# THE CHOICE OF GROWTH CURVE

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## INTRODUCTION

The paper is dealing with the application of knowledge of mathematics that students of agriculture and agricultural economics get in the calculus course that precede statistics, in teaching of growth curves with an asymptote, known as S-shaped curves.

The most known growth curves are Gompertz curve and logistic curve. These curves are characterized by period of an increasing growth rate followed by a period with a decreasing one. The point of inflection of the growth curve is that point where the curve turns from concave to convex. The important property of asymptotic growth curves is approaching an upper limit called asymptote and to lower asymptote of zero. Many time series in different fields like in demography, biology, and marketing are characterized by this pattern. The authors present two methods that can help in discriminate Gomperz and logistic curve that are based on mathematical characteristics of curves that students can easily prove.

## METHODS

Method 1. The mathematical properties of curves that students may easily prove are that both curves increase with decreasing relative changes $y_{t+1}/y_t$. The first differences $\Delta_1 y_t = y_{t+1} - y_t$ increase, reach maximum and than decrease. The growth increments of the logistic are symmetrical and close to normal curve, whereas those of the Gompertz curve are skewed. The difference between these models is that in the case of Gomperz curve the ratio of the successive first differences of $\log y_t$ is constant i.e. $\frac{\Delta_1 \log y_{t+1}}{\Delta_1 \log y_t}$, while this is not true for logistic curve. On the other hand in the case of logistic curve the ratio of the successive first differences of reciprocal values $\frac{\Delta_1 (1/y_{t+1})}{\Delta_1 (1/y_t)}$ is constant.

Method 2. A simple selection method based on sample data was introduced by Franses (1994, 1998). Applying logarithm transformation on difference of logarithms on Gomperz (1) and logistic curve (2) it follows that

$$\log(\Delta_1 \log y_t) = \beta^* + \gamma^* t \qquad (1)$$

$$\log(\Delta_1 \log y_t) = \beta^{**} + \gamma_1 t + \gamma_2 t^2 + \gamma_3 t^3 .... \qquad (2)$$

The methods of discrimination between Gomperz and logistic curve are illustrated by examples with simulated and real data using R program.

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