OVERCOMING OBSTACLES TO SUPPORTING SECONDARY TEACHERS' STATISTICAL CONTENT KNOWLEDGE FOR TEACHING

Sandra Madden
University of Massachusetts Amherst, United States of America
smadden@educ.umass.edu

Supporting teachers' statistical content knowledge for teaching poses a number of challenges. Many grades 6-12 mathematics teachers' prior experiences with statistics insufficiently prepared them to support students grappling with big ideas of statistics. Teachers' view of statistics tends to be procedural and cookbook-oriented. Many mathematics teachers feel that statistics has little place in their instruction, thus professional development is unnecessary. Finally, many teachers have not experienced learning statistics in an environment in which student-centered discourse, learner-centered technology, and worthwhile statistical tasks has been the norm. Research from two professional development projects in which significant changes in teachers' understanding of statistical big ideas occurred is presented. Potential solutions to obstacles, tasks with potential for engaging and challenging teachers, and lessons learned are discussed.

INTRODUCTION

The National Council of Teachers of Mathematics (NCTM) Principles and Standards for School Mathematics (PSSM) recommends that data analysis ands probability be part of the curriculum experience for all students across grades preK-12. The Guidelines for Assessment and Instruction in Statistics Education (GAISE) (Franklin, et al., 2007) augment the NCTM's recommendations by specifically addressing statistical ideas developmentally and especially with a focus on the nature of variability and its relationship to statistics. Both the NCTM's recommendations and those detailed in GAISE, suggest the importance of the use of technology for supporting student learning of mathematical and statistical ideas as they explore the generation of statistical questions, collect and analyze data, and interpret and communicate results and conclusions. For this vision to be enacted in classrooms, critical to the mission is that teachers are equipped with personal understanding of statistical concepts and connections, and facility with tools for supporting the exploration of such concepts and connections by students.

Teaching mathematics requires both common mathematical knowledge and specialized content knowledge in order to support students' mathematical learning (Hill & Ball, 2004). Common mathematical knowledge may be considered as the type of knowledge developed in a mathematics class (e.g., know how to solve problems, computing correctly, using logical arguments), whereas specialized mathematical knowledge includes knowledge that teachers need to attend to the teaching of mathematics (e.g., multiple solution methods, constructing worthwhile mathematical tasks, recognizing and supporting student mathematical reasoning). Groth (2007) extended this work by proposing a framework for thinking about aspects of statistical knowledge for teaching. He differentiated between common and specialized statistical knowledge and within both, he accounted for mathematical and nonmathematical knowledge needed by teachers. While teaching statistics in a mathematics classroom context, teachers' specialized content knowledge must allow them to connect statistical big ideas within the domain of mathematics while being sensitive to the stochastic versus deterministic differences of the disciplines, all the while understanding issues of student learning, curriculum, technology, equity, and assessment.

Teachers' professional preparation to support the advancement of students' learning of data analysis and probability is likely inadequate (College Board of Mathematical Sciences, 2001; Madden, 2008). Though statistics has made its way into national curricular recommendations (e.g., NCTM, GAISE) and state level curriculum framework documents, a cursory examination of textbooks used in many schools reveals a strong disconnect between what visionary documents intend and the actual opportunities students have to learn statistics. When teachers hold procedural views of statistics, even when they read curricular recommendations, they may fail to fully appreciate what is really being asked of them. When their instructional resources further exacerbate this issue, teachers may believe that the familiar procedural approach to supporting students' statistical understanding is appropriate.

In C. Reading (Ed.), *Data and context in statistics education: Towards an evidence-based society. Proceedings of the Eighth International Conference on Teaching Statistics (ICOTS8, July, 2010), Ljubljana, Slovenia*. Voorburg, The Netherlands: International Statistical Institute. www.stat.auckland.ac.nz/~iase/publications.php

The literature is filled with calls to action to support the teaching of statistics (e.g., Cobb, 2007; Garfield & Everson, 2009; Groth, 2007; Shaughnessy, 2007), but there is little guidance for how to do this with more than a small handful of teachers at a time. Garfield and Everson (2009) made the following recommendations for supporting teachers of statistics and with which I agree:

- Emphasize statistical literacy and develop statistical thinking;
- Use real data;
- Stress conceptual understanding rather than mere knowledge of procedures;
- Foster active learning in the classroom;
- Use technology for developing conceptual understanding and analyzing data;
- Use assessments to improve and evaluate student learning.

By introducing teachers to the GAISE framework and through simultaneously conducting statistical investigations, teachers may begin to shift their thinking about what it means to do and learn statistics. Specifically, attending to the processes of formulating statistical questions, collecting data, analyzing data, and interpreting results, across the developmental levels A, B, and C, and with intentional emphasis on the nature of variability helps teachers to develop a more coherent vision for what is possible. Additionally, by exposing teachers to innovative curriculum materials that include a strong statistical component (e.g., Connected Mathematics—middle school, or Core-Plus Mathematics—high school), teachers may begin to develop a sense that instructional materials are available to help support their work with students. Investigations from materials like these may provide teachers with valuable opportunities to grapple with statistical ideas as learners. Curriculum materials with a strong statistical thread may assist teachers with reconsidering the many ways that statistics can be woven into students' mathematical experiences.

There are logistical and substantive statistical issues that make supporting teachers' statistical content knowledge for teaching challenging. Assuming one can overcome logistical issues, still there remain substantive statistical issues including teachers' fragile understanding of important statistical ideas and lack of familiarity with tools to support learning of statistics. This paper explores one researcher's attempt to overcome the latter obstacles in the context of professional development for middle and high school mathematics teachers.

RESEARCH SETTINGS

Participants

Two separate research projects provide the backdrop for this discussion. The first project involved a four-day intensive professional learning experience with 56 Midwest high school mathematics teachers in 2006, while the second involved 11 Northeast middle school mathematics teachers working for five days in 2009. Each of these projects had, as its core mission, the intent to support teachers' statistical content knowledge for teaching and with that, a strong focus on developing facility with dynamic statistical software. *Fathom2* was the primary statistical tool used with high school teachers, whereas *TinkerPlots* was the main tool used by middle school teachers. High school teachers were participants in a larger three-year, improving teacher quality professional development initiative, whereas the middle school teachers were self-selected volunteers for a one-time professional learning experience. Incentives for teachers' participation included daily participation stipends, site licenses for software, and professional development credit hours or graduate credit.

Methodology

Design research methods were utilized and a four-day (20 hour) (five-day, for the middle school project) hypothetical learning trajectory was constructed with the intention of supporting teachers' statistical understanding. A working hypothesis guiding the design of the learning experiences was that without strong understanding of important statistical big ideas and their connections, teachers would be unable to support student learning of these big ideas. Specifically, *comparing distributions* was conceptualized to connect the statistical big ideas of distribution, variability, and sampling distributions, all seen as pivotal ideas needed to reason statistically (see Figure 1) (Madden, 2008). Learning experiences were engineered for teachers to simultaneously explore statistical content and processes while developing facility with technology to support their own learning.

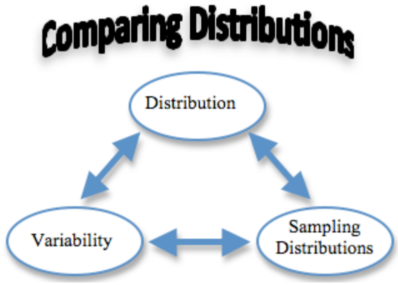

Figure 1. Conceptual relationship among big ideas related to comparing distributions

Pre- and post-statistical content and belief assessments, written reflections of learning, and video-tape of the professional learning experiences were collected. Extensive pre- and post-interviews were conducted for the high school project. Ongoing and retrospective analyses of the data were done using a combination of inductive and deductive qualitative methods as well as quantitative methods for statistical comparisons.

RESULTS

The following quotes are typical of the teachers at the beginning of both projects as they described the nature of their prior undergraduate or graduate statistics course experiences:

This stats course almost turned me off of math altogether, which was unfortunate as it seemed to be a bridge point to the social sciences I was getting more deeply embedded in. It was a large stadium course, taught with all the creativity a Powerpoint presentation can muster. It used computers, but only for problem sets (middle school teacher).

It was lecture format. Come in, 40 minutes of a lecture, 40 minutes to an hour, go home, do your homework, come back. Take a quiz. No technology integrated. It was like board and paper work the whole time. No excel spreadsheets or anything like that. Mostly theory (high school teacher).

These descriptions provide a stark contrast between how teachers described their statistical learning experiences and recommendations for courses intending to prepare teachers of statistics. Interviews with teachers consistently signaled their procedural view of statistics and a limited view of what statistics in the secondary curriculum may entail. A typical teacher response to what statistics high school students should have opportunity to learn was, “Students should know mean, median, and mode, and they need to learn how to make and read basic graphs.” This also contrasts sharply with recommendations that learners have opportunity to study big statistical ideas such as data, distribution, trend, variability, models, association, samples and sampling, and inference (Garfield & Ben-Zvi, 2004).

None of the teachers in either project reported having any experience with *Fathom* or *TinkerPlots* prior to participating in the study. By the end of four or five days, every teacher had developed moderate to strong facility with one of these tools to explore and analyze data and to conduct simulations. Evidence for this claim comes from the video artifacts of the professional learning sessions, post-session task-based interviews, and teacher self-reported comfort-level.

From pre- to post-assessment, teachers’ statistical content knowledge improved significantly on each item and overall, teachers claimed to have learned a great deal about statistics and technology, and their ability to use statistical language appropriately to make and support statistical arguments improved dramatically. Figure 2 illustrates that though most high school teachers (48/56) reported taking at least one statistics course in college, the number of courses taken did little to explain their understanding of comparing distributions on the pre- or post-assessments (scores based on four-level scale).

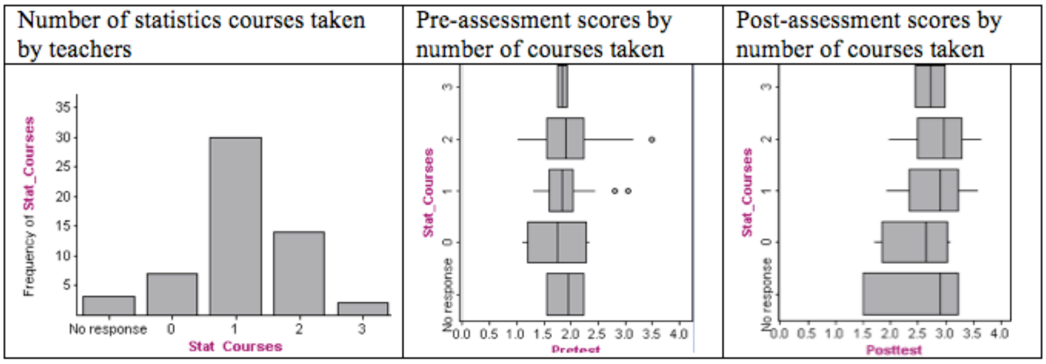

Figure 2. Comparisons among statistics courses taken and pre- and post-assessment results

Teachers scored typically at or near Level 2 on the pre-assessment, indicating they could compute basic statistics, read basic graphs, and use context to make comparisons between distributions. Their understanding of variability and sampling distributions was comparatively weak. At the end of a four-day professional development session, teachers' assessments indicated they had made progress to Level 3. At Level 3, teachers could 1) compare distributions with sensitivity to shape, center, spread, and sample size; 2) move flexibly among representations and measures; 3) describe and interpret variation within and among distributions of samples; 4) reason about samples and collections of sample statistics; 5) begin to understand the Central Limit Theorem; 6) understand the logic of hypothesis testing; 7) draw acceptable conclusions based on informal inference; and 8) communicate using the language of statistics. Fifty-four of 56 teachers' assessment scores *and* self-reported level of statistical understanding improved. These results are generally consistent with those of the middle school teachers. Teachers made strong gains in areas that have been historically challenging for learners (e.g. sampling distributions). Collectively, these data support the view that the professional learning experience was broadly accessible and effective at perturbing and supporting teachers' statistical understanding.

During retrospective analyses, while attempting to understand and test conjectures regarding mechanisms which may help to explain some of the teachers' learning gains, especially given their limited statistical understanding and lack of facility with technological tools, two important factors emerged: 1) the use of randomization testing and statistically worthwhile tasks, and 2) reasoning with dynamic statistical software and making connections to familiar mathematical objects. Each of these ideas will be illustrated.

Randomization Testing and Statistically Worthwhile Tasks

In both professional learning environments, the opening investigation required teachers to work in small groups to design an experiment to test two competing delivery vehicles (modified Orbital Express investigation (Key Curriculum Press, 2005)). Their challenge was to determine whether one of the two vehicles landed closer to the target. The vehicles were crumpled pieces of two different types of paper (e.g., construction paper and paper towel).

Teachers were not explicitly taught about experimental design at the time, but drew on prior knowledge to make suggestions for the group to consider. They were not taught how to represent or summarize data from their experiment, yet they were able to create posters with a variety of graphical representations to communicate their results to the whole group. These representations varied widely. They were not taught about how to make a decision as to whether one vehicle was better than another, but they publically argued, on the basis of their evolving informal inferential reasoning skills to stake a claim expressing varying degrees of uncertainty.

This initial exploration set the tone for the entire professional learning experience in both projects. It was crucial as it provided opportunity for teachers to make their statistical thinking visible and public and begin to develop a safe environment in which sense making and reasoning were privileged and "right answers" were replaced by evidentiary-based arguments. The activity exposed the fragile ways in which teachers knew to reason statistically, yet provided the scaffolding structure on which to build. It precipitated the introduction of "statistical significance" in relation to whether differences observed in the experiment were likely just the result of "chance variation." From the beginning, seeds of experimental design issues, measurement variability, sampling variability, induced variability, and chance variability were planted and teachers continued to explore and reason in increasingly sophisticated statistical ways.

In the context of this investigation, teachers were introduced to randomization testing to compare distributions through physical simulation and then using technology. In a very short period of time on the first day of each session, teachers began to develop a sense of what statistically significant meant, what a *p*-value was, and what was meant by chance variability. This was the first of 19 statistically, technologically, or contextually provocative (Madden, 2009 (submitted)) *comparing distribution* activities explored over the professional development experience. Follow-up activities included exploring a Core-Plus Mathematics unit designed for high school students to develop statistical reasoning using randomization testing. Randomization testing as a statistical process appeared accessible to these teachers and the evidence of its appearance in high school mathematics textbooks convinced teachers of its curricular relevance.

Teachers communicated convincingly during and after the sessions their understanding of this process while implementing and interpreting it appropriately. Randomization testing was a tool for teachers to confirm or refute conjectures based on comparing distributions numerically or graphically. Across multiple contexts, its use convinced teachers that making the call for significant differences is complicated. Randomization testing was one vehicle that allowed teachers to coordinate issues of shape of distribution, sample size, variation, sampling distribution, signal, and noise. It was an evidence-based tool used for reasoning. Most importantly, strong evidence from content tests, interviews, video, and teacher reflections indicate that teachers were making sense of these concepts as they used language and representations to communicate.

Reasoning with Dynamic Statistical Software and Making Connections with Familiar Mathematical Objects

In a *Fathom* or *TinkerPlots* environment with mathematics teachers, it is useful to design tasks in which statistical investigation is the goal, yet the opportunity to connect to more familiar mathematical objects is also relevant. For example, when generating the machinery to explore empirical sampling distributions in *Fathom*, generating formulas to create collections requires a type of algebraic reasoning as does the generation of statistical measures. Reasoning from tables, graphs, and symbols requires fluency across hot-linked representations. Interrogating these representations for meaning requires sense making and communication skills. Navigating within and among hierarchies of objects necessitates connections among mathematical objects. These mathematical and statistical connections may be considered specialized statistical content knowledge for teachers. When teachers can see broad usefulness of a tool to teach statistics *and* algebra, the potential for learning and using the tool dramatically increases. As a specific example, after experiencing the use of sliders as parameters in mathematical function plots to explore the behavior of the normal probability density function, teachers immediately voiced excitement at the use of this tool for supporting the teaching of algebra. One teacher said, "This [*Fathom2*] is much niftier [than a graphing calculator] . . . because I have precalc . . . I think it would help me. The first time I taught it, I didn't completely . . . understand" (high school teacher). This activity signified teachers connecting this tool's use to their classroom practice and curiosity about the capability and value of the tool for the work of teaching and learning.

Considerations for designing technological learning experiences for teachers should take into account task and tools (Doerr & Pratt, 2008), but also the environment. In both professional development projects, one of the non-negotiable norms of participating in the classroom was, "supporting statements with evidence." This was taken to mean that whenever a claim was made, someone in the room would generally ask something like, "What is your evidence for that statement?" or "How do you know that?" or "Why do you think that?" This was not simply the work of the facilitator, but ultimately an agreed upon and communally upheld norm that placed reasoning at the forefront of the intellectual responsibility of the members of the community. With tools and multiple representations and measures at their disposal, teachers made predictions and conjectures, tested them, refined them, and convinced themselves and others of their ideas. In an environment where the authority for the knowledge in the classroom is shared and distributed among all members, and where provocative tasks and flexible software tools are employed, teachers can improve their statistical knowledge for teaching.

In a technologically rich learning environment, teachers can quickly develop enough facility with the tool(s) that upon completion of the professional development experience, they can continue to explore statistical and mathematical relationships at their leisure. When funds are available, as they were in both of the projects I have described, teachers left with site licenses for the software to use in their schools.

CONCLUSION

Engaging middle and high school mathematics teachers in worthwhile statistical professional learning opportunities is essential if they are expected to do so with students. Important considerations include providing reasonable incentives for teachers' participation and then designing learning environments which simultaneously and synergistically support teachers' understanding of big statistical ideas and connections among them; facility with statistical tools, technology, and their connection to mathematics; familiarity with contemporary curriculum materials; and sharing authority for knowledge in the classroom. The research described here illustrates that it is possible to productively perturb teachers' understanding of important statistical ideas in support of their statistical content knowledge for teaching, in as little as four days. By focusing on statistical big ideas and their connections in a technologically rich learning environment, teachers can develop robust connections among statistical concepts and demonstrate understanding of historically challenging statistical concepts (e.g., sampling distributions).

REFERENCES

Cobb, G. W. (2007). The introductory statistics course: A Ptolemaic curriculum? *Technology Innovations in Statistics Education, 1*(1), Article 1.

College Board of Mathematical Sciences (2001). *The mathematical education of teachers*. Providence, Rhode Island: American Mathematical Society.

Doerr, H. M., & Pratt, D. (2008). The learning of mathematics and mathematical modeling. In M. K. Heid & G. W. Blume (Eds.), *Research on technology and the teaching and learning of mathematics: Volume 1 Research syntheses* (Vol. 1, pp. 259-286). Charlotte, NC: Information Age Publishing, Inc.

Franklin, C., Kader, G., Mewborn, D., Moreno, J., Peck, R., Perry, M., et al. (2007). *Guidelines for assessment and instruction in statistics education (GAISE) report: A pre-K-12 curriculum framework*. Alexandria, VA: American Statistical Association.

Garfield, J., & Ben-Zvi, D. (2004). Research on statistical literacy, reasoning, and thinking: Issues, challenges, and implications. In D. Ben-Zvi & J. Garfield (Eds.), *The challenge of developing statistical literacy, reasoning and thinking* (pp. 397-409). Boston: Kluwer Academic Publishers.

Garfield, J., & Everson, M. (2009). Preparing teachers of statistics: A graduate course for future teachers. *Journal of Statistics Education, 17*(2).

Groth, R. E. (2007). Toward a conceptualization of statistical knowledge for teaching. [Research Commentary]. *Journal for Research in Mathematics Education, 38*(5), 427-437.

Hill, H. C., & Ball, D. L. (2004). Learning mathematics for teaching: Results from California's mathematical professional development institute. *Journal for Research in Mathematics Education, 35*(5), 330-351.

Key Curriculum Press (2005). *Exploring statistics with Fathom2 Dynamic Data Software*. Emeryville, CA: Key Curriculum Press Key College Publishing.

Madden, S. R. (2008). *High school mathematics teachers' evolving understanding of comparing distributions.* Unpublished dissertation, Western Michigan University, Kalamazoo, MI.

Madden, S. R. (2009 (submitted)). Supporting high school mathematics teachers informal statistical inference: Comparing distributions, context, and tools. *Mathematical Thinking and Learning*.

Shaughnessy, J. M. (2007). Research on statistics learning and reasoning. In J. Frank K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (Vol. 2, pp. 957-1010). Charlotte, NC: Information Age Publishing.