A MODEL FOR TEACHER KNOWLEDGE AS A BASIS FOR ONLINE COURSES FOR PROFESSIONAL DEVELOPMENT OF STATISTICS TEACHERS

Thomas Wassong and Rolf Biehler
Institute for Mathematics, University of Paderborn, Germany
wassong@math.upb.de

The paper will report from an ongoing project that develops online courses for the professional development of secondary statistics teachers. The design of the course is to based on an analysis of knowledge and competencies that teachers need in their professional practice. We aim at relating the general discussion on teacher competence to the specific needs that statistics teaching requires. Our paper starts with an analysis of theories of professional development of teachers and their knowledge elements. We will combine content knowledge, pedagogical content knowledge and knowledge for technology use and make this concrete for the knowledge of statistics teachers.

INTRODUCTION

Our ongoing project is developing an online course for the professional development of secondary statistics teachers. This professional development course will be based on a theoretical framework for professional teacher competencies. Therefore we defined four steps for the development of the course: (1) development of a framework for professional teacher competencies, (2) analyzing current literature with the view of the developed framework, (3) selection of the content for the course and (4) design and evaluation of the online-course.

Up to now we finished the first step and started the second one. The design of the course will be based on experiences and design principles we used in two other elearning projects: eFATHOM and VEMA. We will integrate learning a computer tool such as Fathom into our course. eFATHOM is an ongoing project by Tobias Hofmann and Rolf Biehler, which provides an online course for the learning of Fathom for data analysis and simulation. The design of the course is orientated by current research in educational psychology emphasizing self-regulated learning, design principles of elearning material and based on empirical research concerning students' use of Fathom. VEMA is an ongoing project by a group of mathematicians and mathematics educators from the Universities of Kassel and Paderborn for developing elearning material for mathematical bridging courses. In this project we developed a concept for content structuring in modules according to different knowledge types and learning scenarios. Our online-course will reuse and adapt the concepts of these projects (Hofmann, 2010; VEMA, 2010). In this paper, however, we first present our framework for professional teacher competencies. In a second step we choose a chapter from Garfield & Ben-Zvi's (2008) book and analyze it with regard to our framework.

THEORIES OF TEACHER KNOWLEDGE AND COMPETENCIES

Overview

The specific approach we decided to take is basing our future course on theories of the structure of teacher knowledge and teacher competencies. Starting with the famous topology of professional teacher knowledge from Shulman (1986) we develop a structural model of competencies. In addition to this topology, we have to include knowledge components and competencies related to the use of technology in the classroom, as Niess (2005) and Mishra and Koehler (2006) have developed in general. In the context of statistics education the inclusion of technology as part of a model of statistics teachers' knowledge was discussed by Lee and Hollebrands (2008). Shulman describes his categories as categories of knowledge. We expand his categories of knowledge to categories of competencies, like Ball and her team (cf. Ball, Thames, & Phelps, 2008) do. We use the concept "competence" with the following four aspects to consider: First competencies are related to the system of action requirements in a domain. Second they include the cognitive and non-cognitive disposals of the actor to solve exercises or problems concerning the domain. Third competencies are context-sensitive, so they have to be learned in an active way with taking conditions of the context for the purpose of practicing into account. And forth they are also disposals for self-organization of the actor to act in unknown situations (cf. Schaper, 2009). In order to make the origin of our structure obvious we keep the wording of Shulman.

Starting with Shulman's (1986) characterization of teacher knowledge we take into account the research of Ball and her team (cf. Ball, Thames, & Phelps, 2008). We also think that

In C. Reading (Ed.), *Data and context in statistics education: Towards an evidence-based society. Proceedings of the Eighth International Conference on Teaching Statistics (ICOTS8, July, 2010), Ljubljana, Slovenia*. Voorburg, The Netherlands: International Statistical Institute. www.stat.auckland.ac.nz/~iase/publications.php

technology has to be considered as part of teacher knowledge, so we take up the ideas from Niess (2005) and Mishra & Koehler (2006). Figure 1 shows the relations between the components. The outer ring shows the three knowledge domains content, pedagogic (PK), and technology (TK). With Ball and her team we split the content knowledge into common content knowledge (CCK), special content knowledge (SCK) and horizon knowledge (HK).

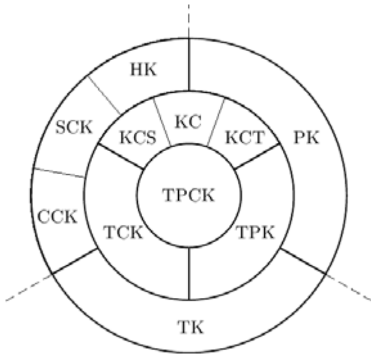

Figure 1. Components of teacher knowledge and competencies

In the middle ring each element of the first three domains is combined with another element of these three domains to new knowledge domain. There are pedagogical content knowledge (PCK), technological content knowledge (TCK) and technical pedagogical knowledge (TPK) be generated. Ball and her team also split the pedagogical content knowledge into knowledge of content and students (KCS), knowledge of curriculum (KC) and knowledge of content and teaching (KCT). The inner circle combines all three domains in one knowledge domain, the technological pedagogical content knowledge (TPCK). So in each step from the outline to the centre the first three knowledge domains are connected stepwise. Our project just focuses on the content "statistics in secondary school", so some of the knowledge domains are not relevant here. In a first step we concentrate on all knowledge domains related to content knowledge: CCK, SCK, HK, KCS, KC, KCT, TCK and TPCK. In the following we will describe the knowledge domains in more detail.

Content Knowledge (CK)

Shulman (1986) describes the content knowledge this way: "The teacher needs not only understand *that* something is so, the teacher must further understand, *why* it is so, on what grounds its warrant can be asserted, and under what circumstances our belief in its justification can be weakened or denied. Moreover, we expect the teacher to understand why a particular topic is particularly central to a discipline whereas another may be somewhat peripheral." (p. 9). Ball, Thames and Phelps (2008) split the content knowledge into common content knowledge (CCK) and special content knowledge (SCK). So teachers need not only to know statistics within the meaning of university statistics (CCK), they also need more content knowledge, only teachers need to know (SCK).

As part of CCK the teachers:

- know the central definitions, properties and theorems and their proofs and apply them,
- know connections between the central concepts and relate them,
- recognize student answers as right or wrong on the basis of subject matter knowledge,
- know relevant applications in- and outside mathematics.

As part of SCK the teachers:

- know different definitions, proofs and plausibility arguments,
- choose different representations for particular purposes and recognize what is involved in using a particular representation,
- have a great collection of neat examples,
- know the importance of the topic in relation to the whole subject and are able to explain this to students and parents (c.f. Ball, Thames & Phelps, 2008, p. 400).

Additionally to CCH and SCK Ball and her team consider the horizon knowledge (HK) as part of the content knowledge. As part of HK teachers know "how mathematical topics are related over the span of mathematics included in the curriculum" (Ball, Thames & Phelps, 2008, p. 403).

Pedagogical Content Knowledge (PCK)

"Within the category of pedagogical content knowledge I include […] the ways of representing and formulating the subject that make it comprehensible to others. […] Pedagogical content knowledge also includes an understanding of what makes the learning of specific topics easy or difficult." (Shulman, 1986, p. 9) This quote shows that Shulman distinguishes two aspects of PCK. Ball and her team revisit these two aspects and name them knowledge of content and teaching (KCT) and knowledge of content and students (KCS). As part of KCT teachers:

- know different sequences of exercises, explanations, definitions and examples and rate them for particular learning groups,
- know different introductions for a particular topic and rate them in their adequacy for particular learning groups,
- adapt their lesson planning concerning changes in the classroom.

As part of KCS teachers:

- rate exercise according to their difficulty for individuals and particular learning groups,
- design a classroom test,
- recognize typical errors and resolve them,
- estimate the processing time of a task for individuals and particular learning groups.

Additionally Ball, Thames, & Phelps (2008) see knowledge of curriculum (KC) as a separate category in PCK. Originally Shulman listed this category besides CK and PCK. In Accordance with later research from members of Shulman's team (Grossmann, 1990) Ball and her team allocate this category to the pedagogical content knowledge, but they are not "sure whether this may be a part of our category of knowledge of content and teaching or whether it may run across the several categories or be a category in its own right." (Ball, Thames, & Phelps, 2008, p. 403) As a part of KC teachers rate schoolbooks and additional materials by their content, representations and definitions, remarks, examples, and exercises for particular learning groups concerning the goals of his class.

Technological Content Knowledge (TCK)

For Mishra and Koehler (2006) "Technological content knowledge (TCK) is knowledge about the manner in which technology and content are reciprocally related" (p. 1028). As part of TCK the teachers:

- know the whole range of software and their educational potential concerning the subject matter and choose the appropriate software for a certain pedagogical or subject matter objective,
- know the limits of the technology and the use of technology concerning the subject matter,
- use technology (software) for problem solving regarding the subject matter,
- know how the use of technology changes the subject matter, their methods and aims, reflect these changes and have an own, founded viewpoint to this change.

Technological Pedagogical Content Knowledge (TPCK)

The TPCK combines the three main knowledge domains content, pedagogy and technology. For Mishra and Koehler (2006) this means the knowledge about the "complex relationships between technology, content, and pedagogy" (p. 1029) and the ability to adjust the other two knowledge domains when the third changes. Niess (2005) describes the outcomes for a teacher preparation program: "(1) an overarching conception of what it means to teach a particular subject integrating technology in the learning; (2) knowledge of instructional strategies and representations for teaching particular topics with technology; (3) knowledge of students' understandings, thinking, and learning with technology in a particular subject; (4) knowledge of curriculum and curriculum materials that integrate technology with learning in the subject area" (Niess, 2005, p. 511).

TEACHERS' KNOWLEDGE ABOUT CENTER – AN EXEMPLARY ANALYSIS

The IASE/ICMI study (Batanero 2008) has focused on teachers' knowledge in statistics. It would be helpful to analyze and structure these papers from a shared model of teachers' professional knowledge in statistics. Up to now a shared model of teachers' professional knowledge has not yet been established. Groth (2007) started an attempt to analyze the statistics teachers' knowledge from a model, but he concentrates only on CCK and SCK. A reasonable goal for the future will be to (1) understand the structure of the literature, (2) identify the strengths and weaknesses of the literature, (3) identify the central competencies for professional teaching and (4) check our framework for gaps and inconsistency. This is beyond the scope of this paper.

Instead we choose the recent book Developing Students' Statistical Reasoning from Joan Garfield and Dani Ben-Zvi as an example on that we apply our framework. This book has the goal to "provide a useful resource for members of the statistics education community that facilitates the connections between research and teaching" (, p. vii). So this book can be interpreted as providing professional knowledge for teachers of statistics. We concentrate on the chapter: Learning to Reason About Center (Garfield & Ben-Zvi, 2008, pp. 187-200) and discuss its content regarding to the eight competency/knowledge categories described above. The chapter on center is divided in two parts: in a first part the authors describe relevant research results and describe their input for teaching statistics. In a second part the authors develop a set of lesson plans with student activities based on the research results described above.

This book has not the aim to teach the subject matter of statistics, so the competence categories for CCK and SCK are not covered explicitly. But implicitly we can reconstruct the main CCK competencies from the aims for student competencies in the suggested lessons. For CCK we identify on a more technical level the ability to define and calculate the mean and the median for given data, construct a distribution with a given mean or median, choose an appropriate measure of centre depending on the given type of data and distribution, and understand weighted means for calculating the mean of aggregated data, on a more conceptual level we find understanding the importance of examining centre and spread together, understand the "idea of deviations (differences from the mean), how they balance out to zero" (Garfield & Ben-Zvi, 2008, p. 197), and that these deviations explain the sensitivity to extreme values, understand that the median is not sensitive to extreme values, and "understand the differences between mean and median in their interpretation and properties" (Garfield & Ben-Zvi, 2008, p. 197). These competencies can be found mainly in the description of students learning goals, it is clear that the teachers have to know this themselves as well.

SCK is difficult to distinguish from CCK. Looking at the text we find the ability to identify the "mean as typical value, mean as fair share, mean as data-reducer, and finally mean as signal amid noise" (Garfield & Ben-Zvi, 2008, p. 192). It is also mentioned, that the deviation idea begins to connect ideas of center and variability. It is unclear whether we should count this as SCK or CCK. Anyway, these interpretations of the mean are often not part of university statistics book.

Horizon Knowledge is explicitly mentioned in this chapter in a few places, mainly in The importance of Understanding Center (Garfield & Ben-Zvi, 2008, p. 188). Here the authors define the understanding of center as a component of understanding distributions and interpreting data graphs and analyses. Also the use of center by considering residuals in a regression analysis and that center and spread are revisited for statistical inferences are mentioned. We miss mentioning that the idea of center is also found in probability as the expected value. This is typical in the following sense: Horizon knowledge is dependent on what we consider as fundamental ideas in statistics. From the perspective of stochastic, the relation to expected value is central, if we come from the perspective of applied statistics, this relation is less important (Burrill & Biehler, 2010).

The Knowledge of content and students is one of the major topics in this chapter. The authors list a lot of research results regarding the misconceptions and levels of understanding and reasoning with mean and median. They distinguish between research for students' problems of understanding center as its own and students problem of understand center as "signals in noisy processes". For Understanding Means (Garfield & Ben-Zvi, 2008, p. 189) it is mentioned that the balance model is a helpful tool for students, the two properties "the mean as a data point between extreme values of a distribution" and "the sum of the deviations about the mean equals zero" are better understood than the two properties "when the mean is calculated, any value of zero must be taken into account" and "the average value is representative of the values that were averaged" (Garfield & Ben-Zvi, 2008, p. 189f.). One of the greatest problems for students is the conceptual understanding of the mean, they often easily only acquire the skill to calculate the mean. There are also problems in comparing two different-sized sets of data with the mean and in understanding the concept of a weighted mean. For Understanding median (Garfield & Ben-Zvi, 2008, p. 191) only two problems are mentioned: the difficulty of determining the medians of graphically presented sets and of unordered sets.

In a study from Mokros and Russell (1995) students were asked for the typical value of a set of data. Students respond with (1) locating the most frequently occurring value, (2) executing an algorithm, (3) examining the data and giving a reasonable estimate, (4) locating the midpoint of the data or (5) looking for a point of balance within the set. These responses show that only an algorithmic understanding of mean were developed, but not a conceptual understanding. But these conceptual definitions are necessary to develop a sophisticated level of statistical reasoning. In the signals in noisy processes context, which was elaborated by Konold and Pollatsek (2002), the authors report that in the history of statistics the mean as a reliable indicator of signal has not been accepted from the beginning referring to Stigler (1986). They call for more research in such contexts. Such knowledge can help to advise instructions.

In the context of choosing an appropriate measure of center Groth and Bergner (2006) use four levels based on the SOLO taxonomy from Biggs and Collis (1982) to describe the students ability to compare and contrast mean, mode, and median. This is referred to by Garfield and Ben-Zvi. (1) On the unistructural level students just use the definitions to contrast them, (2) the multistructural level allows students to response with vague notion that these three are all tools to analyze a set of data. (3) On the relational level students recognize that all these three measures the center or typically value of a set of data, (4) the extended abstract level includes also the ability to discuss these three measures concerning their usefulness in a certain set of data.

The Knowledge of Content and Teaching is the second main topic in this chapter. First it is reported about the Using the History of Measures of Center to Suggest Instructions (Garfield & Ben-Zvi, 2008, p. 193). Here five ideas are mentioned: (1) the estimation of large numbers can help students to get an intuitive notion of mean, (2) using the midrange as a precursor for a more advanced notion, (3) repeating measurement may develop an understanding of the mean, (4) construct the median visually in a dot plot, with equal number of dots left and right from the median and (5) use skewed distributions to underline the necessary of the median (Bakker & Gravemeijer, 2006). We assign the results of Bakker and Gravemeijer in KCT because they see it as relevant for activity design in schools. Their approach is based on Freudenthal's (1983) didactical (and historical) phenomenology of mathematical structures: mathematics educators look into history and into applications of mathematics in order to identify situations from that mathematical concepts can be developed as problem-solving tools. But it is unclear to us, where to locate this type of knowledge in our model of teachers' professional knowledge.

Based on the research results described above the authors made some Implications of the Research: Teaching Students to Reason About Center (Garfield & Ben-Zvi, 2008, p. 194). The implications start with an informal estimation and reasoning about typical values, followed by exploring the characteristics of the mean and the median and how they are affected by different types of distributions. The balance model of the mean and the use of visual, interactive activities and technology are important tools for establishing a conceptual understanding of mean and median. Finally, the idea of center as a signal in a noisy process has to be developed. Therefore it has to be re-cognized that data are generated by a process as Konold and Pollatsek (2002) have argued for. In the last pages of this chapter the authors present two evaluated lessons, which transfer the suggested structure to real activities. The descriptions of the lessons are a little vague, no concrete exercises are given. The Knowledge of Curriculum, the Technological Content Knowledge, and the Technological Pedagogical Content Knowledge are not mentioned in this chapter, which focuses on the center. There is a chapter about using technologies in statistical courses, but this chapter only focuses in a more general way to these topics, the problems with using software for a special exercise or how to introduce the use of the software or a new function is not mentioned here.

CONCLUSION

In this paper we described a model for competencies for professional teachers. We tested and concretized this framework by analyzing a piece current literature. The central competencies are described above. It is interesting that the main competence categories KCS and KCT can be found in separate parts of the analyzed chapter. We regard this result as an evidence for the usefulness of our framework. The assignment of results of the didactical phenomenology in the sense of Freudenthal is an open problem to us. Moreover the CCK may be a fiction in the case of statistics, as there are different communities of practice in statistics and the CCK of applied statisticians is different from the CCK of mathematical statisticians. Analyzing the structure of the chapter we recognized some weaknesses of the material: the descriptions of the lessons are too vague for that a teacher can use it. Also there are only general comments about TCK and TPCK, but comments for the concrete use of technology in certain situations are missing. Finally, the collection of KCS is extensive, but concrete ideas for using this in classrooms are missing. The strength of this book is its extensive collection of KCS and KCT and their application to structure the two lessons.

The next step of our project will be analyzing more literature focusing on topics around center in the first place to understand the structure and the main competencies and to check our framework. With the results of this work we will begin to design and implement our own course.

REFERENCES

Bakker, A., & Gravemeijer, K. P. E. (2006). An Historical Phenomenology of Mean and Median. *Educational Studies in Mathematics, 62*, 149-168.

Ball, D. L., & Thames, M. H. & Phelps, G. (2008). Content Knowledge for Teaching: What Makes It Special? *Journal of Teacher Education, 59*(5), 389-407.

Batanero, C., Burrill, G., Reading, C., & Rossman, A. (2008). *Joint ICMI/IASE Study: Teaching Statistics in School Mathematics. Challenges for Teaching and Teacher Education. Proceedings of the ICMI Study 18 and 2008 IASE Round Table Conference*.

Biggs, J. B., & Collis, K. F. (1982). *Evaluating the quality of learning: The SOLO taxonomy*. New York: Academic Press.

Burrill, G., & Biehler, R. (2010). Fundamental ideas in the school curriculum and in training teachers. To appear in C. Batanero et al. (Eds.) *Teaching Statistics in School Mathematics - Challenges for Teaching and Teacher Education: A joint ICMI/IASE Study*. New York: Springer.

Freudenthal, H. (1983). *Didactical Phenomenology of Mathematical Structures.* Dordrecht: Reidel.

Garfield, J. B., & Ben-Zvi, D. (2008). *Developing Students' Statistical Reasoning*. New York: Springer.

Grossman, P. L. (1990). *The making of a teacher: Teacher knowledge and teacher education*. New York: Teachers College Press.

Groth, R. E. (2007). Toward a Conceptualization of Statistical Knowledge for Teaching. *Journal for Research in Mathematics Education, 38*(5), 427-437.

Groth, R. E., & Bergner, J. A. (2006). Preservice elementary teachers‘ conceptual and procedural knowledge of mean, median, and mode. *Mathematical Thinking and Learning*, *8*, 37-63.

Hofmann, T. (2010). *Lernumgebung EFATHOM*. Online: http://efathom.math.uni-paderborn.de.

Konold, C., & Pollatsek, A. (2002). Data Analysis as the Search for Signals in Noisy Processes. *Journal for Research in Mathematics Education, 33*(4), 259-289.

Lee, H. S., & Hollebrands, K. F. (2008). Preparing to teach data analysis and probability with technology. In C. Batanero, G. Burrill, C. Reading, & A. Rossman (Eds.), *Joint ICMI/IASE Study: Teaching Statistics in School Mathematics. Challenges for Teaching and Teacher Education. Proceedings of the ICMI Study 18 and 2008 IASE Round Table Conference*.

Mishra, P., & Koehler, M. J. (2006). Technological Pedagogical Content Knowledge: A Framework for Teacher Knowledge. *Teachers College Record, 108*(6), 1017-1054.

Mokros, J., & Russell, S.J. (1995). Children’s concepts of average and representativeness. *Journal for Research in Mathematics Education*, *26*(1), 20-39.

Niess, M. L. (2005): Preparing teachers to teach science and mathematics with technology: Developing a technology pedagogical content knowledge. *Teaching and Teacher Education, 21*, 509-523.

Schaper, N. (2009). Aufgabenfelder und Perspektiven bei der Kompetenzmodellierung und messung in der Lehrerbildung. *Lehrerbildung auf dem Prüfstand, 2*(1), 166-199.

Shulmann, L. S. (1986). Those Who Understand: Knowledge Growth in Teaching. *Educational Researcher, 15*(2), 4-14.

Stigler, S. M. (1986). *The history of statistics: the measurement of uncertainty before 1900*. Cambridge, MA: Harvard University Press.

VEMA (2010). *Multimedia-Vorkurs Mathematik*. Online: www.mathematik.uni-kassel.de/vorkurs.