REVIEWING AND PROMOTING RESEARCH IN PROBABILITY EDUCATION ELECTRONICALLY

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In the topic study group on probability at ICME 11 a variety of ideas on probability education were presented. In an international, collaborative, project after the conference, many of these papers have been developed much further, incorporating novel electronic techniques, which also feature in this paper, with links to many references rather than a simple listing. This joint presentation provides a summary of the main threads of research in probability education across the world; there is a linked paper on possibilities of electronic communication (The future of interactive electronic research). This paper provides an incisive and reflective summary on which researchers can build, while the latter enables developments relevant for other areas of research too. Hyperlinks are included throughout.

BACKGROUND

The paper is linked to another paper intimately; the two papers serve rather distinct yet important purposes to guide and direct future research. One is to present a synthesis of current research efforts in the teaching of probability as discussed at ICME 11, where there were ideas from across the world, including the increasingly influential Spanish work. The other is to discuss the influence of new technology in how research is presented and how this changes even the nature of research. These papers use the technology via hyperlinks, making access to work direct and straight-forward for readers and researchers. At ICME, it is salutary to note that there were two separate study groups, on probability *and* on statistics. This reflects a shift of interest from the last years when statistics has taken a leading role and probability has decreased in attention from educational research: a development, which was paralleled by the curricula in schools where probability has been reduced in favour of statistics at all levels. The revived interest in probability comes from recent developments to integrate concepts of chance and risk into teaching. The main themes which occupy researchers currently are: fundamental ideas; pre- and misconceptions; conditional probability and Bayes' theorem; use of technology.

OVERARCHING THEMES IN PROBABILITY EDUCATION RESEARCH

Research in probability education started over thirty years ago, and became established as an area of research with the first ICOTS in Sheffield in 1982. There have been a number of key books published. Perhaps the seminal work was by Tversky and Kahneman, for which the Nobel Prize was awarded. Since that time there have been four key publications presenting and synthesising important research ideas: Fischbein (1987); Kapadia and Borovcnik, 1991; Jones 2005; Borovcnik and Kapadia 2009. This paper focuses on the last publication in order to provide direction for research in the next decade. It is a bold claim, but justified by the range and quality of research below. Whilst some of the ideas are not 'new' in the conventional sense, and no empirical or experimental work is described, there is nevertheless much 're-search' involved to focus on an older and equally relevant interpretation of research. Readers will have to judge for themselves whether the view of the authors is correct: that the whole as presented here is more than the sum of the parts, particularly with the accompanying incisive and reflective commentary.

The challenge is to teach probability in order to enable students to understand and apply it. As the frequentist and subjectivist views of probability are the two main interpretations and schools of probability, they should be incorporated into teaching. In accordance with endeavours to focus on modelling and applications (the main stream in teaching in reaction to the "New Math") one should search for connections of probability to practical applications. With respect to didactical aids, visualization of abstract concepts is important in teaching; for probability, it has also become standard to visualize the consequences of models by simulation. There are other possibilities to facilitate learning. The use of technology helps to reduce the technical calculations and focus the learner on the concepts instead. The world of personal attitudes and intuitions is another source for

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success or failure of teaching. This might be true for other disciplines of mathematics; however, it is especially important for probability, as there seem to be strong emotions connected to the underlying concepts and random situations, which decide also whether students *accept or ignore* what they learn. The reader might miss "risk" (see Gigerenzer, 2002) among the main themes; while risk formed the backbone of several papers it was explicitly dealt with in only one.

Conditional Probability and Bayes' Theorem

Conditional probability and Bayes' theorem are important ingredients of probability and should not be left out of any standard course in probability at school and at university, including for non-mathematical students. The concepts overlap with diverging private conceptions, which are also causally interwoven. These concepts also stand at the "border" between the two different theories of probability, the objectivist conception with mainly the frequentist interpretation and the subjectivist conception with probability as degree of confidence. For such reasons, to learn only the mathematics of these concepts does not suffice to understand and apply them adequately.

Many different types of errors have already been investigated in isolation. According to Diaz & Batanero (Spain), however, there is neither a study investigating connections between various types of misconceptions, nor an analysis whether misconceptions are related to mathematical knowledge. Consequently, they have developed a test with (mainly familiar) items, and administered it to university students. Data is analyzed by means of factor analysis. They describe some phenomena, which remain even with higher mathematics education, but in general a significant decrease in misconceptions is found with a higher level of mathematics. For interrelations between several misconceptions, the result is less optimistic as these misconceptions seem to be quite isolated. As a consequence of this investigation, endeavour in mathematics education in probability has to be fostered while the types of misconceptions still have to be singly put to the fore in teaching again and again in order to facilitate students' understanding.

Huerta (Spain) describes a mathematical *structure* of "ternary problems" and classifies 20 different types of problems with conditional probabilities of which only *one subclass* has been used in existing research. From this, it is questionable as to how to generalize from previous findings. He uses graph theoretic methods to describe the world of all conditional probability problems. Each single problem amounts to a subgraph of the full graph. Solving the problem is mapped by a successive extension of this graph until the target point is connected to it. Thus, he can visualize the steps of solution and the complexity or difficulty of a special problem at hand. Other papers also refer to conditional probability and Bayes' theorem, including those by Vancsó (Hungary), Martignon & Krauss (Germany), and Trevethan et al. (Mexico).

The School Perspective: Pre- and Misconceptions

The rules of probability might be simple. However, they all involve probability, a sophisticated concept, not developed until the seventeenth century. What does a probability of ½ or 0.2 really mean? The individual's world is full of diverging private conceptions connected to probability statements. Even if a student has adopted a frequentist interpretation and even if the probabilities are moderate (i. e., in a range of 0.1 to 0.9) and even if there are no higher losses or wins at stake (which also might involve intuitions about their utility), there remains much to clarify by teaching including the law of large numbers (and the many inadequate conceptions to it), or the task of using such probabilities for decisions. Single trials, the overall outcome, and connections between the two are also important. Abstracting a general law from various single outcomes (which also involves mathematical concepts) is not easy to understand, nor applying such a general law again to single outcomes etc.

Technology might help to build stable intuitions, as explored by Ireland & Watson (Australia) but private conceptions are so unstable that using technology to simulate a random experiment repeatedly and analyze it by inspecting the results from various angles might not help. To perform an experiment with "real world devices" might be very different for younger students from performing such experiments on the computer that they would develop completely different attitudes and conceptions thereby. The reader may remember that younger students often find it difficult to accept that to throw two dice simultaneously amounts to the same experiment as to throw a single die two times. K. Lysø (Denmark) presents ideas on how to develop students' intuitions in the initial phase of teaching: he uses instruments from empirical research (written tests, in-depth interview with the class) to make them aware of the discrepancy with the normative way to deal with such problems. S. Anastasiadou (Greece) analyses the influence of different representations and comes to the conclusion that students often only learn ideas very closely linked to the representation used. Higher levels of understanding, however, are only reached if students are able to recognize the common aspects in different representations.

Abrahamson (USA) analyses a learning environment in which real experiments are mixed with computer experiments on the binomial distribution. While such concepts are beyond what an 11 year old child could do, the learning environment establishes some stable intuitions that later should pave the way for the binomial distribution. The target for the research was to seek the crucial stages of such learning processes: where and when does the child make substantial learning steps, wherein do obstacles lie and how could they be remedied by suitable embodiments of the concepts? His experiment is based on a single child and in-depth interview after a teaching phase in a classroom environment where an urn experiment was replaced step by step by the computer environment. He is careful to let the interplay of the different embodiments of the same notion work as the main input for learning (and not the teacher or the interviewer).

Since Kahneman & Tversky's seminal experiments with probabilistic situations there have been many other investigations. As we now know, a good strategy is often extended to cases where it is inappropriate. Among the heuristics used are negative and positive recency. L. Zapata investigates teachers of mathematics and how they anticipate difficulties for their students. Of course, a good teacher who foresees students' difficulties may be prepared to cope with them. Interestingly, the teachers reveal almost the same problems with perceiving the situations; only those with more teaching experience seem to "have learned their way". As a consequence, one might speculate about the education of teachers at university. Does it suffice to teach them the mathematics of the discipline or should they also follow courses, in which they learn how to teach the subject, as well as about specific learning problems and obstacles in the subject. While this may be important for calculus, it seems vital for probability.

Chiesi & Primi deal with the *development* of heuristics with age. They compare 9, 11, and 25 years olds in order to imitate a longitudinal survey. From a bag with blue and green marbles, one is drawn repeatedly with replacement. "All marbles of the same colour" is presented as the result. The marbles are not actually drawn; their numbers in the bag are known. According to the "negative recency", people predict a change: with 4 blue they would predict "green". With the "positive recency", they predict that colour, which continues the series "observed". How frequently are these heuristics used, and do they depend on the composition of the colours in the bag? Interestingly, the study shows an increase of the normative (correct) solution from 9 to 11, which then drops down at 25; "negative recency" develops in the opposite direction. More in-depth investigations would help to clarify whether such a "development" can be confirmed.

K. Rolka and S. Prediger (Germany) present a study of 12 year olds playing a game (ludo) with tokens moved forward on a playing board by the result of a die. The die used had more red sides whence it favoured the red token. The discussion amongst the children reveals how they fiercely argue in favour of their strategy until they *jointly* agree on a strategy. The *common* struggle for a strategy seems to generate a better understanding (or at least a better acceptance) of the value of their strategy – they seem to be much more aware of the chance that a token of a different colour could still win; much more than they were aware by simply playing or *observing* the game. The remark about understand and accept what one learns is not trivial: even if (s)he understands a concept, a child might not be willing to apply it. In school, students perform the "glass bead game" of Hesse, privately retaining intuitions, despite abiding by the teacher's rules. The social construction, the joint struggle for a solution is a teaching strategy to counter such a phenomenon.

Impact of Technology

Technology can be viewed in at least two very distinct ways. One aspect is the media used such as PowerPoint or interactive use of computers by students. The other aspect relates to the software tools used, such as Excel and Fathom. Some software is generic (Excel) and some software is designed specifically for probability such as ChanceMaker. In practice there is more software relating to statistics, though probability software is growing.

The reader's imagination about what can be done is aroused by the ideas of Pratt & Kapadia (England) on shaping the experience of naïve probabilists. Utilising one of the principal features of probability (namely its unpredictability), they search for possibilities to shape intuitions about probability by a "fusion" of partial control over some parameters of the distributions underlying an experiment and the graphical representation of the empirical consequences. Randomness is what "escapes" your control. Abrahamson also uses Logo as a tool to explore binomial distributions. Ireland & Watson (Australia) use Tinkerplots in order to develop their teaching units on the law of large numbers; they analyse the build-up to understanding by in-depth interviews, available in the electronic version of the publication). S. Inzunsa (Mexico) uses Fathom to enhance connections between empirical and theoretical distributions by simulations; a crucial point is a solid perception of small tail probabilities, which are used for evaluating significance of observed results. R. Peard (Australia) has developed a course on games of fortunes with EXCEL. He considers games of fortune not as abstract situations but–in the face of the huge business–as *applications*.

Fundamental Ideas

Kapadia (England) presents tasks from the national tests and concludes from the poor achievement of students therein that teaching compared to 20 years behind has not improved substantially. This may be rooted back to recent trends favouring data handling at the cost of probability in the curricula. Teaching still focuses narrowly on equal likelihood and experimental probability; work on misconceptions is rare and risk as a related concept is hardly discussed. In judging probabilities, people seemingly have a strong bias towards "equal probabilities", especially when they are (or feel) confronted with two possibilities. The fundamental idea of judging probabilities and risks subjectively has still not found a sustainable form of teaching. Such issues often involve major consequences, utilities and small probabilities. This combines potential (fictional) adverse consequences (which have not occurred or are very rare) with misleading intuitions hindering rational thought. M. Borovcnik (Austria) develops fundamental ideas by examples; *thinking probabilistically* goes beyond thinking in mathematical models. It might be the time to replace Heitele's (1975) fundamental ideas, which resemble the main chapters of a probability textbook by an approach which looks at the concepts more from a non-mathematical perspective. Y. Wu presents ideas from China, which are familiar and evoke the question: Are intuitions about probability free of cultural background? A feature of stochastics generally is that problems are easily formulated but – even for the experts – it is not always sensible to seek a closed solution as one would expect within mathematics. The question remains on how to fill the gaps by intuitions, analogies, or other methods, yet still retaining a sound approach to probability.

CONCLUSION

The aspects of research in probability dealt with in the electronic project after ICME 11 will be particularly valuable to younger researchers who could also see the material in the background as specimen for their own future work. The ease of access is of particular note and importance. The articles show that the community regains interest in probability education. Future generations of students might thank the authors as they would profit by understanding probability.

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