WHERE'S YOUR EVIDENCE? CHALLENGING YOUNG STUDENTS' EQUIPROBABILITY BIAS THROUGH ARGUMENTATION

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Students come to formal schooling with prior probabilistic conceptions developed through informal experiential events. One such concept is that of chance outcomes being inherently equiprobable, even when not the case. In the design-based research described here, a class of 3rd Grade students was posed an inquiry problem embedded with non-equiprobable outcomes: What is the best addition bingo card? Argumentation was employed as a pedagogic approach to challenging students' equiprobable beliefs, with students supported to develop an evidence-based argument in response. Students initially experienced conflict with the realisation of unequal frequencies, then developed representations to act as theoretical evidence. A shift from conceptualizing equiprobable outcomes towards responses reflecting theoretical distribution was observed. This exploratory research suggests potential for an evidentiary focus to challenge probabilistic conceptions.

INTRODUCTION

Children's beliefs, language and experiences outside of formal learning can lead to the construction of informal probabilistic conceptions, including primary intuitions, heuristics and biases, which are inconsistent with accepted theoretical understandings (Amir & Williams, 1999; Khazanov, 2008). In the case of young students, these experiences are often generated through children's games which incorporate aspects of chance; for example, card games, games involving dice or spinners, and similar chance activities which usually have equally likely outcomes (Amir & Williams, 1999). One specific difficulty, among others, which may stem from this, is a tendency to assign the same probabilities to all outcomes in all situations (Lecoutre, 1984, in Lecoutre, Durand, & Cordier, 1990). These intuitions are shaped over time, and through repeated experiences which confirm and reinforce the conceptions held (Fischbein, 1987). This suggests that students need early exposure to activities and experiences which align with the prior experiences that enabled the establishment of intuitions in the first instance, given there may be potential for unrelated activities to add to the schema rather than challenge it.

Lecoutre ascribes equiprobability bias to students' lack of perception of compound results with students not recognizing that results can be achieved in multiple ways. This is an issue of significant interest for educators globally for several reasons: these early probabilistic experiences are common across many cultures (Amir & Williams, 1999); even adults with a background in probability theory often demonstrate equiprobability bias (Lecoutre et al., 1990); and, these beliefs are highly resistant to change (Fischbein, 1987). Lecoutre's extensive research indicated neither experimental modelling nor explicitly showing the compound structure of outcomes acted to support students to confront alternate probabilistic conceptions (Durand, 1989, in Lecoutre et al., 1990). Thus finding other, more successful, approaches to designing learning experiences that enable students to bridge from existing conceptions to construction of new ones would be essential.

Argumentation is described here in terms of its potential as a pedagogical tool in teaching and learning mathematics and statistics. Essentially, argument describes both a product and a process: the content and structure of the argument (product) and the act of discursively presenting the product in either oral or written format (process). McNeill and colleagues (McNeill & Martin, 2011; Zembal-Saul, McNeill, & Hershberger, 2013) provide a student model of argumentation, adapted from Toulmin's seminal work (Toulmin, 1958; Toulmin, Rieke, & Janik, 1984), which incorporates claim, evidence and reasoning. Essentially the claim provides a clearly-positioned statement to address a particular problem, the evidence provides the grounds upon which the students feel capable of making such a claim, and the reasoning provides the link that justifies the claim based upon the evidence (Figure 1). This link incorporates mathematical/statistical rules or assumptions and may address aspects of the context. Finally, research suggests that students applying themselves to argumentation in mathematics find the addition of qualifiers necessary and

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useful in order to explicate parameters to the problem solution or to express uncertainty when working with informal probabilistic reasoning (Wells, in preparation).

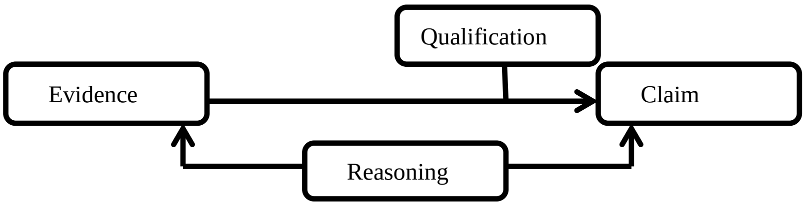

Figure 1: Simplified model of mathematical argument

The term *argumentation* described here refers to *epistemic argumentation* (Lumer, 2010; Siegel & Biro, 1997). Epistemic argumentation theories propose argumentation discourses as those which collectively seek 'truth'. While the goal is to reach consensus, it is a qualified, justified consensus, where all parties not only share the final opinion but—ideally—also their subjective justification for it. Essentially, epistemological argumentation requires that the validity of an argument be evaluated through discipline-based criteria as distinct from, for example, persuasive devices (Biro & Siegel, 1992).

Berland and Reiser (2009) propose three hierarchical levels of explanation and argumentation—understanding, explanation and persuasion. These levels are largely determined by the goals of sense-making, articulation and persuasion, respectively. The goal of understanding is to develop a personal sense of the phenomena under investigation. While evidence is at the core of sense-making, it is aligned with inwardly focused belief systems and the evidence may stem from observed phenomena, prior experiences, or one's own attempts to incorporate or align new knowledge with existing knowledge; as such, these beliefs remain internalised and thus unchallenged. In this instance, this first stage of argumentation can be seen to encompass personally held, 'invisible' understandings about probabilistic outcomes. The goal of explanation is to construct and articulate claims, evidence and reasoning to an audience. Putting students in a position that requires them to 'go public' situates them in such a way that they need to make their links between claim, evidence, and reasoning clear. This 'visibilising' of student beliefs enables the identification of any primary conceptions the students may have. The final level, persuasion, differs in that the goal is to *convince* others of the discipline-based acceptability of the reasoning advanced and to seek to develop the most robust explanation of the phenomena (Berland & Reiser, 2009).

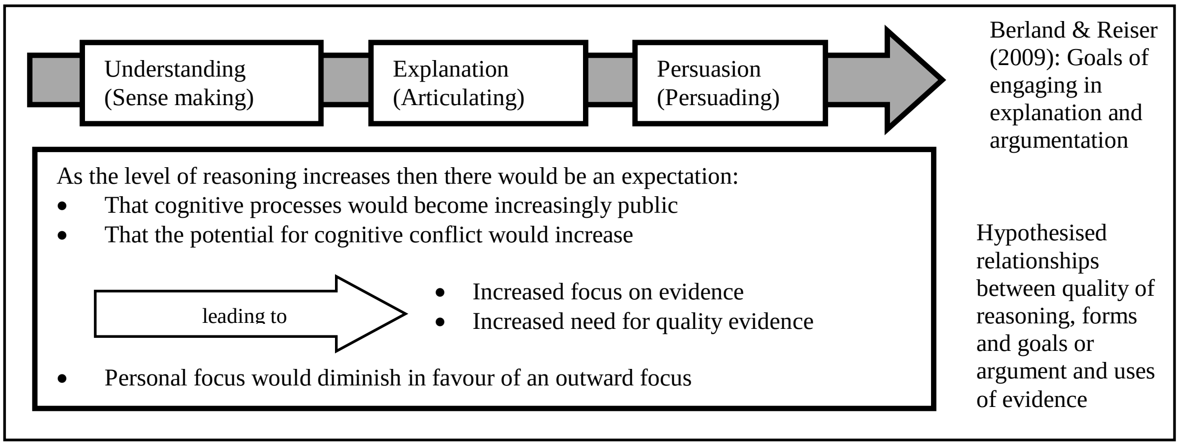

Figure 2: Model indicating potential intereactions between reasoning and goals of argumentation and the use of evidence.

Based on literature and pilot work, initial hypotheses were incorporated into a model (Figure 2) to predict possible implications for evidence as used by students. For example, as students move towards persuasion, evidence would likely need to stand up to increasing levels of external scrutiny and therefore increase students' need for quality evidence. The model suggests the essential nature of evidence in this process, anticipating that as students find weaknesses in their own and others' evidentiary reasoning, they will see that claims must be derived from evidence which stands up to scrutiny by others. Hence, as students are confronted with challenges to their existing intuitions, they will need to incorporate these challenges into increasingly sophisticated schemas which align more closely with normative statistical expectations.

The research question that is the focus of this paper is: *What potential do mathematical argumentation practices demonstrate in challenging equiprobability bias in primary students?*

METHOD

The participants reported in this paper were a Year 3 class (7-8 years of age) from a suburban public school in Australia. The teacher was experienced in inquiry-based mathematics; however, taking an argumentation focus within inquiry was less familiar, having only engaged the class in one prior argumentation-focused geometry unit. A design-based research approach (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003) was utilised with the teacher and researcher jointly developing a unit around a game of 'addition bingo' (Allmond, Wells, & Makar, 2010) to address the inquiry question: *What is the best card for playing addition bingo?* In addition bingo, each possible combination of the sum of two numbers (1 to 10) is written on a slip of paper and placed in a box (100 sums). Children have a card with a 5 x 5 array of self-selected numbers (their predictions based on a theoretical expectation). As each paper is drawn (e.g., 9+6) from the box, children mark off the sum (in this case, 15) once if it appears on their card. A player wins the game if they are first to mark off all of the numbers on their card.

If students' expectations were of equiprobable outcomes, their predicted Bingo numbers could be expected to approach a uniform distribution (Figure 3a). However, if the students recognised that the outcomes would be represented by irregular frequencies, they could be anticipated to select numbers more closely approximating the actual distribution (Figure 3b).

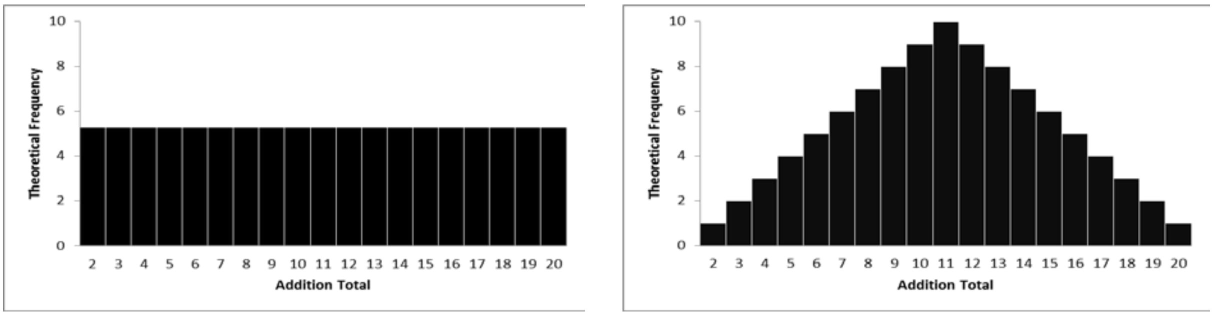

Figure 3: (a) Uniform (equiprobable) distribution (left); (b) Binomial distribution (right)

Data included student work samples, transcribed videotapes of each lesson, and field notes. Analysis was undertaken through several iterations of coding to first provide a general picture and identify salient moments that might provide insight into both the argumentation process and students' development of probabilistic reasoning. A second, deeper level of analysis then took place with identified episodes and artefacts examined using a combination of frameworks (Corbin & Strauss, 2008, p. 40) and mapped against existing literature. In addition, students' Bingo cards were collected and analysed after each of the four iterations to identify shifts in student thinking.

RESULTS

Students were initially presented with the inquiry problem and asked to fill a 5 x 5 Bingo card with the numbers they thought would most likely give them a winning card. The numbers the students chose can be seen as a collated group in Figure 4a below. The distribution of these numbers more closely reflects the uniform (equiprobable) distribution seen in Figure 3a than the theoretical distribution represented in Figure 3b, suggesting students were thinking equiprobably.

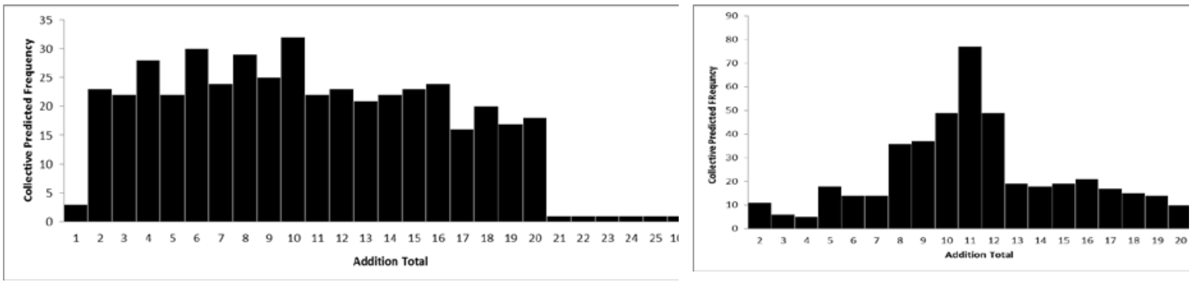

Figure 4: (a) The collated student selected bingo numbers at the commencement of the unit (left); (b) the collated student selected bingo numbers at the completion of the unit (right).

As the class played the first game, several students quickly realised they had selected numbers that could not be called (1, 21-25, and 100). As they continued, the students demonstrated growing awareness of the unequal distribution, initially noting extremes and then frequencies:

Glenn	I can't play anymore. I did two 2's.
Jess	Only 1 + 1 = 2 and that has already been pulled out.
Paul	And I've got two 20's.
Troy	There's lots of 12's. I wish I had made the whole thing 12's.

Playing the game had enabled students to test their cards against experimental outcomes and began to challenge their initial equiprobable conceptions. Many of the students indicated a preference for numbers they had heard called numerous times, suggesting they were focussing their evidence on experimental data [1, 3, 8]; however, Sirena raised a link between the experimental frequencies and the theoretical distribution [4]:

1	Troy	Mostly put 12s on it!
2	Paul	But it might not be mostly heaps of 12s.
3	Glenn	I put five 12s on it.
4	Sirena	Lots were called out and there's lots of ways to make 12. And lots of 12s were called out so I did 5.
5	Teacher	Are there any other numbers that would be called out a lot? That there are lots of different sums that add up to. Are there any others?
6	Gideon	15
7	Teacher	Why 15?
8	Gideon	Because I heard a lot.
9	Teacher	Is there a way of working out which numbers are going to come up most?
10	Paul	…Think about the most ways to make that number win.
11	Teacher	Could you do that? Remember we have been talking about that word 'evidence'? Evidence. If you come out and tell me that twelve comes up all the time, I would like some evidence of that. I need evidence. Can you prove that 12 comes up the most?

The teacher used this discussion [9,11] to challenge the students to consider the evidence they might need to reason to the class that these numbers were more likely to occur in practice. The students were motivated to explore this problem of evidence and used a variety of representations (e.g., lists, tallies, tables). One student, Paul, represented his tally on a number line, effectively constructing a dot plot (Figure 5) which colloquially became known as 'Paul's Mountain'.

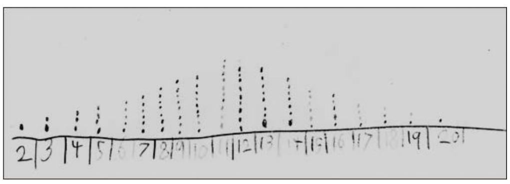

Figure 5: Paul's number line tally (Paul's Mountain)

At the completion of the activity, students crafted a second 'best' addition bingo card. After playing a round, they shared their numbers, evidence base, reasoning, and perceived success with the community. A noticeable shift towards dominant usage of central numbers occurred; however, over-reliance resulted with many students only using a narrow range of central numbers.

12	Gen	All I have 12, 11, 10, 9 and 8. And I wrote - they are all highly likely, that's why.
13	Teacher	OK. Paul, what numbers did you have last time?
14	Paul	11, 11, 11, 11, 11, 10, 12, 9, 12, 13, 13, 9, 13, 9, 10, 12, 12, 9, 11, 9, 9, 10, 11, 10, 13.
15	Teacher	And why did you choose all those numbers again?
16	Paul	11 was most, the others were the second most, likely.
17	Teacher	How many 11s did you have on your card, Clay?
18	Clay	Twelve.
19	Paul	But there is only ten 11s!

Once the issue of over-reliance of central numbers had been raised, the students were asked to produce a third 'best' card, explicating their evidence, and justifying their choices. The students' role was to *convince* the teacher and class that they had selected the best numbers they could, and *provide evidence* for selecting those numbers. This task was aimed at observing whether students acknowledged the theoretical distribution in their responses. Of 17 students who worked on the activity, ten provided a dot plot as evidence for their chosen numbers; of these, two provided source information (an addition table and a table of possible outcomes) for the data in their dot plot. Two students provided addition tables, and five offered no evidence. In terms of reasoning, all 17 students put forth reasoning, with 14 making reference to the theoretical distribution (Table 1). A discernible shift was also noted in the numbers selected by the students for their cards. Figure 4b above shows the combined selection of numbers for the final Bingo cards created by the class. This distribution more closely represented the theoretical distribution in Figure 3b than the original attempt, suggesting that students were now selecting numbers based on envisaged non-equiprobable outcomes, although an over-reliance on the central number, eleven, was still clear.

Table 1: Students' reasoning for final bingo card numbers

Underlying Reasoning		**Sample student comment**
Theoretical Distribution		
Central tendency	11	*I chose 11's because the chance to get 11 is 10/100 which is the best chance of the game. All the numbers near have a high likelihood of being pulled out.*
Spread	2	*I chose three 10's, 11's, and three 12's because they are the three most popular numbers. I chose every other number once because they aren't so frequent.*
Randomness	1	*Uncommon numbers and common numbers because it is a random pick*
Empirical Data	1	*I chose the other numbers because they are from my other Bingo card that has been crossed out.*
Other	2	*I copied Gideon's card*

DISCUSSION AND CONCLUSION

Literature suggested that shifting tenacious, unchallenged probabilistic concepts may be facilitated by creating initial cognitive disequilibrium, and then maintaining that disequilibrium sufficiently to enable students' existing conceptions to be replaced by successively more mature concepts. The research described here sought to bring about that initial disequilibrium by providing an opportunity for students to test their predicted outcomes (their first Bingo cards) with the outcomes of hands-on experimentation (the outcomes of the game). The challenges the students encountered as they sought to refine their cards, develop evidence, and convince others of their evidence and reasoning, served to maintain the disequilibrium and keep the students looking for better models. However, as distinct from Khazanov's observations of teachers "*helping* students build an appropriate model" (2008, p.184, emphasis added), in this instance students were supported and challenged to determine one for themselves and then *convince* the teacher and their peers that they had provided appropriate evidence. In the activity described in this paper, the students had difficulty at times articulating their reasoning. However, their choice of numbers for their 'best' card suggested they were thinking progressively in terms of a theoretical distribution. Young students experience multiple exposures to equiprobable outcomes prior to formal learning, and then often in the early years of schooling; Fischbein (1987) suggests these experiences over time reinforce these beliefs. A possible implication of this research may be to challenge equiprobability bias early, before probabilistic belief systems are heavily entrenched, or at the very least, ensure students have multiple, early experiences with games or activities that do not have equally-likely outcomes. A second implication may be that argumentation as a pedagogical tool has the potential to weaken strongly held schemas and engage students cognitively at a depth that enables more normative schemas to develop. Certainly, both ideas warrant further research.

Maintaining the challenge may also be key. The argumentation model proposed in Figure 2 suggests that as students share their beliefs, they are more open to challenge: whether by others, by responsively crafted activities, or by the students themselves as they struggle to articulate their understandings. The focus on evidence, and the challenge for quality evidence, may work to support new cognitions and focus students away from personal beliefs towards those collectively developed.

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