DEVELOPING PRE-SERVICE TEACHERS' TECHNOLOGICAL PEDAGOGICAL CONTENT KNOWLEDGE (TPACK) OF SAMPLING

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This research sought to explore and improve pre-service primary school teachers' statistical knowledge for teaching informal ideas related to samples and sampling through the conduct of a two-phase exploratory study. In Phase I, a randomly selected sample of n=42 teacher candidates completed an open-ended questionnaire designed to assess their reasoning about sampling concepts and their understanding of student thinking in this area. Insights gained informed the design and implementation of a teaching experiment within an undergraduate mathematics methods course, which aimed at enhancing the participating primary school teachers' (n=8) technological pedagogical content knowledge (TPACK) of sampling. Findings from Phase II indicate a positive impact on participants' TPACK of sampling and other key ideas related to informal inferential statistics.

INTRODUCTION

Traditionally, statistical inference is presented in the classroom as a set of formal tests and procedures, through which information contained in sample data is used either to estimate the values of the respective population parameters (i.e., construct confidence intervals), or to check claims made regarding the values of parameters (i.e., perform hypothesis testing). Given the conceptual difficulties involved in understanding formal inferential methods, exposure to statistical concepts at the lower levels of schooling had been restricted to basic descriptive statistics. In recent years, however, it has been recognized that the foundations for statistical reasoning, including fundamental ideas of inferential statistics should be laid in the earliest years of schooling. Because formal statistical inference ideas and techniques are beyond the reach of young learners, an informal approach to statistical inference is necessary in the early years of schooling (Ben-Zvi, 2006). Developing children's informal inferential ideas is a topic of current interest to many statistics educators (e.g., Gil & Ben-Zvi, 2011; Meletiou-Mavrotheris & Paparistodemou, in press).

The adoption of an informal, data-driven approach to statistical inference undoubtedly provides new opportunities for introducing powerful statistical concepts early in the school curriculum. Recognizing that teachers are in the critical path to the implementation of any innovation, the study reported in this paper aimed at investigating a group of pre-service primary school teachers' understanding of key of sampling ideas and of student thinking in this area, and at using the findings of this investigation to engineer learning experiences for improving the participants' TPACK of sampling. Enhancing teachers' TPACK of sampling is of paramount importance to the early introduction of informal inferential statistics, because building connections between sample and population lies at the heart of informal statistical inference (Pratt et al., 2008).

STUDY METHODOLOGY

To respond to our research objectives, we conducted a two-phase exploratory study. In Phase I, a randomly selected sample of *n=42* teacher candidates at a university in Cyprus completed an open-ended written assessment designed to assess their reasoning about sampling concepts, and their understanding of student thinking in this area. The questionnaire, shown in Table I, consisted of five open-ended questions, each requiring students to justify their responses. The first three questions were designed to provide information about pre-service teachers' understanding of the meaning and role of sample, sample size, method of selection, and sampling bias. The remaining two questions focused on investigating participants' understanding of student thinking regarding sampling issues, and their ability to recognize common student misunderstandings. Pre-service teachers were asked to comment on some hypothetical student responses to sampling tasks.

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Table 1. Tasks included in Phase I assessment

Q1. Two surveys were conducted to determine the proportion of schools in Cyprus recycling. In the first survey, postcards were sent to all school principals (*n = 800*), and about half responded, 91% stating that their school recycled. The second survey used a medium sample size (*n = 150*) and a random sampling method, and found that 57% of the schools recycled. Which survey do you prefer and why? What do you think is the best estimate of the proportion of Cypriot schools recycling?

Q2. Georgia is going to flip a coin 50 times and record the percentage of heads she gets. Her friend Fotini is going to flip a coin 10 times and record the percentage of heads. Which girl is more likely to get 80% or more heads?

Q3. Students in a school conducted a survey to determine the proportion of children in their school recycling at home:
Survey 1: George asked 60 friends (75% yes, 25% no).
Survey 2: Zoi got the names of all 600 students in the school, put them in a hat, and pulled out 60 of them. (55% yes, 45% no)
Survey 3: Andreas asked 60 students that participated in the Environmental Club. (100% yes).
Survey 4: Eleni sent out a questionnaire to every kid in the school and then used the first 60 that were returned to her. (80% yes, 20% no)
Survey 5: Anastasia wanted the same number of boys and girls and some students from each grade. So she asked 5 boys and 5 girls from each grade to get his total 60 students. (45% yes, 55% no)

(a) What do you think about the way that each survey was conducted? Was it done in a proper way? Do its results give a good picture of how many students in the school recycle at home? Explain why or why not.
(b) If you were to pick one of the five ways to do the survey, which one(s) would you choose? Explain why.
(c) If you were to pick one of the five ways to do the survey, which one(s) wouldn't you choose? Explain why.
(d) Which do you think is the best estimate of the percentage of kids recycling at home?

Q4. A questionnaire administered in a Grade 6 classroom in the context of a research study, included Q3 above. Some of the students' responses to Q3.a, where they had to indicate the data selection method(s) they would choose if they were to do a similar study, include the following:

- I would choose George's method, because in his study 45 children recycle and only 15 do not
- In my opinion, they should draw a raffle (Zoe's method) to randomly select children. Otherwise, if they choose their friends, it would be unfair for the rest of the children.
- I would use Andreas' method because he asked children in the Environmental Club, where it was almost certain that they recycle at home, and would therefore respond with a "Yes".
- I would prefer Eleni's method because she sent the questionnaire to all students in the school, and so all children were given the opportunity to participate in the survey.
- I would choose Anastasia's method because students were randomly selected, and also she ensured an equal number of boys and girls from all classes.

Evaluate each of the responses, explaining why you agree or disagree with the student's reasoning.

Q5. A questionnaire administered in a Grade 6 classroom, included the question: *"Have you heard the word sample before? Where? What does it mean?"* Some of the children's responses are the following:

- Sample means a small dose of something, like when we take a blood test.
- Sample means taking a small quantity for testing purposes. For example, someone tells you: "Would you like to try a sample of our new cheese?"
- The word sample *["deigma" in Greek]* has the same meaning as the word example *["paradeigma" in Greek]*.
- Sample is part of a whole. It shows how the whole looks like.

Evaluate each of the responses, explaining why you agree or disagree with the student's reasoning.

The questions included in the assessment were adapted from previous studies investigating middle and/or high school students' understanding of sampling issues (Jacobs, 1999; Watson & Moritz, 2000; delMas & Garfield, 1990; Meletiou-Mavrotheris & Paparistodemou, in press). Responses to all of the questions were coded using similar coding schemes as those employed in previous studies, to provide a point of reference and comparison for our findings. To increase the reliability of the analysis, we first coded the data independently. Differences in coding were debated and consensus was reached after several cycles of discussion.

Findings from Phase I guided a teaching experiment that took place during Phase II of the study, within the undergraduate methods course *Integration of Modern Technology in the Teaching of Mathematics*. Utilizing insights gained from Phase I, we re-designed a course unit on technology-enhanced statistics learning, so as to focus on the early development of informal inferential reasoning. Eight pre-service primary school teachers, mostly in the final year of their studies, were enrolled in the course. Their mean age was 22 years, and they had all completed an introductory statistics course at the college level. The first author was the course instructor.

Multiple forms of assessment were used to collect and document evidence of changes in pre-service teachers' technological pedagogical content knowledge (TPACK) of sampling and other related ideas of informal statistical inference, as a result of participating in the course: written assessments, video-records of group sessions, mini-interviews of selected pre-service teachers (interviewed during group work), field notes, and classroom observations. Using the findings of Phase I as a baseline, we sought to identify shifts in participants' reasoning about key sampling issues and about student thinking in this area as a result of participating in the teaching experiment.

RESULTS

Phase I

The open-ended written pre-assessment allowed thorough investigation of the breadth and coherence of this group of novice primary school teachers' pedagogical content knowledge of sampling. A small discussion of the results follows.

In Q1 (adapted from Jacobs, 1999), which asked participants to compare the quality of two surveys, both conducted to determine how many schools in Cyprus are recycling, the majority of the pre-service teachers (64%), expressed preference for the random sampling method employed in the second study *"although it was based on a smaller sample size, because the sample was randomly selected."* However, a considerable proportion of the participants (19%), failing to recognize the dangers of self-selection, wrote that they preferred the first study *"because we have a larger proportion of all schools participating, so we get a more complete picture of the whole population."* A few others (12%), while acknowledging that the first method is biased, concluded that it is better to use its results since it utilizes a larger sample size.

Pre-service teachers did very poorly in Q2 (adapted from delMas & Garfield, 1990) asking them to decide who, among two friends, is more likely to get 80 percent or more heads: Georgia, who is going to flip a coin 50 times, or Fotini, who is going to flip the coin 10 times. Forty percent of the participants, confusing the independence of a single event with the long-term frequency of random events, argued that since each coin flip is a separate event, the probability of heads is not affected by the number of times flipped, and thus *"it is equally likely for both girls to get 80% or more heads."* Another sizeable proportion (31%) argued that *"Georgia is more likely to get 80% or more heads, because the bigger the sample size the greater the variability in results."* Only 29 percent recognized that it was more likely for Fotini to get an extreme result because of the smaller number of tosses, arguing that *"the more times you throw the coin, the closer you get to 50%."*

Q3 investigated student teachers' informal understanding of sampling issues in the context of evaluating sampling methods. The same question, which was an adapted version from Jacobs (1999), had been administered by Meletiou-Mavrotheris and Paparistodemou (in press) to a group of 11 year-old students in Cyprus. Findings from that study, which concurred with the findings of Jacobs (1999), indicated that although children tended not to like restricted sampling, they evaluated positively self-selected methods. In addition, whereas they liked stratified random sampling because it allowed them to specify the mixture of the sample, they mistrusted the "unknown nature" of simple random samples. When asked to indicate the method(s) they would not choose if they were conducting the survey themselves, two-thirds (67%) selected simple random sampling as the method they would definitely not use. In the current study, although the mistrust of simple random sampling's ability to produce a representative sample was observed in a few pre-service teachers, the vast majority evaluated Survey 2 positively, and 60 percent chose it as their preferred method, arguing that *"giving everybody the same chance to be selected should result in a sample more representative of the school."* Stratified sampling was also evaluated positively, with all but two participants approving the stratified random sampling scheme employed in Survey 5, emphasizing the *"good diversity in age and gender"* that is guaranteed by this method. Three students did point out that Survey 5 *"assumes that each grade has an equal number of students, that boys and girls are also the same in number, and that children are randomly selected from each stratum,"* and thus its quality depends on whether these assumptions hold.

Participants' responses to Q3 also revealed that while they were much more likely to recognize the potential for bias in restricted sampling than younger learners, a sizeable proportion were still ignoring or not identifying the risk of bias in self-selection. Specifically, while only one student expressed preference for a restricted sampling method (Survey 3), one-third of the students (33%) considered Survey 4, which was based on self-selection, as the best sampling method. In the last part of Q3, where they had to indicate their best estimate of the proportion of all children recycling at home, while the majority (55%) focused on survey quality and drew their conclusions based on the information provided by the surveys they had evaluated positively, several participants either aggregated all surveys regardless of quality (26%), ignored the survey results altogether and relied on their personal experiences and judgment to draw conclusions (12%), or refused to draw any conclusions (7%) because "*the provided information is inadequate.*"

Pre-service teachers' preference for random sampling methods, but also their tendency to reject restricted sampling methods but to accept methods employing self-selection, was also obvious in Q4, where they had to evaluate hypothetical student responses to each of the sampling methods described in Q3. The vast majority agreed with the second hypothetical student response, which argued in favor of Zoe's use of simple random sampling. A few of them (15%) also noted that the second child's rationale for preferring Zoe's use of a random sampling process suggests that he/she might also be preoccupied with issues of fairness in the colloquial sense: *"In a statistical study, we don't base our methodological decisions on sentiments (fair-unfair), but on the need to get a sample which is representative of the whole population."* The majority also agreed with the fifth hypothetical student response, emphasizing the *"good representation by age and gender"* guaranteed by Anastasia's use of a stratified sampling procedure, although some disagreed with the factors used for stratification: *"I don't think that a student's gender makes a difference as to whether they recycle or not."* With the exception of one participant, everyone dismissed the first and third student's rationale in Q4, pointing out that the restricted methods favored by these children lead to *"results that are 'cooked' by the researcher according to his or her own beliefs or wishes."* By contrast, two-thirds (67%) failed to detect the bias involved in self-selection, and agreed with the rationale of the fourth student who preferred Eleni's method. Several gave explanations echoing children's preference for self-selection due to pre-occupation with issues of fairness: *"This method is the most 'democratic' one, since it gives everyone the chance to provide their opinion."* Ten students (24%) even characterized the sample as random and/or nonbiased.

In Q5 (adapted from Watson & Moritz, 2000), where students had to evaluate four different hypothetical student responses to the question *"Have you heard the word sample before? Where? What does it mean?"* almost everyone agreed with both the first and the second response, making arguments such as the following: *"A sample is a subset of a population. A person's blood is the population, and a sample of this population is the blood they take during medical testing. We can use this sample to make generalizations about all of our blood. The same goes for taking a small piece of cheese as a sample of the whole cheese.*" Only 40 percent of the participants did point out that these two responses reflect the colloquial use of the term sample, where the purpose behind sampling is to show the homogeneous quality of an item, while in statistical sampling the purpose is to get a representative picture of a population in which there is clear variation among data values: *"The term sample has several different meanings. The first student explained the term sample correctly based on their own everyday experiences and so did the second student. Their explanations, however, do not explain what statistical sample means;" "The examples chosen in responses 1 and 2 are wrong because blood and cheese are homogeneous, so a small sample suffices."* With the exception of two participants, everyone else dismissed the third student's hypothetical response that the word sample (*"deigma" in Greek)* has the same meaning as the sound alike word example *("paradeigma" in Greek),* noting that the student's definition did not capture any of the relevant aspects of the concept of sample. Finally, everyone approved of the fourth student's rationale, stressing that his/her explanation "*approximates the statistical meaning of sample, because it describes the sample as being a representative subset of the population."*

In sum, findings from Phase I revealed that all pre-service teachers had at least partial understanding of sampling ideas and tended to reason with a higher level of sophistication than what was observed in previous studies of middle and secondary students. At the same time, the majority seemed to lack sufficient knowledge of sampling issues and to be in need of further development. Although they had all completed an introductory statistics course, they shared some of the difficulties in reasoning about samples, and of the reasoning biases that are common among younger students. They tended to struggle with issues of sample size and representativeness, to focus on a notion of fairness that often went beyond representativeness, and to fail to recognize potential sampling bias in self-selected methods. Moreover, they seemed to lack sufficient knowledge of children's patterns of informal thinking about sampling ideas and of the conceptual difficulties they are likely to experience (Noll, 2007).

Phase II

Using the insights gained from Phase I, we re-designed a unit on technology-enhanced statistics learning, within *Integration of Modern Technology in the Teaching of Mathematics*, a course designed to offer high-quality professional development experiences to pre-service teachers that would enable them to effectively integrate technology with core curricular ideas. The course design was guided by the technological pedagogical content knowledge (TPACK) conceptual framework (Mishra & Koehler, 2006). The TPACK framework, which builds on Shulman's (1986) idea of Pedagogical Content Knowledge, emphasises the importance of teachers developing integrated and interdependent understanding of three primary forms of knowledge: technology, pedagogy, and content. The aim is to move teachers beyond techno-centric strategies that focus on the technology rather than the learning.

The unit on technology-enhanced learning of statistics, which lasted for three weeks (9 hours), introduced pre-service teachers to the rationale and context for employing technology in statistics classrooms. They experienced some of the ways in which an informal approach to statistical inference supported by technology could help students internalize key statistical concepts related to statistical inference across the primary school curriculum while at the same time improving their attitudes towards the subject. During the unit, we capitalized upon the positive aspects of pre-service teachers' reasoning about sampling ideas identified in Phase I to help address some of the challenges also identified in Phase I. Participants were provided with an inquiry-based learning environment designed to offer ample opportunities for formulating and evaluating informal, data-based inferences, using the dynamic statistics data-visualization software TinkerPlots® (Konold & Miller, 2005) as an experimentation tool. The instructional sequence included both the collection and analysis of real sample data, and computer-based simulations of data samples. The emphasis was on enriching pre-service teachers' TPACK of informal inferential statistics by exposing them to similar kinds of learning situations, technologies, and curricula to those they should employ in their own classroom. Moreover, there were discussions focusing on children's learning and what is required to involve them in informal inferential reasoning. We explored a broad range of topics of interest to the statistics teacher, including curriculum issues (e.g. role of informal/formal inferential statistics in the national and international mathematics curricula) and statistics education research (development of informal inferential reasoning in children, common student misconceptions regarding sampling issues, etc.). The pre-service teachers engaged in conversations about the teaching and learning of sampling and other concepts related to informal statistics, in the analysis of scenarios on common student solution strategies and errors, and in scaffolded reflection. They were introduced to a research-based hypothetical learning trajectory (HLT) and instructional sequence designed to support middle school students' emerging views of samples and sampling (see Meletiou-Mavrotheris & Paparistodemou, in press). At the completion of the unit, they were asked to design, using the provided HLT as a guide, a technology-enhanced lesson plan on sampling targeting a primary school grade.

Analysis of data collected during the teaching experiment, indicated a positive impact on pre-service teachers' TPACK of sampling and other key ideas related to informal inferential statistics. Participants' experimentation with various sampling tasks, and their immersion in the process of data-based statistical inference helped them to better coordinate the complex issues pertaining to sampling and its related ideas. Moreover, their engagement in activities and discussions around student thinking and learning, led to better understanding of the pedagogical issues surrounding learning of sampling ideas, and of the ways in which innovative technological tools and instructional strategies can support the development of young learners' informal inferential reasoning.

CONCLUSIONS

The expanding use of data in modern society for prediction and decision-making makes it a priority for mathematics instruction to help students build sound foundations of inferential reasoning at a young age. Drawing reliable inferences about a population based upon evidence obtained from a sample is the cornerstone of modern statistical methods. Sample size and sampling method are the main determinants of the validity of statistical inferences. Because “statistical inference is almost by definition imperfect—all sampling introduces some error” (Jacobs, 1999), students need to be aware of the potential threats to valid statistical inferences. However, as this and previous studies (e.g., Noll, 2007) have indicated, not only students but also their teachers often lack sufficient knowledge of sampling issues, and tend to share some of the conceptual difficulties and biases in reasoning about samples that are common among young students. This has implications for both pre-service and in-service teacher training, pointing to the need for appropriate professional development opportunities that can help teachers address some of the shortcomings in their reasoning about sampling concepts and their instruction.

Recognizing the central role of teachers in educational reform, the current study investigated pre-service teachers’ statistical knowledge for teaching informal ideas related to samples and sampling, and used the insights gained to design and implement a teaching experiment for improving teachers’ TPACK of sampling ideas. Findings indicate that the approach used in the teaching experiment did have a positive impact on the pre-service teachers’ TPACK of sampling concepts and other fundamental ideas related to informal inferential statistics. It seems that providing teachers with opportunities to experience the whole statistical problem-solving process through the collection and analysis of real and simulated samples, and engaging them in activities and discussions about the teaching and learning of sampling conceptions helps them to improve their content knowledge of sampling concepts, but also their understanding of the pedagogical issues surrounding the teaching and learning of sampling ideas.

REFERENCES

Ben-Zvi, D. (2006). Scaffolding Students’ Informal Inference and Argumentation. In A. Rossman & B. Chance (Eds.), *Working Cooperatively in Statistics Education: Proceedings of the Seventh International Conference on Teaching Statistics*. Voorburg, The Netherlands: International Statistical Institute.

delMas, R., & Garfield, J. (1990). The Use of Multiple Items to Identify Misconceptions in Probabilistic Reasoning. In J. B. Garfield (Ed.), *Research Papers from the Third International Conference on Teaching Statistics.* University of Otago, Dunedin, New Zealand.

Gil, E., & Ben-Zvi, D. (2011). Explanations and Context in the Emergence of Students' Informal Inferential Reasoning. *Mathematical Thinking and Learning*, *13*(1), 87-108.

Jacobs, V. R. (1999). How Do Students Think About Statistical Sampling Before Instruction? *Mathematics Teaching in the Middle School, 5*(4), 240-263.

Konold, C., & Miller, C. D. (2005). *TinkerPlots: Dynamic Data Explorations*. Emeryville, CA: Key Curriculum Press.

Meletiou-Mavrotheris, M., & Paparistodemou, E. (in press). Young Learners’ Reasoning about Samples and Sampling in the Context of Informal Inferences. *Educational Studies in Mathematics: Special Issue on Learning to Make Informal Statistical Inferences from Samples*.

Mishra, P., & Koehler, M. J. (2006). Technological pedagogical content knowledge: A framework for teacher knowledge. *Teachers College Record*, *108*(6), 1017-1054.

Pratt, D., Johnston-Wilder, P., Ainley, J., & Mason, J. (2008). Local and Global Thinking in Statistical Inference. *Statistics Education Research Journal*, *7*(2), 107-129.

Noll, J. A. (2007). *Graduate Teaching Assistants’ Statistical Knowledge for Teaching* (Unpublished Doctoral Dissertation). Portland State University, Portland, OR.

Shulman, L. S. (1986). Those Who Understand: Knowledge Growth in Teaching. *Educational Researcher, 15*(2), 4-14.

Watson, J., & Moritz, J. (2000). Developing Concepts of Sampling. *Journal for Research in Mathematics Education, 31*(1), 44-70.