BANISHING THE THEORY-APPLICATIONS DICHOTOMY FROM STATISTICS EDUCATION

Larry Weldon
Department of Statistics and Actuarial Science, Simon Fraser University, Canada
weldon@sfu.ca

The math-stat versus applications dichotomy in statistics courses has had a regressive influence on modernization of statistics education. Statistics theory involves much more than mathematics, and safe application of statistical methods requires an understanding of the theory behind the methods. Courses which focus on theory alone, or on applications alone, are lacking the linkages needed for a useful knowledge of the subject. But while all statistics courses should include guided experiences in applying statistics theory to application contexts, the choice of contexts should reflect the needs of the particular group of students targeted. In this paper, the implications of a context-dependent taxonomy of courses is explored. This apprenticeship approach has advantages for student motivation and for the authenticity of student learning.

INTRODUCTION

The math-stat versus applications dichotomy in statistics courses has had a regressive influence on modernization of statistics education. Statistics theory involves much more than mathematics, and safe application of statistical methods requires an understanding of the theory behind the methods. Courses which focus on theory alone, or on applications alone, are lacking the linkages needed for a useful knowledge of the subject. But while all statistics courses should include guided experiences in applying statistics theory to application contexts, the choice of contexts should reflect the needs of the particular group of students targeted. In this paper, the implications of a context-dependent taxonomy of courses are explored. This apprenticeship approach has advantages for student motivation and for the authenticity of student learning.

MATHEMATICAL STATISTICS IS NOT THE THEORY OF STATISTICS

"Theory" and "Applications" attached to statistics courses usually really mean "Math Stat" and "Service Stats" respectively. The dichotomy on the basis of math content is a consequence of the math roots of the development of the subject. However, statistics has outgrown its origins, and now encompasses all those concepts and tools that help researchers extract information from data. With the emphasis on the vast majority of statistics students being the practice of statistics, the "Theory of Statistics" might be defined as all those generally applicable tools and concepts that help to extract information from data. This is a much broader theory than the mathematics of statistics. Data quality, audience sophistication, psychology of perception, establishment of causation, assessment of user bias, judgment of non-response, illusions of randomness, practical significance, and many other issues, have theoretical importance for statistics but not much to do with mathematics. While mathematics is a key technology for the development of statistical methods, it does not embrace the entire discipline. Thus the discipline of Statistics is not merely an application of mathematics. A corollary is that no mathematical theory can be the theory of statistics. Put another way, Mathematical Statistics is not the same thing as the Theory of Statistics. Consequently, *if* we wish to provide some students with more "Theory" and less "Applications" and other students the reverse, the amount of mathematics involved in the course is not the way to vary the content. However, I will argue that we do not want to vary the theory-applications split for most students. But first I need to describe an alternative to the Math-based sequence that is so common in our undergraduate programs.

ALTERNATIVES TO THE MATH-BASED SEQUENCE OF STAT COURSES

Lecturers prefer student groups to be homogeneous in their preparation for a course. Different courses are defined to suit different degrees of preparation. However, curriculum designers who have been trained in mathematics naturally think of the preparation of students in terms of the amount of mathematics they have mastered. This has typically led to two streams of statistics courses: math-based courses and service courses, the latter emphasizing methods but with

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a minimum of mathematics. But this dichotomy has caused problems in both streams. The math-based courses often do not instill in students the intuition they need to judge the applicability of the various techniques they learn. And the service courses often teach “methods” to students who do not appreciate model assumptions underlying the methods they are learning. Both streams fail to involve students in the linkage between the theory of statistics and the constraints imposed by real world applications.

Once statistics is appreciated as a separate discipline from mathematics, the level of preparation needed for a given statistics course should be determined by the degree of experience the student has with statistical analysis. The novice should be shown the breadth of statistics tools and concepts so that they appreciate the nature of the discipline. For example the first course could include some simulation, time series, design of experiments, Bayesian decision-making, and perhaps some trivariate graphics, in addition to a few of the more traditional topics. The next course could include some more methods, preferably in the context of applications that are of interest to students. The focus of this course should be to improve the capability of practitioners to extract information from data, to understand the strategies and requirements of useful data collection, the use of graphics for data analysis, and the various strategies for relating variables in a multivariable data set. Additional courses would give students the mathematical tools to develop new methods for special situations. But the students taking such math-stat courses would have already gained an understanding of the role of intuition and judgment in statistical applications. Even experts need to know how to use statistics strategies on data!

EVERYONE NEEDS BOTH THEORY AND APPLICATIONS

Proper use of statistics requires both conceptual understanding and application experience. Of course, different amounts of statistics education will prepare students for different roles in the use of statistics, but whatever the amount, the mix of theoretical concept and practical experience should be about the same. A student who has mastered the requirements of a typical “Service” course will not add much statistical expertise to an employer: how much experience would such a student have in the judgment of data quality, sampling shortcomings such as non-response, awareness of perception problems with graphical displays, verbal explanation of the rationale of methods, adjustment for the statistical sophistication of the audience, degree to which correlation information can imply causation, awareness of the practical significance of statistically significant results, familiarity with the impact of model assumptions, need or lack of it for parametric models, illusions of randomness, etc.? Of course the student who has mastered the requirements of a typical “Math-Stat” course is probably even worse off in approaching these same key issues, since they will have had almost no data analysis experience.

Now imagine a student who has been guided by an experienced instructor through a series of real-world data-analysis experiences. Suppose that the instructor has introduced concepts and techniques as the experience required. Would this student have encountered the “theory” issues alluded to above? Perhaps not all of the issues would be mastered, but some would be. The difference between a student training to be an expert in statistics, and one simply hoping to make sensible use of some of the more widely used techniques, is the amount of guided experience they have had. The experiential model of teaching and learning has the feature that it actually guides students in the real-world application of statistical theory. See Weldon (2009) for a full discussion and additional references. The techniques that a student does encounter will automatically be ones that arise in real-world application: the technique content of such a learning model is automatically “authentic”. This does not mean that all required techniques will have been learned, but rather that those that are learned are likely to be genuinely useful in practice. Moreover, the crucial step of implementation in a real-world setting will have been experienced, and the required attitudes and intuitions absorbed. A single course produces a student with some appreciation for the most common strategies of statistics. The creation of the expert is accomplished by taking several such courses.

Thus, the “level” of a statistics course is determined by its position in a sequence of courses, rather than by its mathematical content. The expert needs “more”, not “different”.

APPRECIATION -> PRACTITIONER -> EXPERT

What is the relationship of the mathematical sophistication of a student group and the "level" of a statistics course? It is clear that issues of maximum likelihood, multivariate analysis, Bayesian inference, combinatoric strategies in nonparametrics and experimental design, asymptotic theories, spectral theory for time series, etc. require sophisticated mathematics. The requirements of higher-level courses in statistics, such as would be required of experts, should have considerable prerequisites in mathematics to enable forays into these areas of advanced statistics. However, the statistics courses themselves should not be spending large portions of their class time covering this mathematics – rather the mathematics instruction should be left to the math faculty. Statisticians always complain that user departments should not be teaching statistics when the experts in statistics are in the statistics departments. By the same logic, statistics courses should not be teaching mathematics! We need to use mathematics, but concentrate on its proper use in the context of extracting information from data: i.e., in statistics.

MATH FOR STATS

A practical question is: at what level should a statistics course require students to have calculus? Many universities have a "service" stream of three courses that do not require calculus– in fact they do not require any advanced mathematics beyond high school math. This suggests that at least three courses could be reasonably taught without a math prerequisite–usually they would be basic descriptive and inferential stats, regression and anova, and experimental design. Of course, it is not really necessary to offer the three courses with these techniques specified: it is likely that three experiential courses would cover those techniques anyway, given an instructor with adequate data analysis experience. So perhaps the fourth course in the sequence might have stronger math preparation as a prerequisite: calculus, discrete math, and linear algebra.

A related issue is the need for probability–especially applied probability models. Applied probability models can involve simulation and graphics, but need not involve data analysis. So presumably this material could be taught as mathematics, or perhaps "applied" mathematics. Students need to know the basic models, like gamma, beta, lognormal, etc., but it is important they know what sort of phenomena they model, and how the parameters relate to typical applications. Things like Poisson processes and simple queues need to be understood well by statistics practitioners. It seems reasonable to have this material taught by mathematicians, as long as they have some modeling interest and experience. This material would certainly support the subsequent experiential statistics courses which would inevitably involve simulation and graphics. Of course, the applied probability material could also be taught by statisticians, but the content need not be combined with data analysis content. There is a lot to understand about the models without complicating the discussion with the dilemmas of inference.

ADAPTING COURSES TO SPECIFIC STUDENT INTEREST GROUPS

Courses for Statistics Practitioners

A big advantage of experiential teaching over technique-based teaching is that the context of experiences can be tailored to the interests of the student group. Although as generalist statisticians we may bemoan the fact, it is nevertheless true that care-givers tend not to be interested in engineering applications, science students tend not to be interested in government policy, and business students tend not to be interested in biology. When a student is interested in the context of a data-based study, they are likely to be interested in extracting the information from that data, and may be pre-disposed to absorb the concepts and tools needed to root out the information. For this reason, courses should be segregated by student interest group: life science examples for life science majors, social science examples for social science majors, engineering examples for engineering majors, etc. In addition to providing for a more-motivated student group, this strategy allows statistics departments to avoid the unpopular huge classes of undifferentiated students. The unpopularity of these huge classes is not simply something to be overcome with an entertaining lecturer, since the quality of learning is likely lacking even if the students seem happy with the lecturer. See Naftulin, Ware and Donnelly (1973) concerning the enthusiastic student response to an empty, but entertaining, lecture: the Dr. Fox effect. Moreover, without demonstrations of effective use of statistics in the fields of interest to students, the reputation of the discipline will suffer.

Courses for Statistics Experts

We have discussed the use of experiential teaching for students in majors other than statistics itself. But what examples should be used for the statistics majors? This is one group for which it is not obvious which contexts would be of most interest. In fact, there is certainly a subgroup of statistics majors that would rather not have any applications at all! These are the ones that are currently delighted with the traditional math-stat courses. One wonders why these students are not in pure math majors? One explanation frequently heard is that a statistics major is easier to market in the job market than a math major. This may be true, but is an application-free course really a proper training for a statistician? Whether the math-stat major becomes an academic or an employee of business or government, to be most useful the student will need a great deal of application exposure. One can only hope that the work-environment provides the opportunity for this exposure. In particular, it is a concern that many new statistics instructors will not have application experience, and they will suppose that their own training is appropriate for their students.

The math-stat majors need to be immersed in applications to complete their education in statistics, even if they are not keen on the immersion. The contexts useful for this group should be diverse and unspecialized: neither life-science-oriented nor natural-science-oriented nor social-science-oriented. A deviation from this would be an undergraduate major in “biostatistics”. Perhaps this kind of specialization could occur at the graduate level, as long as the more varied experience has been met at the undergraduate level. For consider, as an example, that the strategies met in industrial quality control (reduction of variability saving cost and maximizing profit) may well apply to hospital management (standardization of management across hospitals and maximizing net revenues available for improvement of care). Or marketing strategies such as customer satisfaction surveys could be applied in the context of an engineering consulting firm. The best preparation for a career in statistics requires an exposure to a wide range of application contexts. So rather than an absence of applications in a particular area, statistics majors should have exposure to applications in as many different areas as possible. One limitation for statistics majors is that the sophistication of the application context may have to be constrained: stat majors would not be experts in any applied area even if they had been introduced to applications from several areas.

How is it possible to involve the statistics major in a broad variety of applications and, at the same time, include a rigorous training in mathematical statistics? If the curriculum must include courses in mathematical statistics, then they should come after the applications courses. Arguably, the mathematics of statistics need not be taught at all if the student has sufficient courses in mathematics itself. The use of mathematics in statistics courses will have the same status as the use of every other kind of thinking that the student has been trained in. The student needs “common sense”, logical thinking, awareness of potential biases, judgment of the effects of randomness, ability to consider the limitations of an audience in designing informative displays, judgment of the amount of time available for devising an appropriate analytical method, etc. Thus statistics course should teach statistics, broadly, and not use the excuse of the need to teach mathematics at the same time, in order to avoid the non-mathematical aspects of statistical problem-solving.

IMPLICATIONS FOR THE TAXONOMY OF STATISTICS COURSES

The proposal is to have just one kind of statistics course at the undergraduate level: the kind where students are guided through solution of statistical applications, concepts and techniques being introduced as they are required by the application. Of course the degree of sophistication of the concepts and techniques will be limited by the parallel training of the students in other subjects: calculus, linear algebra, combinatorics, and possibly in-depth training in an area of application, such as psychology, business, or chemistry.

However, courses would be differentiated by the context of the applications introduced. Statistics majors would be introduced to applications with relatively simple details from the various application areas. Bioscience majors would have applications from bioscience. Business majors would have applications from business. And so on. In other words, a list of undergraduate course might look like this:

- Statistics 1 (life) Statistics 1 (social) Statistics 1 (natural) Statistics 1 (general)
- Statistics 2 (life) Statistics 2 (social) Statistics 2 (natural) Statistics 2 (general)
- Statistics 3 (life) Statistics 3 (social) Statistics 3 (natural) Statistics 3 (general)
- Statistics 4 (general)
- Statistics 5 (general)
- Statistics 6 (general).

And of course there would be more application-area divisions and more levels in situations where the numbers allow the additional specialization.

OPPORTUNITY FOR FACULTY GROWTH

If the scheme proposed here is put in place, it may lead to a repatriation of statistics courses from user departments, and growth in numbers for the statistics department. However, the “growth” in faculty capability might even be more dramatic. Imagine the creative explosion caused by faculty addressing new problems of interest to user departments. Imagine the new awareness of techniques needing development. Imagine the enhanced opportunities for faculty consulting and collaborative research.

CONCLUSION

In 1925, Harvard University Business School introduced the Case Study Method of Teaching. It is always reported as a very successful method. It is time we tried to bend this idea toward statistics education!

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