

ELEMENTARY SCHOOL STUDENTS' UNDERSTANDING OF CONCEPT OF ARITHMETIC MEAN

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The research (spring of 2008) investigated the understanding of concept of arithmetic mean of 208 4th and 6th grade students of elementary school, in Thessaloniki in Greece. The result agrees with the conceptions of Mokros & Russell (1995, 22) that "the arithmetic mean is a mathematical object of unappreciated complexity". The students participated in research were able to calculate the mean using the arithmetic algorithm, although they couldn't use the metre to summarize data neither applied it in real life problems, that is interpreted as absence of statistical thought (Phannkuch & Wild, 2003). Children limit themselves in the simple knowledge of the algorithm (computational knowledge) that constitutes the lower level of instrumental understanding of the concept (Pollatsek et al, 1981). With regard to the comprehension of representativeness students recognized this attribute but they didn't lead to the confrontation of data as an entity.

INTRODUCTION

The most commonly used concept of statistics - the mode encountered in everyday life and included in most curricula of general education is the arithmetic mean, (Goodchild, 1988, Watson & Moritz, 1999, Van De Walle, 2007). The arithmetic mean is common in the vocabulary of many children, even before the formal statistical knowledge they get from school. Phrases such as average height, average age, average score, used by teachers and books before even the students can understand its meaning.

In this study, we tried to search the degree of understanding of the concept of average, between children aged 10 and 12 years, corresponding to the 4th and 6th grade of primary school.

Questions that led us in dealing with the issue were: whether the average is a concept easy to understand, how children acquire functional knowledge surrounding the concept and use it to solve problems of everyday life. Also, we tried to see whether students learn both the computing and the statistical dimension of the concept during their school education. Additionally we searched to find possible misunderstandings, informal ideas related to the meaning of the average, existing in children before teaching and the ways that these ideas were used in teaching for a proper understanding of the concept.

In this research the sample consisted of 208 students from six schools of the Thessaloniki region, so the sample reflects to some extent the socio-economic and cultural status of students. The questionnaires collected were completed by 109 students of 4th grade and 99 students of the 6th grade.

RESULTS

The students participating the survey were asked to answer whether they had heard the expression of the average or mean. With following questions we tried to record the previous knowledge on the average, from the natural and social environment of children. From the responses we collected 62% of pupils of 4th class and 89% of pupils in 6th class, replied positively. But when students were asked to indicate an example, using the concept of average, an important weakness appeared, especially in the 4th grade students. Note, that at this grade, students are not taught in school about the average. The question "Write an example of average", 4th grade students were able to give a good example from everyday life did not exceed 39% of the sample, while the 6th graders percentage rose to 64% for the same question.

In order to see an understanding of the average concept in everyday life, we asked the children to indicate how they interpret the phrase: "The average age of the children of this school is 9 years". The answers we received were classified into the following categories: algorithm (Cai, 1998), expected value (Goodchild, 1988), fare share (Cai, 1998), individual age-measuring individuals (Konold & Pollatsek, 2002), most, lower price (Watson, 2006), measure of location

(Goodchild, 1988), median (Mokros & Russell, 1996), mode (Mokros & Russell, 1996), representative number (Goodchild, 1988; Mokros & Russell, 1996), total price, point of balance (Mokros & Russell, 1996) (Table 3). When asked about half of the 4th grade students, 10% of them gave their interpretation based on the maximum value (most), 10% on fare share (Cai, 1998), 6% on individual age, 5% on the median, 6% on the mode and 4% on the lowest price. 6th graders respondents to the same question up to 70% and these the biggest part responded based on the mode (37%), followed by the answers of the measure of location (10%), of the fare share (7%), of the algorithm (3%), of the median (3%).

These results indicate that a significant number of pupils of the 4th grade declare their knowledge on average (62%) from the experiences obtained from school, family, social networks and electronic media. Many however were those who find it difficult to link these experiences with the facts and give an example of an everyday use of the term. So either they didn't respond (48%) or gave the wrong example (13%). Also from the responses to the question "What is the meaning of the phrase: The average of the school pupils is 9 years?" -a question that explores the way to understand the term in an everyday life example has shown the absence of the concept of the totality of data and thus they tend to focus on individual values (higher value, personal value) (Table 3). A significant part of students (10%), understand the average as fare share, or as an equivalent deal, to a common value to be had if all elements were the same. In 6th grade children indicate a higher percentage (89%) has similar experiences mainly from school and less than their social circles, but here the rate drops significantly when it comes to make an example (64% correct answers to 31% no response and 5% wrong answer). Unlike the students of 4th grade, the part of the problem of understanding the concept of average, is greater than the mode responses (37%), measure of location (10%) and the fare share prices (7%), indicating that students are able to handle data as a whole.

The investigation sought to identify strategies that children use to determine the price of average. For that reason they were asked to estimate the value of average choosing from a given set. The survey recorded that nearly 9 out of 10 students of the 6th grade, who had just been taught the chapter introducing the concept of average, used the correct algorithm to calculate the price, in an everyday life example, where its use was not explicitly defined (Gal, 1995). Further processing of this finding shows that pupils of 6th class of the Greek education system though achieved higher performance than the investigation of Gal et al, (1990), had similar performance compared to the 9th graders. In contrary they found it difficult to use the concept of average using unfamiliar examples. Interpreting this phenomenon as Mokros & Russell, (1995), argued that students who use the algorithm, such as they were taught in school, do not recognize the concept of representativeness, generally fail to use it correctly in any application.

On the other hand, pupils of the 4th grade, which had no formal knowledge, used the algorithm (35%) and of these only 1 out of 4 calculated the average price. The others used alternative, or other spontaneous approaches to calculate the average, as proper (logical) value, mode, most, current value, modal clump and number of values (Table 1).

As the youngest pupils were concerned, lack of formal knowledge lead them to search elsewhere for the calculation and use of the average term in everyday life examples. Particular interest shows the use of approximate average term as appropriate (reasonable) price, which is considered by Mokros & Russell (1995), including the status of the representation of data and the appropriate teaching method that seems to lead to a comprehensive understanding of the term.

Students of both grades showed in majority that they had already heard in some way the term: mean or average, and many of them were able to make an example. This allows us to conclude that they had some experiences regarding the concept under consideration. Source of these experiences was school, mainly for 6th graders and its surroundings for the students for 4th graders.

For the interpretation of the term of average and an everyday life example, about 4 out of 10 6th graders, use the term mode.

The concept fare share was rarely used by 6th graders (7%). The interpretation of the average as fare share leads to the heart of the statistical nature of the concept. Such an understanding of the average is considered easy (Van de Walle, 2007) and provides a conceptual

platform to connect with the algorithm (NCTM, 2007), while giving students an opportunity to create alternative ways of calculating (Foreman & Bennett, 1995).

For the same example 4th graders showed varying interpretations of the average, with equal distribution of fare share values (10%), the expected (typical) number (2%) and the measure of location (a value close enough, but not exact) (2%). The Goodchild, (1988) survey, identified the use and the interpretation of average in children 13 and 14 years old, with the form of the measure of location and the expected number, as spontaneous ideas related to the concept of average. According to Van de Walle, (2007), the average price of a standard value could be a good starting point for teachers to connect children's pre-existing knowledge.

The results of the processed data showed that 49% of 4th graders and 71% of 6th graders responded that in their opinion, there is a number that has the status of representativeness, which the 17% assigned themselves on average, while 52% of the 6th graders did the same. The remaining responses were divided into values such as current, maximum, minimum, stand. However rates reduced by an indirect determination of representativeness, together with different approaches of problem solving that Mokros & Russell (1995), associated with the concept. These approaches are reasonable price, the midpoint and the mathematical point of balance.

Table 1.

<i>What strategies were used to calculate the average?</i>	<i>Grade 4th</i>	<i>Grade 6th</i>
Algorithm	34,86%	86,87%
Proper (logical) value	11,01%	0%
Maximum value (most)	6,42%	2,02%
Mode	17,43%	5,05%
Median	0,92%	0%
Modal clump	3,67%	0%
Number of values	1,83%	0%
Arbitrary value	7,34%	1,01%
No answer	16,51%	5,05%

CONCLUSION

Attempting an assessment of the results in relation to the assumptions of the investigation, it seems that they are significantly verified. So regardless sex, children of primary school learn easily to calculate the average of a data set. This is not simultaneously accompanied by the opportunity to use this measure to summarize the data and make comparisons of data sets. Even for the older pupils, showed a difficulty to use school knowledge to solve unfamiliar problems. This is interpreted as a lack of statistical thinking, due to lack of understanding of the conceptual schema required to solve larger problems (Skemp in Pollatsek et al., 1981). It seems children are limited to mere knowledge of the computational algorithm of the average, which is the lowest level of organizational understanding of the concept.

For the purposes of representation, although many students have shown to recognize this nature in the average term, this realisation did not seem to get them to address the data as a whole, suggesting the failure of conceptual understanding.

From the research it results that the concept of mean is complex and conceptually exigent process. The students appear that they face difficulties in comprehension of mean and show a tendency to calculate the algorithm so that they resolve statistical problems. Consequently the

instruction will not be supposed to give the main weight in the learning of calculating algorithm, but in the growth of comprehension of ideas of representativeness and fair share. For this matter Mokros and Russell (1995) supported that the teaching of the concept of mean should be imported only when the students have developed a powerful base for the idea of representativeness. The previous knowledge of students constitutes the base in order to build the necessary ideas that will lead them to the functional comprehension of statistical concepts and to statistical literacy that constitutes the final goal of statistical education.

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