TOWARD ASSESSING UNDERSTANDING OF PREREQUISITE KNOWLEDGE FOR SAMPLING DISTRIBUTIONS

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Sampling distributions play a vital role in understanding how statistical inferences are made, yet students often fail to achieve proficiency in this important topic. The goals of our project are to examine the literature on sampling distributions and concepts that may be prerequisite knowledge for sampling distributions - sampling, variability and distribution, develop an assessment tool that can be used in a timely fashion to provide instructors with feedback on the understanding and misconceptions that students have about sampling distributions and prerequisite concepts, and better understand misconceptions to improve the teaching and learning of basic statistics. We developed new and modified existing assessment items to study the relationship between comprehension of sampling distributions and mastery of each prerequisite area. Open ended questions were refined through a pilot study and we share our experiences in developing items.

INTRODUCTION

Sampling distributions are one of the most difficult concepts for students in introductory statistics to grasp, yet without a solid understanding of sampling distributions students may be unable to develop a complete understanding of confidence intervals and hypothesis testing, the crux of statistical inference. Chance, delMas, and Garfield (2004) carried out a series of research studies to investigate the difficulties that students face when learning about sampling distributions and to examine the impact of interaction with software simulation tools on this learning. Throughout these studies, the authors found that many students possess an incomplete understanding of related concepts which may hinder understanding of sampling distributions. They suggest that prerequisite knowledge includes the ideas of variability, distribution, and sampling and put out a call for additional assessment tools to better reveal the complexity of student reasoning about sampling distributions.

Inspired by Chance, delMas, and Garfield, our research group, formed as part of the Making Outreach Sustainable grant from the Consortium for the Advancement of Undergraduate Statistics (CAUSEmos), decided to investigate the prerequisite knowledge concepts for learning about sampling distributions and to develop an assessment tool to evaluate how these concepts are related to understanding of sampling distributions. Specifically, we aim to investigate whether these concepts are prerequisites and what types of problems in understanding sampling distributions arise from deficiencies in these prerequisite concepts.

While recognizing the advantages of extended teaching experiments and interviews that much of the research in Statistics Education is based upon, these types of interactions are often impractical given student and teacher time constraints (Garfield & Ben-Zvi, 2005). Our ongoing goal in this research project is to develop an online assessment tool that can be used in a timely fashion to provide instructors with quick feedback on the understanding and misconceptions of their students.

In this article we describe the process we are undergoing to develop or modify assessment items, show a few examples of test items we've piloted so far along with types of responses obtained, and share some preliminary results.

PREREQUISITE CONCEPTS – DISTRIBUTION, VARIABILITY AND SAMPLING

In order to define clearly the prerequisite concepts for understanding sampling distributions, we focused on key ideas about sampling distributions that students should understand. Inspired by the work of Chance et al. (2004), key ideas that students should understand about sampling distributions include:

In C. Reading (Ed.), *Data and context in statistics education: Towards an evidence-based society. Proceedings of the Eighth International Conference on Teaching Statistics (ICOTS8, July, 2010), Ljubljana, Slovenia*. Voorburg, The Netherlands: International Statistical Institute. www.stat.auckland.ac.nz/~iase/publications.php

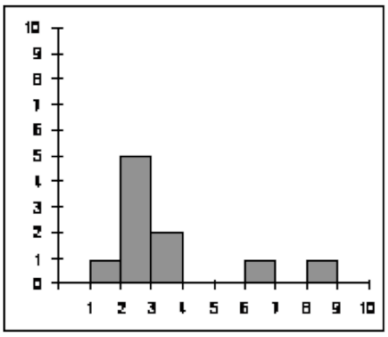
- The distribution of the underlying population is distinct from the distribution of the sample statistic.
- Samples vary and sample statistics will vary, and the sampling distribution is the distribution of all possible sample statistics for a given sample size.
- The mean of the sampling distribution is the mean of the population.
- The variation of sample statistics is predictable, as the sample size increases, the variability of sampling distribution decreases.
- The shape of the distribution of the sample mean becomes normal if either the underlying distribution is normal or if the sample size is sufficiently large.

While considerable research has been conducted into student understanding of variability (Wild & Pfannkuch, 1999; Reading & Shaughnessy, 2004; Watson & Kelly, 2007; Garfield & Ben-Zvi, 2005) and distributional thinking (Bakker & Gravemeijer, 2004; Pfannkuch and Reading, 2006), we focus on those aspects that are specifically related to understanding sampling distributions: ability to read histograms and distinguish between the values observed and the frequencies of those values, understanding of standard deviation, comparing variability across two groups, characterizing shape of distributions, distinguishing between distribution of a population and distribution of a sample statistic, recognizing that the shape of the sampling distribution most often will not resemble the shape of the original population, and appreciation of predictable variation in sample statistics.

In order to assess each of these specific concepts, each member of the research team conducted a background literature review and searched for existing assessment items for one of the four concepts: distribution, variability, sampling, and sampling distributions. We drew from such web resources as ARTIST, WISE, SOCR, and Rice University, as well as statistics education literature and websites of statistics instructors.

For example, the online test begins with a basic question (Figure 1) to gauge student ability to correctly read a histogram, a question originally proposed on the Sampling Distribution Pretest on Bob delMas' website.

The histogram below displays a set of test scores.



1. What do the numbers on the horizontal axis represent?
2. What do the numbers on the vertical axis represent?
3. How many people received a score greater than 4?
4. How many people took the test and have a score represented in the graph?

Figure 1. Basic question to gauge student ability to correctly read a histogram

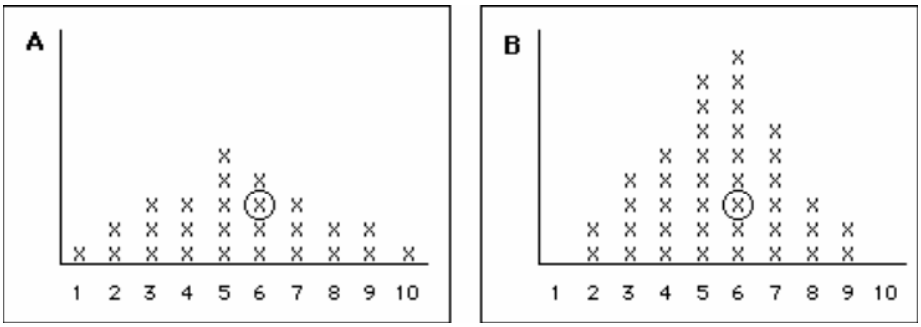
Of the 126 sample pilot responses we obtained, only 54% correctly identified that the horizontal axis represents scores and the vertical axis frequencies. Other common answers included switching the axes (14%) and calling them independent/dependent variables (8%).

The item in Figure 2, modified from the Sampling Distribution Pretest, gauges ability to distinguish between the distribution of the population and that of the sample statistic.

Of the pilot respondents, 76% responded "Yes" but only 36% of respondents provided a correct explanation of their reasoning. Thus, the majority of students were unable to make a clear

distinction between the distribution of the population from which samples were drawn and the distribution of sample means.

Figure A below represents one sample of 26 scores drawn from a population, and Figure B represents part of a sampling distribution of mean scores for samples of size 3. One value is circled in each distribution.



Is there a difference between what is REPRESENTED by the X circled in A and the X circled in B?

Yes

No

Explain your reasoning below.

Figure 2. Item to gauge ability to distinguish between the distribution of the population and that of the sample statistic

A question to investigate student understanding that statistics vary from sample to sample in a predictable way showed that more than half of the respondents expected samples either to vary much less or to vary much more than sampling theory would suggest.

SAMPLING DISTRIBUTIONS – BRINGING IT ALL TOGETHER

The test items illustrated in the section on prerequisites show how we aim to assess each prerequisite idea separately. In order to assess the impact of these on overall understanding of sampling distributions, we include questions on sampling distributions and then examine whether there are correlations in the responses to the prerequisite item responses and the sampling distribution item responses. The item in Figure 3, modified from the Chance, delMas and Garfield Posttest for Sampling Distributions, addresses student understanding of the shape and variability of the distribution of the sample mean.

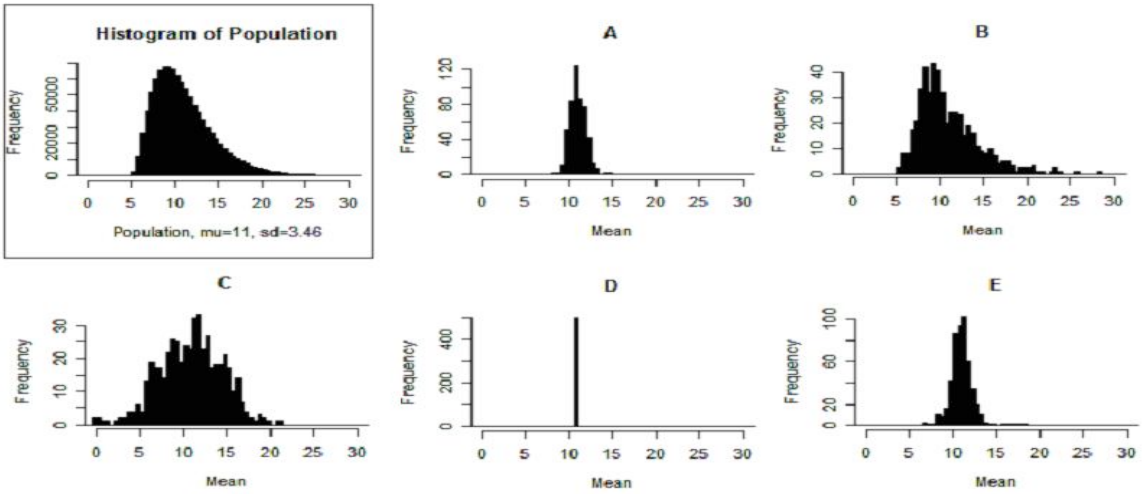
For samples of size 1, 45% of the respondents correctly indicated graph B. However, a surprising 25% indicated graph D, with explanations revolving around the idea that as samples were of size 1, graph D is the only graph with 1 sample, indicating an inability to correctly read histograms. For samples of size 16, 32% correctly answered graph A, but 27%, responded graph E with comments that E is more "compact" than the population but still has the same shape– indicating a misconception that the sampling distribution should have the same shape as the population.

CONCLUSION

Our project to create an assessment tool which provides quick feedback to professors about student understanding of variability, sampling, distribution and sampling distributions is an ongoing process. After an initial "on paper" pilot test of all items at two institutions, we made items available on the WISE website http://wise.cgu.edu to collect data in an easier to evaluate electronic form. We have completed a first round of pilot testing and based on the results of this first round, we are adding items and modifying a few existing items. A preliminary analysis of the pilot results is promising, indicating that student performance on questions about variability and distribution is related to performance on questions about sampling distributions. The evidence for a link between

questions on sampling and on sampling distributions is weak, but we do not know if it is weak due to inappropriate questions or because sampling is less of a prerequisite concept. Our next steps are to develop the tool further, collect a wider sample of data, and do some validity and reliability testing. We welcome suggestions and participation in the piloting of our online tool.

The first graph is a distribution for a population of test scores. Each of the other five graphs, labeled A to E, represents a possible sampling distribution of sample means for 500 random samples drawn from the population.



Which graph represents a sampling distribution of sample means for samples of size 1? Explain your reasoning below.

Which graph represents a sampling distribution of sample means for samples of size 16? Explain your reasoning below.

Figure 3. Item to address student understanding of the shape and variability of the distribution of the sample mean

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