FROM HAT PLOTS TO BOX PLOTS IN TINKERPLOTS: SUPPORTING STUDENTS TO WRITE CONCLUSIONS WHICH ACCOUNT FOR VARIABILITY IN DATA

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A statistical question acknowledges that there is variability in data that needs to be accounted for in the conclusion. Accounting for variability is problematic if students do not have an understanding that a distribution shows patterns and can be described by the centre, spread and overall shape. TinkerPlots provides opportunities to build understandings of spread and measures of centre as students work with distributions, adding and manipulating dividers and hat plots. In this exploratory study, students in a middle school inquiry classroom used hat plots to compare distributions and write justified conclusions. Results suggest that the necessity to account for variability in data within their conclusions presented students with a purpose to transition from hat plots to box plots to provide evidence to answer the question.

To answer a statistical question students are required to understand, explain, and quantify variability in data (Franklin et al., 2007). Unless students have an understanding that distributions show patterns in data and can be described by their centre, spread and overall shape, accounting for this variability is problematic. There is a “need for research that explores the kinds of tools and representations we can provide younger students that allow them to notice and quantify the amount of variability in a distribution” (Bakker, Biehler & Konold, 2004, p. 171). Tinkerplots’ (Konold & Miller, 2005) features allow students to produce, explore and compare the patterns of distributions efficiently, enabling them to focus on statistical understanding rather than the mechanics of representations (Paparistodemou & Meletiou-Mavrotheris, 2008). This paper reports on an exploratory study in Australia aimed at understanding the processes of students’ learning to account for variability in conclusions of statistical questions, moving from informal measures of centre and spread with hat plots to standardised measures of centre and spread using box plots.

LITERATURE

The purpose of statistical analysis is to account for variability in data (Frankin et al., 2007). Variability can be acknowledged in both standard and non-standard ways (Ben-Zvi, Aridor, Makar & Bakker, 2012; Makar & Confrey, 2005) with formal statistical measures (e.g., IQR), qualitative or informal statistical tools (e.g., modal clumps) or the language of uncertainty (e.g., estimating an average as “*about* 2 seconds”). Statistical interpretations must seek to make sense of this variability within the problem context. While the mature practitioner reflects upon all aspects of a statistical investigation when interpreting results (question, data collection methods, analysis and context), beginning students cannot be expected to make these linkages (Frankin et al., 2007).

Box Plots and Hat Plots

Box plots are widely used for representing data as they explicitly show the centre, spread and shape of distributions, making them easier to compare. However, students (and even teachers) can find box plots difficult to interpret (Pierce & Chick, 2013; Watson, 2012). Bakker et al. (2004) argue that these difficulties likely stem from lack of visible access to individual cases, proportional reasoning needed to interpret density, and difficulty in understanding the median. They question the wisdom of introducing box plots until after middle school and suggest that when box plots are introduced, the underlying dot plots remain visible to help students relate the number of points in a section of the box plot with how their density is represented. Students often interpret box plots by listing “facts” (e.g., location of quartiles) but may neglect sense-making in applying box plots meaningfully within a problem (Pfannkuch, Budgett, Parsonage & Horring, 2004). Neglect of sense-making can signal that students are asked to compute and represent without making clear a purpose or utility of doing so (Ainley, Pratt & Hansen, 2006). A focus on the “modal clump” of data can assist students to think about data as an aggregate (Konold et al., 2002). This led to the development of a “hat plot” (Konold, 2007) as a pre-cursor to box plots to capitalise on learners’

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natural tendency to think of values in three categories: low, middle and high. If the "middle" is defined as the middle 50%, then the hat plot is precisely a box plot without a median (Watson, 2008). Rather than introduce box plots directly, a stepwise support from unorganized data to conventional box plots is needed (Bakker et al., 2004). Although this was likely an intent of hat plots, there has not been much research looking at this transition, particularly given the benefits of encouraging a focus on both variability and aggregates.

Tinkerplots as a Tool for Sense-making in Statistical Investigations

Programs like Tinkerplots have many benefits in providing tools such as hat plots to encourage this more stepwise approach to box plots. Tinkerplots allows students to "participate in the construction and evaluation of methods by providing a graph construction tool for young students who can invent their own elementary graphs, whereas most other tools provide only a readymade selection of standard graphs" (Biehler, Ben-Zvi, Bakker & Makar, 2013, p. 698). Having such a set of tools can give children access to powerful statistical topics such as inferential statistics and the broader process of statistical investigation, by removing computational barriers. This opportunity can shift the focus of school statistics from tools and procedures towards more process-oriented approaches that go beyond techniques. Therefore it seems appropriate to research the use of Tinkerplots' hat plot as a transition to the more challenging box plot.

METHOD

The participants were 27 children (age 12) representing a diversity of achievement levels in a 1-to-1 laptop class (i.e., each child had a laptop) from a government school in Australia. The study used exploratory action research to respond to the following research question:

- How can technology-supported experiences with statistical inquiry assist students to transition from hat plots to box plots and provide meaningful quantitative evidence required to answer statistical questions?

A unit (17 lessons) was designed to develop an appreciation of distributions for analysis and as evidence to justify conclusions in two statistical investigations modified from the literature. Most of the class had little experience with statistical investigations so were not fluent with representing and interpreting data to provide answers to statistical questions. In the first investigation, students developed a method to estimate (single value and interval) the time it would take for a paper helicopter to drop (cf., Ainley, Nardi & Pratt, 2000); in a second investigation, they compared jumps of origami animals (cf., Scheaffer, Watkins, Gnanadesikan & Witmer, 1996). A focus on variability (natural, measurement, induced) and the use of statistics to make sense of problems in context were key drivers in the unit. Space prohibits detailed description of the lessons and classroom culture, although these were clearly important aspects.

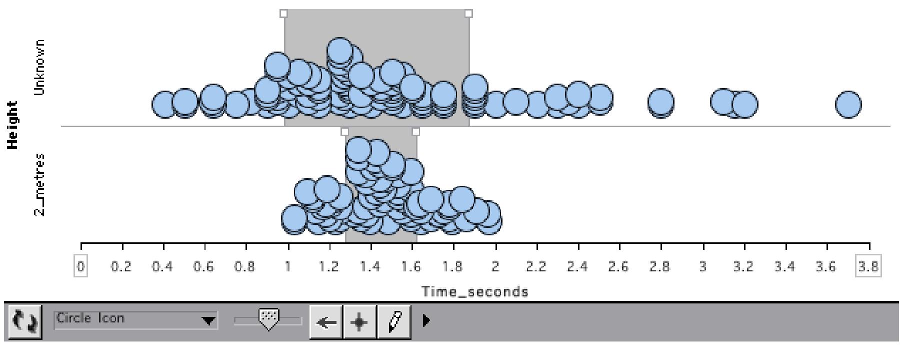

Figure 1: Student representation in Tinkerplots using dividers to isolate the modal clump.

Tinkerplots was used to informally interpret data, construct statistical tools to assist with their analysis, calculate measures of centre and spread, and represent data with box plots as evidence for conclusions. Students initially represented data with hand drawn dot plots and visually identified point and interval estimates to account for variability. The data were then imported into Tinkerplots to explore data flexibly, manipulating dividers and hat plots to create point and interval estimates (Figure 1). The second investigation motivated a need to standardise interval estimates (fixing the interval estimate at the middle 50% and adding the median as a point estimate).

After each investigation, students addressed an individual task designed to formatively assess progress with near and far transfer. Tasks required them to analyse data and provide a justified conclusion; students were not told to account for variability in the data, but doing so had been an expectation in class. At the end of the unit, students were asked to reflect on how technology assisted them to provide the evidence required to answer statistical questions, as previously they had not used statistical software. The focus of this paper is on data collected from these two tasks and the final reflection in order to gain insight into the children's expressions of variability and use of evidence to justify their conclusions with technology.

Students' analyses and conclusions were categorised, focusing on how students accounted for variability. Excerpts were selected to illustrate each category. The intent was not to compare tasks, but to consider students' development as they moved from informal to more formalised statistical tools. Their reflections were categorised and examples located to illustrate categories. Again, the purpose was not about the categories, but to understand opportunities of technology-based tools for children's early access to powerful statistics in an inquiry-based environment.

RESULTS

Below we provide a glimpse of students' developing attention to acknowledging variability in their data analyses and conclusions for the two tasks, as they moved from informal to more formal means of accounting for and representing variability. The diversity of results in Task 1 is summarized to save space; students' responses were more consistent in Task 2.

Task 1

In Task 1 (at the end of the first investigation), students were given two sets of data within the same context of their investigation consisting of times that a helicopter remained in the air for multiple throws at a known induced height (fixed at 2 m) and at an unknown (variable) height. The induced height would have less variability than when the height was not fixed (Figure 1). The task was heavily scaffolded with a class-constructed topic sentence: "The evidence supports the claim that the method that induces height produces a more accurate measure of the time a paper helicopter stays in the air." Students were then asked to individually build on the response by providing evidence to support and represent their (informal) analysis. While student responses varied in complexity, most (83%) attempted to account for variability patterns in their analysis in one of three ways (Table 1), through their choice of measures of variability and description of spread patterns (clumping) in the data.

Table 1: Students' focus on variability in Task 1 data analysis

Category (%)	Example
Range only (40%)	• *The data is ranging between 0.4 and 3.7 secs in the unknown but in the known it ranges between 1.3 and 1.8 secs. (Lily)*
Range, qualitative spread (13%)	• *The range for the 2 m height (1-2 secs) is much smaller than the unknown height (0.4-3.7 secs). The 2 m height is a lot more clumped and closer together than the unknown height. (Jo)*
Range, quantitative spread (30%)	• *The induced data shows a smaller range and more clumping whereas on the unknown data there is a longer range and less clumping. With the induced height data it's easier to make a point and interval estimate. The range of the induced data is 1-2 secs with an interval estimate of 1.3-1.6 secs whereas in the unknown data the range was 0.4 to 3.7 with an interval estimate 1.1 – 1.8 secs. (Darren)*

A joint class topic sentence was also constructed to assist students with their conclusions ("*The time a paper helicopter stays in the air depends on the height the paper helicopter is dropped from.*"). In their conclusions, 87% provided some evidence of students' understanding that a statistical question anticipates there will be variability in the data that needs to be accounted for in the answer. Students accounted for variability with either interval or point estimates, additionally expressing variability through language of uncertainty (Table 2).

Table 2: Students' focus on variability in their Task 1 conclusion

Category (%)	Example
Interval estimate (48%)	• *If the paper helicopter is dropped from 2 metres the time would be 1-2 secs.* (Jo)
Point estimate (39%)	• *From a height of 2m a paper helicopter generally stays in the air around 1.3 secs* (Darren)

With informal and visual tools within Tinkerplots, students were able to provide reasonable evidence, with support, that showed they were developing skills that acknowledged variability in addressing a statistical question.

Task 2

In Task 2 following the second investigation, students were asked to respond to a question in a context that differed from their investigation: "*Do Year 5-10 students spend more time working on homework or playing sport?*" Students were supplied with two box plots (using data from Watson et al., 2011); they were only lightly scaffolded with no jointly constructed topic sentence, requiring greater initiative and independence. However, in the lessons, students had been working with box plots and were taught the type of evidence expected for their analyses and conclusions.

All students provided a detailed analysis of the context in their responses that acknowledged salient features of the box plot and were confident interpreting the box plot even though the data points were hidden. Students' analyses were more sophisticated in Task 2, with responses indicating significant development in using the box plot to interpret the range and spread of the data. Remembering that these are evidence of analyses not conclusions (which are described later), descriptions frequently included both relevant and irrelevant aspects and facts about the box plots rather than interpretations. Nearly all students (91%) acknowledged and quantified range, spread and centre. Students did not necessarily interpret the IQR in terms of the density of the data, although many did articulate the evidence meaningfully in context.

> *On the hours of homework per week box plot there is a range of 0 – 30 hours of homework covered by the box plot. The inter quartile range of the data is 7 hours which ranges from 1 -8 hours spent doing homework. The median (centre of data) is 4 hours which is typically how long grade 5 – 10 students spend doing homework. On the hours of sport per week plot there is a range from 0- 32 hours spent doing sport. The inter quartile range is 11 hours and ranges from 3 – 14 hours spent doing sport per week. The median (centre of data) is about 7hours which means a typical student spends around 7 hours a week doing sport. There is a difference of 3 hours more time spent doing sport.* (Sanora)

The remaining 9% of responses quantified the range and spread but ignored the centre and made weaker links to the context.

> *The data is showing that the minimum data value of both representations is exactly the same as it is both 0 hours. The range of the homework per week is 30 hours and the lowest and highest data points are 0 and 30 hours. The range for hours in sport per week is 32 hours. The interquartile range for the hours in homework is 7 hours with the lower quartile 1 hour and the upper quartile is 8 hours. The inter quartile range for hours in sport is 11 hours and the lower quartile is 3 hours and the upper quartile is 14 hours. (The inter quartile range is where 50% of the data is).* (Nick)

All students correctly answered the question in their conclusion. 82% of responses chose medians to justify their conclusion and 41% of responses also quantified the difference in medians.

> *Students ranging from grade 5-10 typically spend about 7 hours doing sport and about 4 hours doing homework per week. Therefore they spend around 3 hours more time playing sport.* (Sanora)

However, without the prompt to provide quantitative evidence to support their conclusion, some students (18%) either gave a very general response or justified the conclusion with background knowledge rather than evidence.

> *The Year 5 – 10 students spend most of their time playing sport than homework because the box plot shows that the sports range is larger [higher] than the homework range.* (Haydn)

Student responses to Task 2 revealed significant development in their ability to use formalised statistical tools in their analysis to give an accounting of the variability in the data and to make the links between the data collection and analysis, the question and the conclusion.

Working with Box Plots in Tinkerplots

At the end of the unit, students were asked to reflect on the benefits of using Tinkerplots in the unit. Students were taught to manually calculate range, interquartile range, and median, create point and interval estimates visually, and to construct histograms, dot plots, hat plots and box plots by hand prior to their introduction to these tools on Tinkerplots. Their reflections suggest that they valued the technology and experienced little difficulty making connections with the tools and their functions. Three common themes emerged from our analysis of students' reflections (Table 3):

Table 3: Themes in student reflections

Theme (% students)	Example student comment
The removal of computational boundaries allowing a focus on analysis (93%)	• *Tinkerplots provided numerical values eg %, mean, median, IQR making it easier to justify conclusions with evidence.* (Gemma) • *Hat plots helped identify where data was skewed (clumped)* (Delmar) • *Use adjustable dividers to manually find (estimate) the cluster of the data and add hat plot to check estimates.* (Charlie)
The ability to operate accurately and efficiently (93%)	• *Tools (hat plots, mean, median) analyse data automatically; if you make a mistake on the calculator [you must] start all over again which is tedious to do and takes an extended time.* (Lucy) • *Hand drawn plots are more prone to human error such as incorrect calculations and incorrect representations.* (Darren)
The flexibility offered (70%)	• *Make different graphs to best support evidence required by changing format turning dot plots into histograms, hat plots or box plots.* (Lucy) • *Does not matter if you have 10 data points or 200 automatically puts on a plot.* (Salome) • *Unlike a hand drawn representation, you have control of the data to organize it and change it whenever.* (Konrad)

The students' reflections suggest that they valued the opportunity to work with the data in Tinkerplots. Although some students preferred the software for its ability to decrease the effort needed, many of them also acknowledged the additional benefits of being able to generate, compare and select the best evidence, and to improve their analysis.

DISCUSSION

This study focused on experiences that would encourage twelve year old students to consider the purpose of accounting for variability in conclusions and the utility to employ the statistics that would enable them to do so (Ainley et al., 2006). Bakker et al (2004) highlight the importance of stepwise support from unorganized data to conventional plots such as box plots. Constructing visual representations and having students estimate the centre (point estimate) and spread (interval estimate) using dividers and hat plots in Tinkerplots before introducing formal measures of centre and spread gave students a feel for the data. This was seen as the first step in the support process to build their understanding of the purpose of the median and IQR.

Opportunities to compare and share data sets helped reveal the difficulties in comparing variability within and between groups if different measures are being used. This provided an interim step in the process, moving students from visual hat plots to a standardized hat plot (as a standardised interval estimate) and a median (as a standardised point estimate, or measure of centre). The final step in the process challenged students to construct a tool that combined their standardised tools for centre (median) with a tool for spread (hat plot to identify and represent the interval estimate). This resulted in a fairly seamless transition to the box plot as a functional tool to express variability in the problem context. By focusing on the purpose and utility of box plots throughout the unit, students appeared to have little difficulty in using them productively as a tool for data analysis and as evidence to justify their conclusions.

The students recognised that using technology enabled them to operate flexibly, clearly visualise the distributions and concentrate on making meaning of the data rather than the mechanics of data construction and analysis (cf., Pfannkuch et al, 2004). Removing the computational boundaries allowed them to operate quickly, accurately and confidently, freeing them to focus on the links between the question, evidence and conclusion.

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REFERENCES

Ainley, J., Nardi, E., & Pratt, D. (2000). The construction of meanings for trend in active graphing. *International Journal of Computers for Mathematical Learning, 5,* 85–114.

Ainley, J., Pratt, D., & Hansen, A. (2006). Connecting engagement and focus in pedagogic task design. *British Educational Research Journal, 32*(1), 23-38.

Bakker, A., Biehler, R., & Konold, C. (2004). Should young students learn about box plots? In G. Burrill & M. Camden (Eds.), *Roundtable: Curricular development in statistics education* (pp. 163-173). International Association for Statistical Education.

Ben-Zvi, D., Aridor, K., Makar, K., & Bakker, A. (2012). Students' emergent articulations of uncertainty while making informal statistical inferences. *ZDM, 44*(7), 913-925.

Biehler, R., Ben-Zvi, D., Bakker, A., & Makar, K. (2013). Technology for enhancing statistical reasoning at the school level. In M. Clements, A. Bishop, C. Keitel-Kreidt, et al. (Eds.), *Third International Handbook of Mathematics Education* (pp. 643-689). New York: Springer.

Franklin, C., Kader, G., Mewborn, D., Moreno, J., et al. (2007). Guidelines for assessment and instruction in statistics education (GAISE). Alexandría, VA: American Statistical Association.

Konold, C. (2007). Designing a data tool for learners. In M. Lovett & P. Shah (Eds.), *Thinking with data* (pp. 267–291). New York: Taylor & Francis.

Konold, C., & Miller, C. (2005). *Tinkerplots* (Software v1.0). Emeryville, CA: KCP Technologies.

Konold, C., Robinson, A., Khalil, K., Pollatsek, A., Well, A., Wing, R., & Mayr, S. (2002, July). Students' use of modal clumps to summarize data. In B. Phillips (Ed.), *Proceedings of the Sixth International Conference on Teaching Statistics*, Cape Town, South Africa. IASE.

Makar, K., & Confrey, J. (2005). "Variation talk": Articulating meaning in statistics. *Statistics Education Research Journal*, *4*(1), 27-54.

Paparistodemou, E., & Meletiou-Mavrotheris, M. (2008). Developing young students' informal inference skills in data analysis. *Statistics Education Research Journal, 7*(2), 83-106.

Pfannkuch, M., Budgett, S., Parsonage, R., & Horring, J. (2004, July). *Comparison of data plots: Building a pedagogical framework*. Presented at the Tenth International Congress on Mathematics Education (ICME-10), Copenhagen, Denmark.

Pierce, R., & Chick, H. (2013). Workplace statistical literacy for teachers: interpreting box plots. *Mathematics Education Research Journal, 25*(2), 189-205.

Scheaffer, R. L., Watkins, A., Gnanadesikan, M., & Witmer, J. A. (1996). Jumping frogs. In *Activity-Based Statistics* (pp. 175-181). New York: Springer.

Watson, J. M. (2008). Exploring beginning inference with novice grade 7 students. *Statistics Education Research Journal, 7*(2), 59-82.

Watson, J. M. (2012). Box plots in the Australian curriculum. *Australian Mathematics Teacher, 68*(3), 3-11.

Watson, J. M., Beswick, K., Brown, N. R., Callingham, R. A., et al. (2011). *Digging into Australian data with TinkerPlots*. Springvale VIC, Australia: Objective Learning Materials.