DEVELOPMENT OF IDEAS IN DATA AND CHANCE THROUGH THE USE OF TOOLS PROVIDED BY COMPUTER-BASED TECHNOLOGY

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To support middle school students' learning of data and chance, we have developed a set of classroom activities along with a probability simulation tool integrated into a future version of the dynamic data analysis software TinkerPlots (Konold & Miller, 2004). The activities and the software were designed to build on students' current intuitions. In this paper, we describe the modeling and simulation capabilities of TinkerPlots and how particular features influence the formation of new ideas as students begin to perceive data as comprising signal and noise.

INTRODUCTION

We have recently designed and tested instructional tasks that involve using the simulation component of a future version of *TinkerPlots* (Konold & Miller, 2004). A main objective is to develop understandings of core ideas in data and chance through modeling not only chance events (Konold & Kazak, 2008), but also modeling aspects of realistic data (Konold, Harradine & Kazak, 2007; Konold, Kazak, Lehrer, & Kim, 2007).

This paper focuses on the probability simulation capabilities of the software and how we have been using it instructionally to develop students' perceptions of data as comprising signal and noise. In Konold and Kazak (2008) we elaborate four core ideas we target in these explorations:

- *Idea of distribution*. We focus on the emergent, aggregate properties of data.
- *Model fitting*. Predictions or expectations allow students to assess relevant data.
- *Signal and noise*. In data distributions, signal reveals relatively stable aggregate properties of distributions and noise is introduced by chance variability.
- *Law of Large Numbers*. As a sample gets larger, its aggregate properties tend to get closer to the corresponding features (signals) of the actual process or population.

Before giving a detailed analysis of one of the problem contexts we have had students explored with *TinkerPlots,* we provide a brief description of the software and the instructional tasks we have been developing.

CONTEXT

In the Model Chance project, funded by the National Science Foundation (ESI-0454754), we have developed simulation component for the data exploration software *TinkerPlots* (Konold & Miller, 2004) and curriculum materials for teaching probability and data in the middle school. A major component of the project has been to add to *TinkerPlots* a general probability simulation capability that would allow not only modeling typical chance events, but also building and running models of realistic data. Our hope is that by allowing students to model and explore both these contexts they can begin to see the chance components of real data.

As part of the project we conducted four rounds of field tests in different classrooms (grades 6 through 8) at Lynch Middle School in Holyoke, Massachusetts. Throughout these field tests, we continued to refine both the software and the instructional activities.

TASKS

The current version of our instructional materials involves three chance-related investigations that are interwoven with activities dealing with data during a ten-week of instruction. After giving brief description of activities, we then focus on one activity (the "Wink Problem") and describe how we attempted in this investigation to develop students' intuitions into more formal ideas of chance.

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Three Problem Contexts

We introduce the Wink Problem as a three-person game that involves blindly drawing a disk twice with replacement from a bag. One disk is labeled with a dot (•) and the other with a dash (-). If both disks are dots, student A wins. If both disks are dashes, student B wins. If two disks are different (•,- or -,•), student C wins the game. We suggest to students to imagine that the symbols on the two disks are eyes, which can either be open or closed. Accordingly, we refer to (•,•) as "Stare," (-,-) as "Blink," and (•,- or -,•) as "Wink."

The Family Problem involves exploring the composition of children's gender in families with exactly four children. We first look at a few specific families with four children that we know and start organizing them into a distribution according to the number of boys (from 0 to 4) in those families. Then we explore how the distribution of number of boys will be shaped as we add more and more data to the distribution.

In the Dice Problem, we ask students to anticipate the distribution of the sums (from 2 to 12) if we roll two dice 1000 times. Early in the investigation, they select from among five possible distributions the shape they expect to get, and then we collect for real data and simulated data to compare to their expectation.

Each investigation described above involves initial predictions about the outcomes, collecting data from the actual situation, modeling the situation in *TinkerPlots* to test predictions, constructing the sample space, and testing how closely the simulated data resemble expectations based on analysis of the sample space.

AN EXAMPLE: THE WINK PROBLEM

Making Initial Predictions

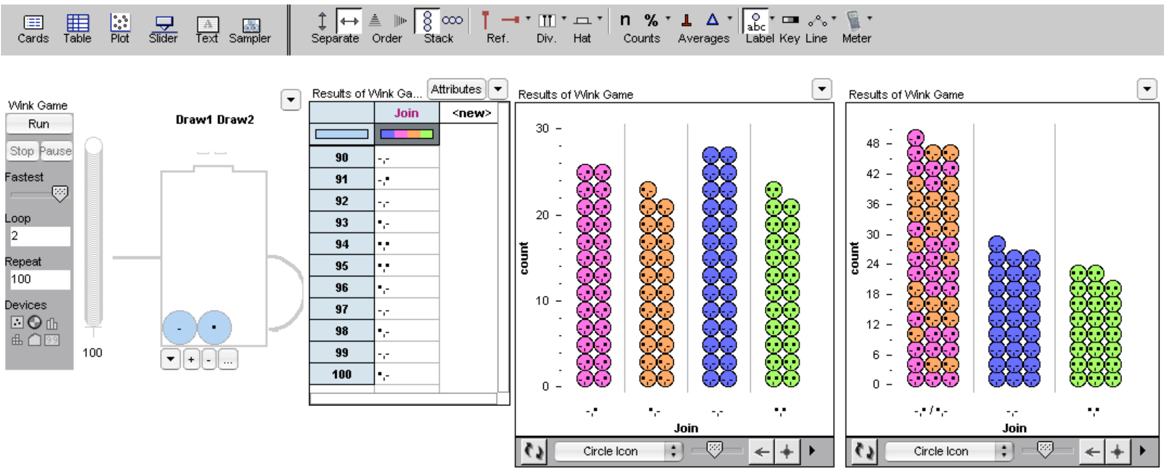
Students come to the class with certain conceptions and beliefs about chance events. When we ask about their initial predictions, they rely on their primary intuitions (Fischbein, 1975), many of which are in conflict with the normative theory, such as representativeness (Kahneman & Tversky, 1972), outcome approach (Konold, 1989), and equiprobability bias (Lecoutre, 1992). For example, before playing the game with three students in front of the class, we ask students whether they think the game, with three players, is fair and to give an explanation. Initially many students believe that it is a fair game. One intuition students have is that because there are three possibilities determined by chance (-,•; •,•; -,-), "you have equal chance of winning." This reasoning does not take into account, of course, that there are two different ways of getting a wink. Another line of reasoning that some students use is that the chances of drawing each disk (- or •) are equal, each result (wink, blink, or stare) is chance of each event is also equal. This could be based on equiprobability bias or a form of representativeness heuristic. In other words, if the simple events are fair, then the combined event is also fair. Even though some students did not consider the game fair, they were initially unable to provide a reasonable explanation.

Collecting Data

Following the initial predictions, we play the game once in front of the class. Then students in pairs play the game 12 times using a bag and two chips and record their results. By manually collecting data, students have an opportunity to understand the process prior to modeling it in *TinkerPlots*. We record the results of 12 trials from each group and the total of wins in a table on the board. Looking at the combined results, the majority of the students will now argue that the game is not fair, as wink will have occurred substantially more often than the other two events. At this point, most students will express the belief that collecting more data will help them confidently decide whether the game is actually fair. Now they need to test their new ideas. We suggest to them that each group playing the game 100 times by drawing from the bag. This suggestion generally provokes a protest and finally a suggestion that we perhaps can use *TinkerPlot*'s Sampler to collect the data.

Modeling

We work together as a class to build and run the first model of Wink game. We ask the class how to change the default settings of the Sampler to play the game. They instruct us to remove all but two elements from the mixer, to label one with a dash (-) and the other with a dot (•), and to draw twice with replacement (see left side of Figure 1). Students seem to be confident that the computer version of the game will give the same sort of results as playing the game with bag and disks. Facilitating this belief, no doubt, is the close resemblance between the two mechanisms (i.e., randomly drawing two objects from a container).



A single mixer device is set to draw twice with 100 repetitions. To the right of the mixer is a table which shows the results of each repetition as they are drawn. The graph next to the table displays the number of outcomes for each event type. In the graph on the far right, the two outcomes (•,- and -,•) are combined into a single bin by dragging one into the other.

Figure 1. Model of the Wink game in *TinkerPlots*

Testing Revised Predictions with Simulated Data

The model of the Wink game built in the Sampler enables students to draw large samples. Looking at these data, they rather quickly become convinced that, contrary to their initial thinking, the game is not fair. Although they offer that wink seems to be about twice as likely as either blink or stare, they cannot at this point give a compelling explanation for why this might be. It is at this point we introduce the sample space idea to students as it provides an explanation that they need to understand why wink occurs twice as often. We ask them to list all different possible results that can happen in the Wink game. We typically see students list either all four simple outcomes or only three possibilities (i.e., •,-; •,•; -,-). The latter list indicates an equal chance of winning for wink, blink, and stare, but this idea is eventually refuted. Some students argue that depending on what you draw from the bag first (either • or -) either the player with Blink or the player with Stare will be eliminated from the game. On the other hand, the player with "Wink" will always still be in the game after the first draw. This insight also helps students conjecture that wink is twice as likely to win. The list of all four possible outcomes makes more explicit that wink occurs twice as often as blink or stare because there are two ways (•,- and -,•) to get it but there is only one way to get either blink (-,-) or stare (•,•).

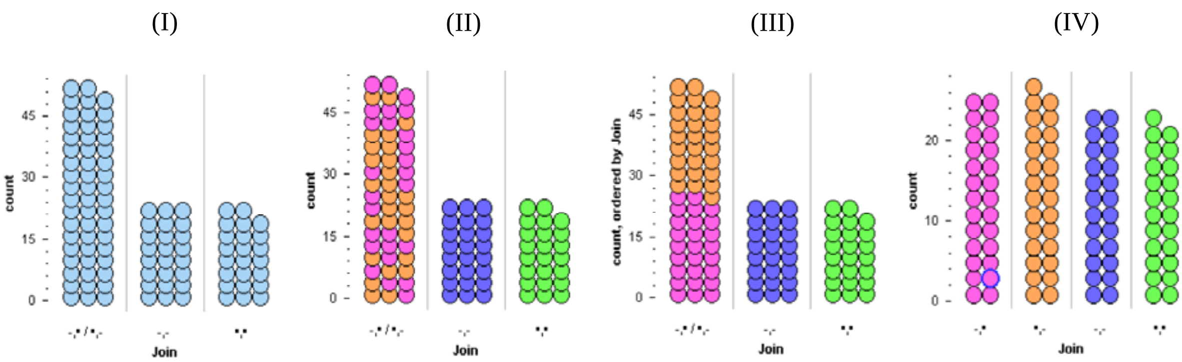


Figure 2. Four different graphs of results from a sample of 100 games in *TinkerPlots*

To test these revised conjectures, we use the model built in the Sampler to generate more data in front of the class. We go through several steps using the various features of the software to provide students insights into why the Wink column in the graph is about twice as high as the others. We first look at the data arranged as shown in graph I (see Figure 2) where the wink outcomes are combined. Next we color the cases by the attribute "Join" (graph II) and ask students what they notice about the Wink stack. Now they can see that it comprises two different colors. We then order the cases as shown in graph III and separate the Wink column into two outcomes by dragging a case icon to the right (graph IV). This last action finally reveals that there are four simple outcomes, two of which form the event "wink."

To make the first graph of the results in *TinkerPlots*, one needs to combine the two outcomes (•,- and -,•). Hence, we initially anticipated that dragging the two outcomes together would focus students' attention on the fact that there are two different ways of getting wink. Surprisingly, only a few students in our classroom tests of this problem have come to realize that the event "wink" is comprised of two outcomes in the process of creating the graphs. The fact that both outcomes (•,- and -,•) are called *wink* perhaps encourages students to think that "they are the same thing" and that the order, therefore, is not relevant. Accordingly, developing the sample space becomes an important component of the activity.

After listing the four possible things that could happen, we rearrange them to form the expected distribution for the three events: wink, blink, and stare (Figure 3). This displays the expected relative frequencies of occurrences of each event. According to this model, wink should occur twice as often as blink or stare. This theoretical distribution helps students perceive the simulation data in a new way – as a noisy version of the expected distribution.

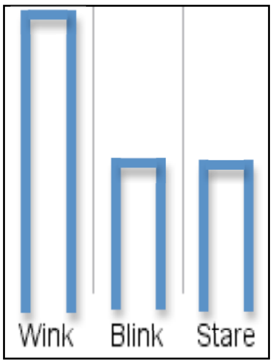


Figure 3. Expected distribution of event types in the Wink game

To support this perception, we introduce sketching as a means to keep record of the empirical results from the computer simulation. This helps students connect expected results with simulated data. We instruct students that the key to making a sketch of a distribution is to pay attention to overall shape and to relative heights of stacks in the graphs rather than to such details as exact frequencies. We take this holistic approach because in our first classroom testing, students tended to focus on exact frequencies in the results and to interpret the small differences in the heights of the stacks as significant even in large samples.

After the introduction to sketching, students use their models built in *TinkerPlots* to draw multiple samples to compare the distribution of results to the expected distribution based on the sample space. They sketch the results they get from five successive samples of 100 and then from 2500 repetitions. To develop the idea of sample-to-sample variability in relation to sample size, we discuss for each trial how closely the actual results from a trial resemble (or fit) the expected distribution. We also run samples of 20 repetitions in front of the class and have students watch the heights of the stacks compared to the expected distribution to see what happens as we increase or decrease sample size. The observation that as the samples get larger and larger, the results will get closer to what we expect helps to establish an understanding of the relationship between the sample size and the fit between the model and observed results. This we view as a qualitative understanding of the Law of Large Numbers.

CONCLUSION

In the investigations of chance events, we initially have students predict and explain the real data, physically collected. Having noticed a misfit between the actual data and their expectations, students typically begin to question their initial expectation. This motivates the need to collect more data and, in turn, the need to using the simulation capabilities of the computer.

The Sampler in *TinkerPlots* allows students to build a model, run large numbers of repetitions in multiple trials, and display the data as they are gathered. Analyzing the situation with more data spurs students to develop new conjectures about the situation. We introduce the sample space as both an explanation for the results they see as well as the basis for predicting what they will see as they collect more data.

Finally, students tend to focus on the differences between the expected distribution based on the sample space and the simulation data. With this new perception, they come to see these deviations as “noise.” By varying the sample size (small vs. large samples) students can observe the noise increasing or decreasing. This helps them develop a general understanding of the Law of Large Numbers—that as the sample size gets large, the observed results tend to get closer to the expected distribution based on the sample space.

In these investigations, the computer tool plays a central role. Without the software it is not possible to gather enough data in the classroom to observe results settling down on expectations as they sample size gets large. Furthermore, the software allows students to see this settling down happening dynamically, in real time. Below we summarize the key aspects of the use of *TinkerPlots* in our chance investigations:

- In building a model of a situation, the software allows students to choose from various Sampler devices, such as mixers, spinners, or distribution objects, and to draw from a single device repeatedly or ” “in-line” from multiple devices (a feature not described above). Students generally can build an appropriate model with little or no support. The device preference usually varies based on the context. For instance, we expect due to the close resemblance to the real situation (randomly drawing two objects from a bag), most students model the Wink problem by draw twice, with replacement, from a single mixer.
- Using the model built in *TinkerPlots*, students can generate large amounts of data quickly over and over. This allows students to articulate their informal theories about chance situations and then put them to the test.
- The *TinkerPlots* environment facilitates students’ visual reasoning via dynamic graphs where the results accumulate as they are generated by the Sampler. Through observing the simulation data from multiple trials coupled with the sketching activity, students can explore the fit between the expected distribution based on the sample space and the empirical data. As a result of these observations, students begin to perceive data as signal and noise.

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