master the specialized content knowledge regarding the variety of situations-problems, languages, structures, arguments and connections for the specific educational level (common knowledge) and the "mathematical horizon", knowledge, i.e., the articulation with the next educational level.

The global epistemic suitability criteria for a teacher training process is the inclusion in the program of a representative sample from the system of the didactic-mathematic knowledge (including understanding and practical domain) that the "mathematics educators community" consider relevant for a suitable teaching of mathematics at the corresponding level.

In summary, the guide given by Godino (2011) to assess teaching mathematics processes (GADS-MI) helps to reveal and classify the specialized knowledge of the didactical-mathematical content involved in mathematics instructional processes and that should be taken into account in the training of teachers. We admit that this system must be specified in each case according to the mathematical content whose teaching is being planned (in our case statistics) and the corresponding educational level (primary, secondary, university). It should be also considered whether other elements in this guide should be adapted, such as: the difference between valuing a global plan (macro-didactic processes) and a local plan (micro-didactic processes), or whether it is a design or implementation of a training process.

FINAL CONSIDERATIONS AND IMPLICATIONS

In this paper we discussed the problem of designing teachers training processes for statistical education. We interpreted and applied the notion of didactical suitability, as well as the instrument "Guide for the Assessment of Didactical Suitability" (Godino, 2011), which it is actually a family of instruments that synthesize the didactical-mathematical principles for teaching and learning mathematical contents in specific levels and given circumstances. These instruments should be mastered by teachers through adequate teachers training processes and would enable them to adapt and develop curricular guidelines in a specific level.

Didactical suitability indicators are inferred of didactical research and curricular guidelines widely spread and accepted in the mathematics education community. In the training of secondary level mathematics teachers usually the mathematics and didactics-mathematics training are separate. Moreover, both contents maybe taught by different educators (from different areas of knowledge), which could make even more difficult the articulation of both formations.

Mathematicians should be responsible for training in the common knowledge and knowledge in the mathematical horizon (Ball, 2000; Ball, Lubienski & Mewborn, 2001) while Didacticians should carry out the training corresponding to the specialized knowledge of the mathematical content, both as regards its common component and its horizon component. But, as suggested by the epistemic facet of GADS-MI components and indicators, the specialized content knowledge is intrinsically connected to the common knowledge and knowledge in the mathematical horizon, since they all represent different and deeper ways to know the mathematics that has to be taught. In addition, the specialized content knowledge also includes issues related to the students' learning of the specific content, so that into this process the common and horizon knowledge get involved again with the cognitive facet. A similar situation happens as regards the use of technological tools (resource facet), which are often specific to the intended contents. This overlap between mathematics and mathematics education is what leads us to introduce the construct "didactical-mathematical knowledge" and to propose the integrated study of mathematics and its didactic in the mathematics teacher training, both at secondary and primary educational level.

ENDNOTE

¹ We understand the idea of statistical problem as a situation that involves a research process which comprises the steps of formulating questions, collecting data and analyzing and interpreting the results to answering the questions (Franklin et al., 2005)

ACKNOWLEDGEMENT

The research reported in this article was carried out as part of the following projects: EDU2010-14947, Ministerio de Ciencia e Innovación (MICINN), and EDU2012-31869, Ministerio de Economía y Competitividad (MINECO).

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