ASSESSING PRE-SERVICE TEACHERS CONCEPTIONS OF RANDOMNESS THROUGH PROJECT WORK

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In this paper we present results of assessing conceptions of randomness in a sample of 215 prospective primary school teachers in Spain. Data were collected as a part of a statistical project where teachers first collected data from a classical experiment directed to assess their intuitions of randomness, then analysed these data and produced a report where they had to justify their conclusions. Conceptions are first analysed from the data collected in the experiments and secondly from the teachers' written reports. Results show a good perception of expected values in a series of experiments and poor conceptions of variation and independence in random sequences. These results also indicate a need for better statistics preparation of these teachers and illustrate the usefulness of working with statistical projects in assessing the teachers' knowledge and improving their statistical and pedagogical knowledge.

INTRODUCTION

New mathematics curricula are being proposed in different countries that place more emphasis on the study of probability and that suggest the need of performing in the classroom simulations and experiments to provide students with a stochastic experience. For example, the new Spanish curriculum for primary education (MEC, 2006) include*: "Random phenomena and related vocabulary"; "describing and quantifying random situations"; "recognizing random phenomena in everyday life"; and "planning and performing simple experiments to study the behavior of random phenomena"*. This curriculum is not an exception, since similar goals are found in recent curricula adopted in other countries. Prospective primary school teachers in many countries enter the Faculties of Education with a very limited statistical competence, and the time available for educating them in statistics and related pedagogical knowledge is very limited. Therefore it is important to find activities where teachers work with meaningful problems related to their professional development and reflect on these activities (Llinares & Krainer, 2006). Moreover we should present prospective teachers with activities based on a constructivist approach to teaching if we want them to use this approach with their future students (Jaworsky & Gellert, 2003). In this paper we analyse one such activity, directed to confront prospective teachers with their conceptions of randomness.

Perception of randomness

Subjective perception of randomness has been extensively investigated (e.g., Wagenaar, 1972; Bar-Hillel & Wagenaar 1991) using a variety of stimulus tasks, which were classified by Falk and Konold (1977) into two main types. In the first type (*generation tasks*), subjects generate random sequences under standard instructions to simulate a series of outcomes from a typical random process, such as tossing a coin. In the second type (*recognition* tasks), people are asked to select the most random of several sequences of results that might have been produced by a random device or to decide whether some given sequences were produced by a random mechanism. Similar types of research have also been performed using two-dimensional random distributions, which essentially consist of random distributions of points on a squared grill. In these investigations, systematic biases have consistently been found. One such bias is known as the *gambler's fallacy*, by which people believe that, after a long run of a same result in a random process, the probability of that event occurring in the following trial is lower. Related to this is the tendency of people in sequence generation tasks to include too many alternations of different results (such as heads and tails in the flipping of a coin) in comparison to what would theoretically be expected in a random process. Similarly, in perception tasks people tend to reject sequences with long runs of the same result (such as a long sequence of heads) and consider sequences with an excess of alternation of different results to be random. In addition to various psychological mechanisms such as *local*

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representativeness that may be in operation and may explain these biases, some authors (e.g., Falk & Konold, 1994; 1997) believe that individual consistency in people's performance with diverse tasks suggests underlying misconceptions about randomness. Other authors, such as Fischbein and Gazit (1984) have also documented children's difficulties in differentiating random and deterministic aspects, and their beliefs in the possibility of controlling random experiments.

Green's (1989, 1991) research with children showed that the percentage of children recognizing random or semi-random distributions actually decreased with age. The study also demonstrated that children were able to describe what was meant by equi-probable. However, they did not appear to understand the independence of the trials, and tended to produce series in which runs of the same result were too short compared to those that we would expect in a random process. Using Green's tasks, Batanero and Serrano's (1999) research with secondary school students showed a mixture of correct and incorrect properties associated by students to randomness. On one hand. students perceived the local variability, lack of patterns in the lineal or spatial arrangement of outcomes, and unpredictability of the random processes. In many cases, the students carried out a statistical analysis of the frequencies for the different events in the random sequences and compared these frequencies with an underlying equiprobability model. Below we describe a statistical project that uses a generation task inspired by one of the tasks proposed by Green (1991) and is directed to assess the teachers' conceptions of randomness and help them overcome some of their misconceptions on the topic.

SAMPLE AND EDUCATIONAL SETTING

Participants were 215 prospective teachers in the second year of study in the Faculty of Education, University of Granada, Spain. These teachers were divided into 6 different groups (30-40 teachers in each group) and followed the same course of mathematics education, with 3 different lecturers, using the same contents, materials and activities. All of them had followed a mathematics course, which included descriptive statistics, the previous year. The project unit analysed here is part of a formative cycle, which is discussed in depth in Godino, Batanero, Roa and Wilhelmi (2008) and that consists of three sessions (each 90 minutes long each). In the first session the prospective teachers were given the statistical project "Check your intuitions about chance", which is described below. After collecting the data and working in small groups they were asked (as an assignment) to complete the analysis at home. They wrote a report with a complete discussion of the project, including all the statistical graphs and procedures they used and their conclusions regarding people's intuitions about chance and their ability to simulate (invent) a random sequence. In a second section the reports were collected and the different solutions to the project given by the prospective teachers were collectively discussed in the classroom. In a third session a didactical analysis, was carried out in order to analyse the pedagogical content knowledge involved in teaching statistics in primary school, the statistical and didactical features of this project, the teaching of statistics through project work and the extent to which the project was useful to teach statistics in the upper level of primary school.

The statistical project: "Check your intuitions about chance"

There were two aims in the project: a) showing the usefulness of statistics to check conjectures and analyse experimental data; b) confronting prospective teachers with their correct and misleading intuitions about randomness. The sequence of activities in the project was as follows:

1. *Presenting the problem, initial instructions and collective discussion.* The session started with a discussion about intuitions; prospective teachers were encouraged to carry out an experiment to decide whether the group had good intuitions on randomness or not. The experiment consisted of trying to write down apparent random results of flipping a coin 20 times (without really throwing the coin, just inventing the results) in such a way that other people would think the coin was flipped at random.
2. *Individual experiments and collecting data.* Each prospective teacher tried the experiment him/herself and invented an apparently random sequence (simulated throwing). They recorded their sequences using H for head and T for tail. Afterwards prospective teachers were asked to

flip a fair coin 20 times and write the results on the same recording sheet (real throwing).

3. *Classroom discussion, new questions and activities.* After the experiments were performed a discussion of possible strategies to compare the simulated and real random sequences was organised. Some participants suggested comparing the number of heads and tails in the two sequences to see whether the average number of heads in each sequence would approach the theoretical value (about 10). This suggestion introduced the idea of producing a graph for the number of tails and heads in the real and simulated experiments for the whole class and then comparing the similarities and differences. The session continued by collecting data about the number of runs and length of the longest run, as students suggested these variables might be compared.
4. At the end of the sessions the future teachers were given a copy of the data set for the whole class, which contained six statistical variables: number of heads, number of runs and length of the longest run for each of real and simulated sequences from each student. They were asked to complete the analysis at home and produce a report.

ANALYSIS OF EXPERIMENTS CARRIED OUT BY THE TEACHERS

Conceptions of randomness were first deduced from the analysis of the data collected by the teachers in their experiments, and by comparing the distributions of some statistical variables that can be deduced from these data. In figure 1 we present the box plots of the variables collected in the experiments and in table 1 the mean, standard deviations, confidence intervals and p-value in the t- test or F- test of differences In these graphs and table, the data collected by the six groups of teachers (n=215) are analysed together.

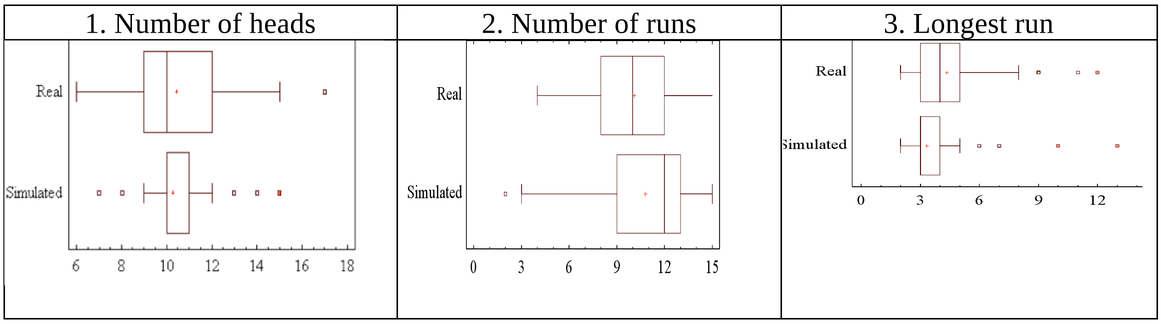

Figure 1. Distribution of variables collected in the experiments

Table 1. Means, standard deviation and 95% confidence intervals for variables in the experiment (n=215)

	Mean	95% confidence interval	p-value	Std. deviation	95% confidence interval	p-value
Number of heads (real)	10.45	[10.13;10.45]		2.05	[1.87;2.26]	0.001
Number of Heads (simulated)	10.29	[10.18;10.73]		1.22	[1.11;1.35]	
Number of runs (real)	10.10	[9.70;10 .49]	0.014	2.90	[2.66;3.21]	
Number of Runs (simulated)	10.78	[10.40;11.16]		2.80	[2.57;3.10]	
Longest run (real)	4.35	[4.14;4.57]	0.001	1.61	[1.47;1.78]	0.001
Longest run (simulated)	3.32	[3.17;3.47]		1.12	[1.02;1.24]	

Results show a good perception of the expected number of heads in the binomial distribution (10 heads) as it is shown in the median and average number of heads in the simulated sequences, and in the non significant difference in averages between real and simulated sequences. However, perception of independence was poor, as prospective teachers produced shorter runs and higher number of runs than expected in a random process (see significant results in t-tests of differences in averages). Perception of variation was also poor, both in the number of heads and number of runs in the sequences produced by the teachers (lower standard deviation; significant *F* tests in the comparison of variances; different spread in the box plots). However, perception of variation was good as regards the number of runs. All these results reproduced those obtained by Green (1991) and Batanero and Serrano (1999) with secondary school students, which is natural, because the statistics training prospective primary teachers receive in Spain is reduced to topics studied in secondary education.

These results also alert of the possibility that these prospective teachers might transmit their incorrect perception of variation and independence in random experiments to their future students. In order to confront these future teachers with their misconceptions, they were given the data collected in their classroom and asked to compare the variables collected from the real and simulated sequences and conclude about the class's intuition of randomness. Sample size for data in each group were smaller (30-40 experiments per group), although the distributions graphs and summaries were very close to those presented in Figure 1 and Table 1. Teachers were given freedom to select graphs or summaries in order to complete their reports.

ANALYSIS OF TEACHERS' REPORTS

A qualitative analysis of the written reports was performed, The majority of participants represented the data with graphs that varied in complexity and that have been analysed in a previous paper (Batanero, Arteaga, & Ruiz, 2009). Many participants also computed averages (mean, median or modes) and variation parameters (range or standard deviation). However, few of them got a correct conclusion about the group intuitions, including a correct judgment of the collective perception of both average values and variation.

Table 2. Frequency (percent) of conclusions

(n=200)	Number of heads	Number of runs	Longest run
Correct conclusion for average and spread	18(9)	3(1.5)	3 (1.5)
Correct conclusion for average only	14 (7)	5(2.5)	3(1.5)
Correct conclusion for spread only	7(3.5)	7(3.5)	4(2)
Incorrect conclusion	107(53.5)	94(47)	83(41.5)
No conclusion	54 (27)	91(45.5)	107 (53.5)

These results suggest that these pre-service teachers did not only hold some misconceptions of randomness, but they also were unconscious of their misconceptions and were unable to perceive them when confronted with statistical data from the experiments. Even when an important proportion of participants correctly computed averages or spread measures and produced statistical graphs where the differences were apparent, few of them were able to translate the differences found in these statistics and graphs to a conclusion about the problem posed in the project. Misconceptions related to runs (length and number) in a random sequence were particularly hard to be perceived by the participants.

These results suggest that these prospective teachers failed to complete the last part of the modelling process (Henry, 1977): they were able to understand a problem of reality (check the intuitions on randomness) and translated the problem to statistical terms (comparing different pairs of distributions); they built and worked with some statistical models that represented the variables deduced from this problem (summaries and graphs), but were unable to translate the results from working with these models to the real situation (they could not understand what these results indicate about the intuitions in the group). Moreover, many participants justified their wrong conclusions by making explicit their own misconceptions of randomness that were parallel to some historical views for this concept. Below we include some examples of these misconceptions, that, although they include some correct properties of randomness, are still incomplete and impede these teachers to fully understand the usefulness of statistics in studying random phenomena.

- *Randomness as unpredictability:* Some participants argued that anything might happen in the distributions, because the process was random. These teachers made explicit their "outcome approach" (Konold, 1991), that is they interpreted some questions about probability in a non probabilistic way and relied on the unpredictability of random events to reject taking a decision: *"You cannot predict the distribution of results, since this is a random experiment"* (AA); *"Results of random experiments cannot be predicted until they happens"* (SG).

- *Randomness as equiprobability.* A few subjects stated that *"Any result is possible, since this is a random experiment; there is equal probability for each result".* These participants related randomness to equiprobability, using a *classical approach* to this concept. For them an event would only be a random if there was the same probability for this event and for any other possible event in the experiment. Consequently they did not perceive the Binomial distribution.
- *Randomness as opposed to causality.* In an early meaning randomness was the opposite of something that had some known causes. 'Chance' was then assumed to be the cause of random phenomena. This vision was shown in some responses: *"I believe the data, both in the simulated and real sequences are due to chance"* (AA).
- *Randomness as variation.* Some pre-service teachers made reference to variation in results as an important feature of random process or either justified randomness based on this variation. *"There is more variety in the random sequence. This is pretty logical, since these results were obtained by a random experiment that involved chance"* (NC); " *In the number of heads there is a difference, … since when we invent the data (in the simulated sequence) results are more even, but real sequence are more uneven, since they are due to chance"* (IE).
- *Randomness as a lack of patterns.* Some participants associated randomness to lack of model or pattern. This conception by held by Von Mises (1952/1928) who indicated that a sequence was random whenever it was not possible to get an algorithm that produced the sequence. However, in fact in the analysis of the project data a variety of models: Binomial distributions, runs, etc. appear, so that randomness can also been interpreted as multiplicity of models.
- *Randomness as lack of control.* Some students viewed randomness as something that cannot be controlled, a view that according Batanero and Serrano (1999) was common until the Middle Ages when random devices (dice, atragalus, etc.) were used to take a decision when fairness was at stake, "*These results appear in the real sequence: despite the inability to control randomness, we got equal number of heads and tails*" (AG). Other participants showed the *illusion of control,* believing they could predict the result of random experiments. For example, one participant classified the students in the group according their capacity for predicting the results: *"Only 21.7% students guessed the number of heads in the experiment; 13% were very close because the had an error of (± 1); the remaining students failed in their prediction"* (LG).
- Other students viewed randomness as lack of order or showed misconceptions of independence, in rejecting the sequence as random because some runs were longer than expected: *"Some students cheated and invented their sequences, since they are too ordered or have too many successive heads or tails to be random"* (EA).

DISCUSSION AND IMPLICATIONS FOR TRAINING TEACHERS

Our analysis of the experimental results and the teachers' reports suggest these teachers present different misconceptions of randomness they could transmit to their future students. It also show the usefulness of working with projects similar to the one described in this report to help these teachers make these conceptions explicit. In order to overcome these misconceptions, after working with the project, it is important to continue the formative cycle. In our experience, we organized a third session, where a didactical analysis was carried out in order to confront teachers with their misconceptions and help them realise the complexity of the meaning of randomness for which different properties need to be understood. We tried to convey in teachers the view that, in fact, it may be preferable to consider the term *randomness* as a 'label' with which we associate many concepts. such as experiment, event, sample space, probability. In this sense, the word 'randomness' refers us to a collection of mathematical concepts and procedures that we can apply in many situations. We should think more about an orientation we take toward the phenomenon that we qualify as 'random' rather than a quality thereof. We apply a mathematical model to the situation . because it is useful to describe it and to understand it. But we do not believe that the situation will be 'identical' to the model. Deciding when probability is more convenient or adapted to the situation than other mathematical models is part of the work of modeling that we should encourage among prospective teachers. Finally, in this last session prospective teachers discussed about the pedagogical content knowledge involved in teaching statistics in primary school and about the statistical and didactical features of this particular project.

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