EXPLORING RELATIONS OF VITRUVIAN MAN TO DEVELOP STUDENTS' REASONING ABOUT VARIATION

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This paper aims to explore reasoning about variation with twenty-five seventh and eighth graders. Anthropometric measures were collected and tasks developed, supported by dotplots. Coding schemes were used to classify students' answers. Higher reasoning about variation levels in the posttest were observed in questions that asked for analyzing a situation by dotplot and by interpretation of mean, both concepts discussed during activities. Before the teaching session, students were asked to describe the size of shoes and the height of the person. We observed that they realized variation before expectation. Some students said that distributions were spread or grouped, used minimum and maximum values and chose to compare groups of male and female in an intuitive way. After the teaching session, students were more engaged in statistics tasks to analyze other variables, showing the importance of teaching statistics in school.

INTRODUCTION

According to Watson and Kelly (2002, p. 1), "variation is at the heart of all statistical investigation, because if there were not variation in data sets, there would be no need for statistics." Primary and secondary students can develop an intuitive notion of variation to solve a problem, for instance, the understanding that data (observations) can vary, which is considered by Garfield and Ben-Zvi (2008) as an informal aspect of the concept of variation. The formal aspect of variation is considered variation based on measures such as range, interquartile range and standard deviation (Garfield & Ben-Zvi, 2008) and it is one of the aims of high school, college and undergraduate statistics courses.

In some papers, participants used the range, in addition to other concepts that may be used individually, such as maximum and minimum values, and mode to explain variation (Ben-Zvi, 2004; Reading, 2004; Silva & Coutinho, 2008; Watson & Kelly, 2002). In paper of Lehrer, Kim and Schauble (2007), participants used unconventional strategies and mean absolute deviation to describe error of measurement. Unconventional terms are used more frequently to explain reasoning about variation, as in *clustered* or *spread out* (Bakker, 2004; Makar & Confrey, 2005).

By using the didactic activity *Vitruvian Man*, the aim of this paper is to verify levels of reasoning about variation of seventh and eighth graders when they are solving problems supported by dot plots. The question is: how can dot plots help students reason about variation and develop intuitive notion of interquartile range?

METHOD

Participants

The sample for this study consisted of 25 students, 11 seventh graders and 14 eighth graders from a public school of Ilhéus, in the state of Bahia (Brazil). The mean age was 15.4 years old (with a standard deviation of 1.0); the youngest student was 13 years old and the oldest was 17 years old. They were slightly older than the expected age of this grade because 21 of them had already failed a grade, with four of these having failed in Mathematics.

In an initial interview, 20 students said that they understood graphs, 9 knew about the mean and 2 knew about the median. No student claimed to know about variation.

Vitruvian Man Activity

Vitruvian man is one of the teaching experiences we are developing in the *Avale Project* (a Virtual Environment for support in developing statistical literacy) to help mathematics teachers to teach statistics. This activity was developed during two days in July (2009), totaling 16 hours, and it was conducted by the first three authors. All activity was recorded in audio and video.

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From reading texts on the mathematical proportions of the human body established by Leonardo Da Vinci, students chose some body measures to study and data were collected (height, arm span, lengths of the palm, hand and forearm, head circumference and shoe size).

The researchers chose to work with shoe sizes because it is a variable that is easier for explaining variation, and with height, which, being a continuous variable, requires the introduction of a variation measure, since mode may be no longer applicable to gain majority.

Students solved seven tasks, in pairs: five of the tasks were before didactic intervention of the researcher and two of them were after the intervention. In this study, we present reasoning about variation in tasks in which dot plots were used to represent data.

Students individually took the pretest, with three questions. In the first one, students were asked to compare three pairs of graphs of grades and decide which classes had done better. In the second task, about weather, students had to explain the meaning of the daily mean temperature in Ilhéus, propose the temperature on six different days of the year, explaining this choice, and interpret three different graphs of Ilheus' temperatures during one year. Both tasks were translated and adapted from Watson, Callingham and Kelly (2007). In the last question, students had to choose one of two dot plots that could represent the number of siblings each student had. The posttest was the same as the pretest and was given at the end of the second day.

Data Analysis

The four hierarchical levels of the SOLO taxonomy (Structure of the Observed Learning Outcome) of Biggs and Collis (1991, p. 65) were used to analyze students' answers: prestructural level (code 0), in which "the learner is distracted or misled by an irrelevant aspect belonging to a previous stage," unistructural level (1), in which "the learner focuses on the relevant domain, and picks up one aspect to work," multistructural level (2), in which "the learner picks up more and more relevant or correct features, but does not integrate them," and relational level (3), in which "the learner integrates the parts with each other, so that the whole has a coherent structure and meaning." Answers were independently analyzed by the first and third authors of this paper. The lower concordance level was 85%.

DEVELOPMENT OF REASONING ABOUT VARIATION

Students explored heights and shoe sizes by using the database described to them as for the factory that would make the new uniforms for the school's students. They also estimated the height and the shoe size of a new student. Most of the responses were based on personal experiences associated with the proportionality of body measures, e.g., estimating the height of a specific friend and inferring his shoe size through the proportionality of these measures.

A *human dot plot* was made, in which students were the points, to stimulate the perception of variation and center measures. They noticed that most boys were taller than girls, although there were exceptions, and they were asked to make graphs of girls' and boys' heights and shoe sizes on transparent paper (Figure 1).

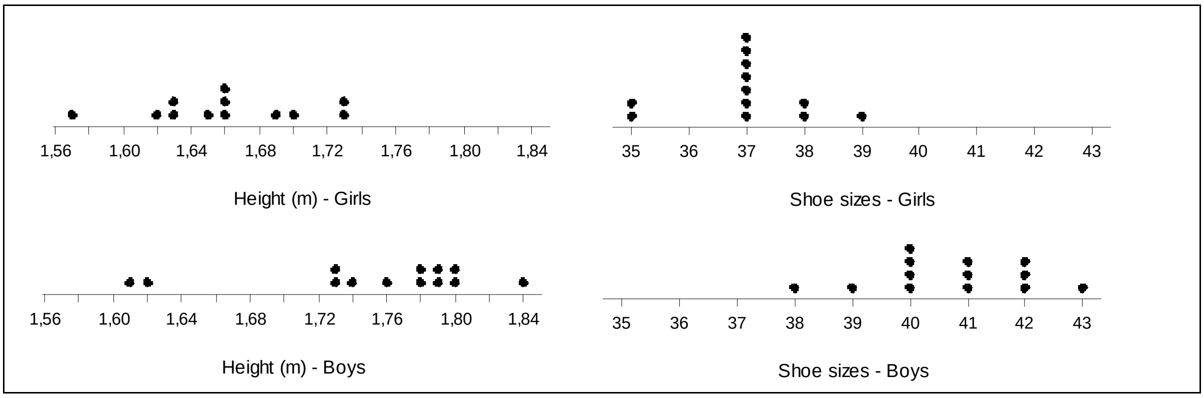

Figure 1. Distribution of heights and shoe sizes of girls and boys

Group comparison has been shown to be an effective strategy for developing reasoning about variation with secondary and high school students (Ben-Zvi, 2004; Watson, Kelly, Callingham & Shaughnessy, 2003) and with undergraduate students (Meletiou & Lee, 2002). However, comparing groups of different sizes requires the use of more sophisticated proportional reasoning to solve the task. Watson et al. (2007) report that only one student (out of 73) reached the highest variation understanding level when comparing two data sets with different numbers of elements.

Coding categories of responses are presented in Table 1 in the "Before teaching" column. Although students had fun with the fact that there were some boys shorter than some girls, three pairs of students presented deterministic answers, such as "men are taller than women" as observed by Ben-Zvi (2004) and were classified at prestructural level. Unistructural level answers used terms such as "more spread out" or "more different," or used maximum and minimum values.

One example of a multistructural level response was presented by AG and IR: *"I think that the difference is slight because girls have smaller shoe sizes than men do, men have larger shoe sizes, from 40 to 44. But girls have shoe sizes from 35 to 38. The difference is that most men are taller than women. Real data for the majority of men is 1.78 and 1.80 meters."*

It is interesting to observe that these students used a central range named by Konold et al. (2002) as a *modal clump*. The same reasoning appeared in papers by Makar and Confrey (2005) and Silva and Coutinho (2008).

Table 1. Coding categories of responses to the task before and after the teaching session

Codes	Level	Description	Before teaching (n=12)	After teaching (n=11)
0	Prestructural	Vague or incorrect responses; no acknowledgement of variation.	3	1
1	Unistructural	Answers with only one aspect of variation: "more," majority, range, maximum and minimum values.	7	3
2	Multistructural	Answers with two or more aspects of variation, without fully interrelating them.	2	5
3	Relational	Correct answer to requested task, with arguments that relate different aspects of variation.	0	2

In the first teaching session, which occurred on the second day, responses from previous tasks were discussed and they were compared with center measures and range, highlighting the importance of variation. After this session, students were asked to compare head circumferences of three hypothetical groups by using dot plots, as presented in Figure 2. Coding categories of responses are also presented in Table 1, in the "After teaching" column.

A pair of boys used a calculator to calculate values for the mean, restricting their analysis to presenting these results, without considering any aspect of variation. This kind of answer was classified as prestructural level.

It is possible to observe that the number of answers at the multistructural level increased, because students had some variation tools for working with the data, without, however, fully interrelating them. At this level, students MC and DN observed the concentration of data and the extreme values and were able to present an answer with an intuitive notion of central density.

The answer of CL and CS was classified at the relational level because it contained a decision about the comparison based on the perception of the size of the groups, the range and the concentration: *Class A has the same number of students as class B and C, each one with 32 points. In class A, people's head circumferences were from 46 to 76. In class B, head circumference was from 51 to 64. In class C, it was from 51 to 60, so comparing them, the head circumferences are less in class C and they measure only between 51 and 60. So, the head circumferences of class C are less than A and B".*

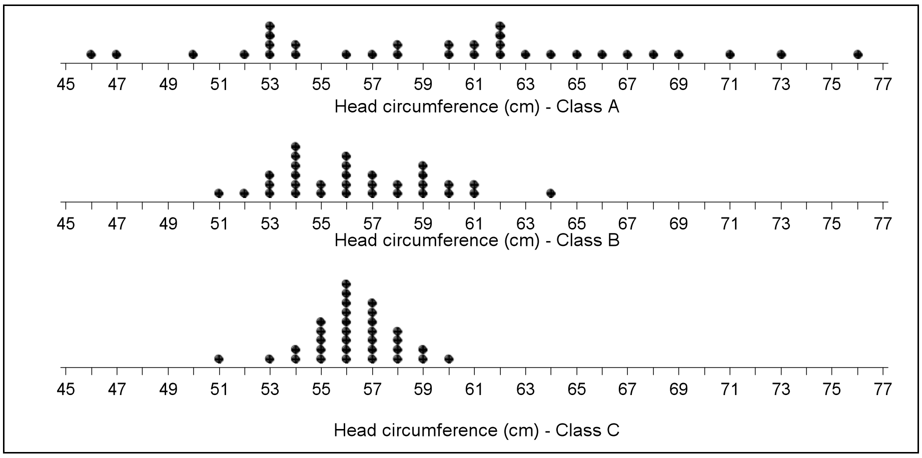

Figure 2. Head circumferences of three hypothetical classes

After the students' presentations, another teaching session was held. Students were asked to pretend that they would make hats for these three classes, but they did not have enough money to produce all circumference sizes and they would need to select a few to fit the greatest number of students. This discussion allowed the students to perceive the important but restricted role of range, making them reflect on the need for other measures to help make a decision.

In the virtual environment, students could see the animation of changing a dot plot to a box plot (Figure 3). Quartiles and interquartile range were discussed by asking students count the number of points inside the box and compute this percentage. Students perceived that 50% or more of the points are in the box and this become easier the task to choose the majority to whom they would make the uniforms.

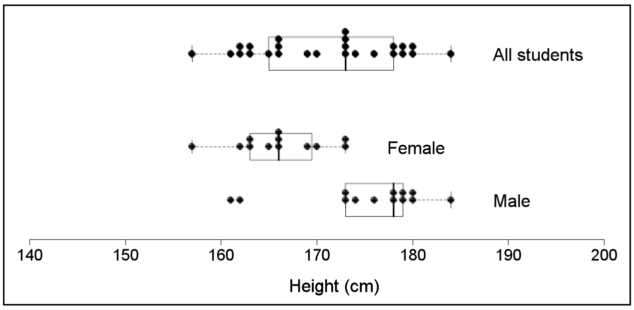

Figure 3. Distribution of height of male, female and all students

The development of reasoning about variation of most students during this experience was similar to the study by Ben-Zvi (2004, p. 52), in which students began by looking at the edges of the distributions, moved on to compare quantities of neighboring values, and finally progressed to the mode, indicating the "first steps towards understanding density in a distribution." The use of median and the percentage inside the box (intuitive idea of interquartile range) was possible by graphs animation in the virtual environment.

PRETEST AND POSTTEST ANALYSIS

A pretest and posttest task asked students to choose one of two dot plots that could better represent the distribution of the number of siblings of students, when compared with the distribution of their fathers' number of siblings. In Brazil, people from older generations tend to have more siblings, and thus, the researchers developed the two graphs of Figure 4.

Answers at the relational level should include the choice of Graph A, using arguments such as mode, range, interquartile range and extreme values. As can be observed in Table 2, no answer was classified at this level, because no students perceived either the extreme value or interquartile range in distribution A.

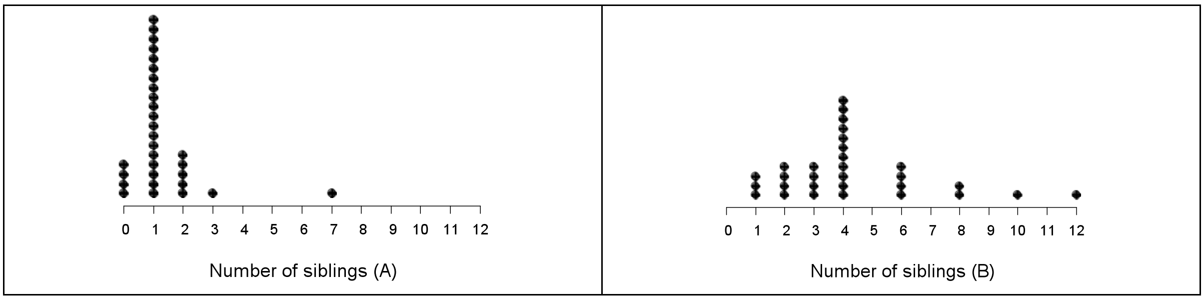

Figure 4. Pretest and posttest question supported by dot plots

Table 2. Frequency of response levels in pretest and posttest

Pretest	Posttest				Total
	0	1	2	3	
0	2	5	0	0	7
1	4	6	2	0	12
2	0	1	0	0	1
3	0	0	0	0	0
Total	6	12	2	0	20

Four students presented answers in which the reasoning level decreased from unistructural in the pretest to prestructural in the posttest. For example, in the pretest CN chose graph B and explained, "*Graph A has a lot of people with no siblings and in graph B most people have siblings*." Although this student did not consider center, he has an intuitive perception of variation. In the posttest, the same student choose graph A because he considered the greater number of siblings. It is possible that he said "*greater number of siblings*" by looking at the higher frequency of students with one sibling and not observing the variation of values of the variable. This misconception of variation is associated with difficulties in reading graphs and it was reported by Meletiou and Lee (2002), when undergraduate students compared two histograms.

The opposite situation can be exemplified with the answers of AM. She presented a vague answer in pretest, classified as prestructural, and in posttest her answer contained one aspect of variation: *"I counted the number of points that are in both graphs and there are 30, but I chose graph B because they are more spread out,"* although she had thought about a different family model than that expected by the researchers.

It was possible to identify that this question was deficient because it is influenced by regional differences. Ilheus, the city where this experiment was held, has very different social, economic and cultural features than big cities, such as São Paulo. The family structure of most citizens of Ilheus is very different from that of the rest of Brazil and it is possible that the students could not agree with the two graphs presented. Because of this, the analysis of the answers was based on the coherency of the chosen graph and the aspects of reasoning about variation that were presented.

CONCLUSIONS AND IMPLICATIONS

Official educational documents in Brazil recommend teaching variation measures in high school, but the development of the perception of variation can be explored beginning in the early grades, as reported in Watson and Kelly (2002). The aim of this study was to verify if secondary school students who have had some learning difficulties during their school trajectories could develop higher levels of reasoning about variation. Results of activities developed during the experience allow us to conclude that they could, thereby confirming the statement by Watson (2005) that children develop an appreciation of variation before expectation. Since our sample is very small, these results must not be generalized.

For this learning to become meaningful, this kind of work must have continuity. We expect that AVALE can provide didactic and conceptual conditions for mathematics teachers to develop similar experiences with their students from a perspective of statistical literacy.

The Vitruvian Man activity fosters the development of covariation reasoning, restricting the possibilities of reasoning about variation for each variable independently, which was only possible with the task of comparing groups.

The dot plot was a very easy graph for students to grasp, and when it was associated with the boxpot, it provided conditions to introduce the concept of interquartile range, but it is necessary to develop activities to measure its effectiveness.

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