CREATING A WORLD POPULATION MODEL TO ANALYZE THE DYNAMICS OF CHANGE

Henry A. Kranendonk
Retired High School Mathematics Teacher and Administrator,
Milwaukee Public Schools, United States
hkranendonk@earthlink.net

The transition from a secondary study of data analysis and statistics to a college level involves more than simply expanding a list of topics. Students' reasoning is expected to more clearly address the questions "What if …?" and "What is the story behind the data?" as they mature. The unit presented, "The World Population Project", *was designed for high school students as a way to explore how students answer those questions when given control of data. Students were provided authentic population data sets of the 10 largest countries. After a study of each country, students are provided opportunities to alter the dynamics of a country's population given various scenarios. By having direct control over the data, they are provided opportunities through a model on a computer to directly answer authentic "What if…?" questions. Their explanations to these questions provide opportunities to give students' feedback in their transition to higher education.*

BACKGROUND

Secondary mathematics and statistics courses are designed to introduce and develop skills, definitions, and applications that provide the foundation for further study in a college or university setting. A continual challenge for secondary teachers is to develop with their students those necessary skills to be prepared for a post-secondary opportunity, along with a realization that mathematics and statistics can impact their future by studying meaningful applications. For example, secondary teachers often introduce probability topics by tossing coins, shaking dice, or drawing cards. Although these applications provide students an introductory understanding of probability, they do little to provide students a sense of the more mature and serious applications of probability. How can teachers provide students meaningful transitions to the relevant applications that launch these post-secondary investigations?

The National Council of Teachers of Mathematics (NCTM) has recently articulated the unique challenges of high school teaching through a series of publications entitled *Focus in High School Reasoning and Sense Making* (NCTM, 2009). One of the goals of NCTM is to provide perspectives in mathematics that demonstrate how students reason and make sense of meaningful problems through carefully guided instructional strategies. To be blunt, the high school situation is critical as students are too often directed to simply redo problems solved by a teacher or by a computer program that require little reasoning. The focus on reasoning and sense making is directed at achieving a balance between the learning of procedures with the equally important understanding of the appropriateness of the procedures, what they accomplish, what results they can produce, and how to apply them productively. The challenge for teachers is to provide secondary students opportunities to understand the reasoning inherent in solving relevant problems.

The goal of this paper is to describe a series of secondary level investigations that extend an introductory understanding of probability and statistics by designing, testing, and re-designing a model of a "world population" scenario. The investigations result in students creating a model that allows them to demonstrate their reasoning and sense making with simply an introductory understanding of probability. As they design their model, students are provided an opportunity to analyze several *"What if…?"* scenarios and "What is the story behind the data?"

Launching the development of this model requires students recognizing several features. Clearly students must understand the dynamics of the project presented. They must also be able to recognize a way to replicate the key features and details discussed in the problem. This replication is based on students understanding the initial conditions of the problem as they articulate and make sense of the implications of these conditions.

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SETTING THE PARAMETERS

After several coin, dice, and card problems, students are presented with an investigation (Figure 1).

The World Population Project:
Consider an imaginary “world population” of 100 people. In time, this population will experience deaths and births. During a given year, each person in this world will age one year. Each person also has a probability of dying during the year. If a person in this world is a female, there is a probability that she will give birth to a new member of the world. Death, of course, will decrease the number of people in the world; births will increase the number of people. Each new birth has a probability of turning out to be a male or a female. Design a model that will allow you to examine the number of people in your world after each year and the distribution of their ages. What is the total population of the world after 100 years based on the parameters you set for death, birth, and gender?

Figure 1. Investigation presented to students

Students were asked to restate the investigation as a way to assess their understanding of the project’s expectations. In restating the investigation, students needed to recognize that at least in this initial set-up, the population under investigation is a total population. They were investigating a “world population” and not the population of a “country” in which the number of people could also grow by immigration or decrease by emigration. The switch to a “country” in which these additional dynamics could be added to the model is addressed later in this paper. In this way, the goal of understanding the whole population before investigating samples is developed.

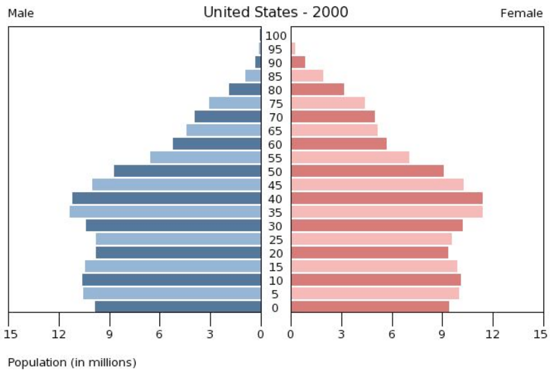
Providing students an initial chart to represent the “world population” was important as a way to organize the changing dynamics of the population. They were given the world population, or 100 people, in the chart shown in Figure 1 (only 20 people of this world are shown for this paper).

Modeling Project: A Population Simulation, Starting “world”										
Age	0	2	4	1	3	4	5	6	7	9
Gender	F	F	F	M	M	M	M	M	M	M
Age	5	7	9	10	12	14	11	12	14	16
Gender	F	F	F	M	M	M	F	F	F	M

Figure 2. Starting “world” Chart presented to students

The above represents 20 of the 100 people in the initial “world.” The 100 people in this “world” were set to proportionally represent the distribution of the United States as represented in the pyramid graph (Figure 3). Later, students were directed to create a new starting population of 100 people (Figure 4) representing one other country from the list of the 10 largest countries of the world.

If a person died in the “world”, the cell representing that person was removed. Initially a death was indicated by simply putting a large “x” over the cell and not counting that person when tabulating the total “world” population. If a person was added to the world, a new cell was added with an age of 0 and a gender based on the result of a probability assignment. Students also increased the age of each person in their world by one year as a result of aging.



Source: United States Census Bureau
International Data Base (IDB)

The initial "world population" is:

Age: 0 – 4	3 females, 3 males
Age: 5 – 9	3 females, 4 males
Age: 10 – 14	3 females, 3 males
Age: 15 – 19	3 females, 3 males
Age: 20 – 24	3 females, 3 males
Age: 25 – 29	3 females, 3 males
Age: 30 – 34	3 females, 3 males
Age: 35 – 39	4 females, 4 males
Age: 40 – 44	4 females, 4 males
Age: 45 – 49	4 females, 3 males
Age: 50 – 54	3 females, 3 males
Age: 55 – 59	3 females, 3 males
Age: 60 – 64	3 females, 2 males
Age: 65 – 69	2 females, 2 males
Age: 70 – 74	2 females, 2 males
Age: 75 – 79	2 females, 1 male
Age: 80 – 84	2 females, 1 male
Age: 85 – 100	2 females, 1 male
Total:	52 females, 48 males

Figure 3. Initial "world " population and United States population pyramid on which it was based

"New people":

Age										
Gender										

Note: the empty set of cells were provided as way to add new people to the "world" who were born by the dynamics of the model.

Figure 4. Proforma to create a new starting population

SETTING THE PARAMETERS

Deaths were discussed by setting up probabilities that would be used to determine if a person lived or died during a given year. The following was presented to the students:

Death parameters – the probability that a person will die during the simulated year is:
____ % if greater than 100 years old.
____ % if between 90 and 100 years old, including 100.
____ % if between 80 and 90 years old, including 90.
____ % if between 70 and 80 years old, including 80.
____ % if between 60 and 70 years old, including 70.
____ % if between 40 and 60 years old, including 60.
____ % if between 30 and 40 years old, including 40
____ % if between 20 and 30 years old, including 30.
____ % if between 10 and 20 years old, including 20.
____ % if between 0 and 10 years old, including 10.

The additions to population in this first development were presented by the following descriptions:

Birth parameters ("new people" to your world)
If a person in the world is female, then the probability of giving birth during the simulated year is:
____ % if greater than 100 years old.

____ % if between 50 and 90 years old, including 90.
____ % if between 40 and 50 years old, including 50.
____ % if between 30 and 40 years old, including 40.
____ % if between 20 and 30 years old, including 30.
____ % if between 10 and 20 years old, including 20.

If a birth took place, then the probability of a new addition to the world was defined by the following:

Gender parameters:
If a new person is born during the simulated year, the proability is:
____ % that the person will be born a female.
____ % that the person will be born a male.

Students were asked to create initial estimates of the above parameters. During the first iteration of the project, students were simply picking percents resulting in extreme increases or decreases in the number of people. To provide some reasoning behind their estimates, students were directed to select one of the following scenarios:

Scenario 1: Over the course of 100 years, design your world model so that the population increases approximately 3% at the end of each year with no more than 1% of the population older than 100 at any given time.

Scenario 2: Over the course of 100 years, design your world model so that the population decreases approximately 3% at the end of each year with no more than 1% of the population older than 100 at any given time.

Scenario 3: Over the course of 100 years, design your world model so that the population essentially does not change at the end of each year with no more than 1% of the population older than 100 at any given time.

DEVELOPING STUDENT REASONING

Students needed to recognize that “wild” assignments to the death, birth, or gender parameters would drastically change the results after each year, but would not allow for the more gradual growth or decline rates described in the scenarios. For example, some students indicated that during a simulated year, the probability that a person would die during a given year was 90% or higher. When they simulated a year using these estimates, the world declined to a population value that resulted in hardly anyone left in the world. (In fact, in some cases, the world was wiped out.) Students who elected to set all probabilities to 0% (or close to 0%) were unable to meet the requirements of limiting the number of people older than 100 years old to 1% or less.

Simulation results were investigated in 5-year increments, thus allowing students to consider altering their parameter assignments. A graphing calculator was used to generate random numbers that determined if a person lived or died, if a person would be born into the world, and what would be the gender of the person born.

Reasoning strategies emerged as students estimated their “death parameters” and “birth parameters” for the initial set of 100 people. Students generally started the process by aging each person one year, followed by simulating whether or not a person died during this year using their death parameters. The next step the students would apply involved determining the number of births by applying their birth parameters to the subset of females using a new set of random numbers. Students would adjust their initial parameter estimates to balance the increases from the births to the losses from deaths, trying to obtain the appropriate expectations of their selected scenarios. Finally, students would experiment with various probability assignments of gender. This last component was the most challenging, as assigning a greater probability of a birth to a specific gender generally does not result in noticeable changes in the total population until later in the simulation. In several cases, it took at least 30 simulated years to observe the implications on the total population resulting from variations in the gender probabilities. Most students simply started their models by assigning a 50% probability that a person born during a simulated year would be male or female. As they continued their experimentation with their model, they were curious in changing this parameter and observing how this change would alter the total population. Students noted that in most countries, slightly more than 50% of the people in the age category of 0 to 4 years old were male based on a study of population data. Several students incorporated that observation in their models by assigning a slightly higher probability to the birth of a male.

Representations of the 10 largest countries of the world were presented using the pyramid graphs provided by the United States Census Bureau, International Data Base. Students were asked to consider at least one other representation of their initial “world” that would proportionally represent another country from this list. Students again selected one of the three scenarios to apply to the revised starting population, and proceeded to develop new parameters in their model. The adjustments in the parameters were very significant depending on which country was selected. A careful study of the pyramid graphs of the countries would indicate the challenges students faced (United States Census Bureau, 2009).

PROJECT IMPLEMENTATION

The development of this model evolved into a meaningful opportunity to observe students address probability as something serious and important. Initially, the students’ reaction to the project was less than enthusiastic. Their reactions were addressed through discussions of how this simulation was a reflection of the dynamics of change in a genuine population. As students started to assign parameters to the death, birth, and gender descriptions, the larger goal of providing them an opportunity to reason through their understanding of probability also grew. Without a lot of prompting, students took the challenges seriously and enthusiastically – incorporating interesting techniques into how to assign their parameter values. One group of students studied obituaries, and determined at what ages most deaths occurred. They created a set of estimates based on the distribution of ages reported in the obituaries. Another group approached these assignments with a more statistical approach. They interviewed a group of family members and friends using a simple survey to determine how many of them over the course of several years were aware of people who died and their ages. They also recorded how many births the people in their sample could recall. Again they tried to obtain estimates of the mothers’ ages when they gave birth. This second approach resulted in a discussion of sampling and what would be needed in a good sample to incorporate with selecting parameters for the simulation. In the next iterations of this project, a more intentional opportunity to reason about setting parameters from a survey and a sample will add another dimension to the development of the model.

After the determination of the parameters, most students proceeded to simply run the simulation (a rather tedious exercise) for a 5-year period, and then organize the counts and readjust the parameters. The initial death parameters students selected were generally too high, and in most cases, students understood that they needed to lower their estimates to more realistic levels. How students determined more realistic levels was part of the reasoning they continually demonstrated in this project. The probability of births was more challenging, and required even more understanding of their model to coordinate the increases from births with the decreases from death.

The gaming aspect of the simulation clearly generated the most interest. Students often developed programs on their calculators or in a computer science class to allow for faster and more extensive executions of the simulation. A group of students with more extensive programming skills created an intricate world population game that incorporated probabilities of national disasters that would significantly increase death rates. Other students, especially as they developed a connection of their world model to the proportions of a different country, developed this more as a country project. Students added into the model the probability of a person immigrating into the country, along with a probability of a person leaving the country or emigrating. The expanded models evolved as a result of students incorporating technology to produce more sophisticated and meaningful simulations. In all cases, the models were evidence of how students applied their introductory understanding of probability to investigate a meaningful challenge.

CONCLUSION

The “World Population Project” outlined in this paper was intended to prepare secondary students for an appreciation and an improved understanding of probability and statistics. Students were given control over the parameters of the project, and over the general design of their model. They were given scenarios, however, to structure their thinking, and to establish and defend their individual decisions. The models they created resembled a game, and allowed them to create unique strategies in both setting their parameters and reaching their goals. In most cases, students expanded the project by adding new parameters and incorporating technology components to expand the number of simulations and the range of parameters.

The project was designed to provide opportunities for students to see topics and applications in probability and statistics as relevant options in their post-secondary plans. By allowing students to be creative in the design of the project, students had to examine the ongoing results and modify their thinking to “win the game” based on the goals of the selected scenario. Students were applying their understanding of the introductory topics of probability and statistics in way that resulted in a more mature understanding and appreciation of these topics. This project was designed to bridge the gap that often exists as students transition from secondary education to the first year of a college or university education.

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