DEVELOPING A FRAMEWORK FOR REASONING ABOUT EXPLAINED AND UNEXPLAINED VARIATION

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As a principal form of statistical thinking, consideration of variation impacts on all aspects of statistics. There has been extensive research about students' reasoning about variation but little research focusing on helping students model variation as a combination of explained and unexplained variation. A study analysed responses to a measurement instrument that was developed to assess tertiary students' informal reasoning about variation, focusing on explained and unexplained variation. Selected students were also interviewed. This paper reports the analysis of the responses that informed the refinement of a framework that describes six components of reasoning about explained and unexplained variation. Implications for researchers and educators will also be discussed.

INTRODUCTION

Recently, there has been an increase in research on developing students' understanding of fundamental statistical concepts that are necessary for inference, as well as research that emphasises the need to help students develop an appreciation of stochastic thinking (e.g., Pfannkuch, 2005; Wild & Pfannkuch, 1999). As a principal form of statistical thinking, consideration of variation impacts on all aspects of statistics. There has been extensive research about students' reasoning about variation but little research focusing on helping students model variation as a combination of explained and unexplained variation. The importance of "an understanding of how variation arises" and of the "uncertainty caused by unexplained variation" for inference is widely acknowledged (e.g., Pfannkuch & Wild, 2004, p.18). However, the concept of variation as a combination of explained and unexplained variation is rarely emphasised in introductory statistics courses and there has been little research into students' reasoning about explained (signal) and unexplained (noise) variation.

Encouraging a view of the measures of centre as the signal in a noisy process is the main focus of Konold and Pollatsek (2004), who stated that "the notion of an average understood as a central tendency is inseparable from the notion of spread" (p. 173). Their work focuses on helping students find the signal amongst the noise. However, they fail to emphasise the need to consider both signal *and* noise (explained and unexplained variability) when dealing with comparisons of groups. One goal for statistics educators should be to encourage students to view the total variation in data as a combination of explained and unexplained variation. This view should underpin all topics in an introductory statistics course (e.g., exploratory data analysis, sampling distributions, inference).

The research presented in this paper was situated in a learning context that aimed to extend Konold and Pollatsek's work by helping further develop students' reasoning about variation through making explicit the need to model variation as a combination of explained and unexplained variation when comparing groups and making inferences. The focus is on the refinement, based on student responses, of a framework of components of reasoning about explained and unexplained variation.

CONTEXT

To describe students' reasoning about explained and unexplained variation, Reid, Reading & Ellem (2008) proposed five components of this reasoning: *noticing and acknowledging variability, acknowledging variability around the signal, looking for causes of variability, controlling causes* and *linking explained and unexplained variability for inference*. Note that both the term *variability* and the term *variation* are used. Following Reading and Shaugnessy (2004), the term *variability* is taken to mean the observable characteristic and *variation* to mean the description or measurement of that characteristic. These components of reasoning about explained

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and unexplained variation evolved from the four components of consideration of variation identified by Wild & Pfannkuch (1999), and a causation hierarchy developed by Reading & Shaughnessy (2004). To investigate the usefulness of the framework, Reid at al. (2008) developed a seven-item questionnaire, including four items specifically designed to address the five components of reasoning about explained and unexplained variation.

METHODOLOGY

The seven-item questionnaire (Reid et al., 2008) was administered to students enrolled in a one-semester tertiary introductory statistics course. The questionnaire was designed to elicit responses reflecting students' reasoning about variation, with a particular focus on explained and unexplained variation. First, these responses were coded, using Reid and Reading's (2008) *Consideration of Variation Hierarchy,* according to the level of consideration of variation. Second, six students were selected for interview because their pre- and post-study responses to at least one of the items differed in the level of consideration of variation exhibited. A key objective of the interview was to prompt the student to expand on his/her reasoning about any of Reid et al.'s (2008) five components of variation. Finally, the responses to both the questionnaire items and the interviews were analysed to determine whether the Reid et al. (2008) five components of reasoning about explained and unexplained variation was a sufficient framework to cover the reasoning exhibited.

RESULTS

While evidence was found of reasoning in all five components of the Reid et al. (2008) framework, the analysis provided evidence of two necessary improvements to the framework: (i) inclusion of a sixth component (*modeling and quantifying explained and unexplained variability - italicized* in Table 1); and (ii) description of the links among the various components. The proposed improved framework is illustrated in Table 1. This section describes how the five pre-existing components were evidenced in student responses and then what features of responses prompted creation of the new component. Note that the titles of two of the pre-existing components have been slightly adjusted for clarity. The discussion section explains how the components are linked.

Table 1. Framework of components of reasoning about explained and unexplained variation

Noticing and acknowledging variability (NAV) Noticing and acknowledging that variation exists in data is the most basic level.	
Acknowledging variability around the signal (AVS) Recognising variability around a signal (mean, trend, etc) is a necessary precursor to quantifying explained and unexplained variability.	***Looking for causes of variability (LCV)*** Looking for and identifying causes can lead to an appreciation of the difference between the concepts of explained and unexplained variability.
Modeling & quantifying explained and unexplained variability (MQV) *Modeling and quantifying the variability (e.g., numerical and graphical summaries) is necessary to better understand and describe explained and unexplained variability.*	***Controlling causes of variability (CCV)*** Considering how to control causes of variability encourages a focus on sources of explained and unexplained variability and how to include them in experimental design and analysis.
Linking explained and unexplained variation for inference (LVI) Making appropriate links between explained and unexplained variation facilitates informed comparisons, conclusions, and inferences.	

Noticing and acknowledging variability (NAV)

The nature of the items used meant that students were required to think about variability, and all responses showed some awareness of variability. When asked “What does variability mean to you?” most students focused on individual variability (unexplained). Students required a more structured context before being able to expand on ideas related to explained and unexplained variability. For example, items that required a comparison of means for a given set of circumstances resulted in responses that referred to differences among groups (explained variability) and within the groups (unexplained variability).

Acknowledging variability around the signal (AVS)

Although students in introductory courses often focus on the signal or trend in data without considering variability around the signal, the analysed responses showed good attention to variability, especially in the items designed to focus attention on variability around a trend. For example, in the two items that included time series plots some responses displayed well-developed reasoning, where reference was made to trends over time and included a detailed description of the variability around the trend line.

Linking explained and unexplained variation for inference (LVI)

The items were not designed to explore formal inference and so responses only exhibited informal reasoning about linking explained and unexplained variation. In one item requiring a comparison of two distributions, with equal means but different variability, some students linked the explained and unexplained variation to draw conclusions about the improved reliability of predictions based on the distribution with the least variation around the mean. One student stated that “Statistical tests … test whether the variability is due to … chance really or whether … there is a true difference in the population” and, when prompted, referred to formal tests discussed during the course, recognising the link between explained (which he referred to as “a true difference”) and unexplained (referred to as “chance”) variation when making inference.

Looking for causes of variability (LCV)

For all items there were responses that identified causes of variability. Responses included references to “factors you can’t control”, and acknowledged that both identifiable (i.e., explained) and unknown (i.e., unexplained) factors may impact on the outcome. Prompting during interviews assisted students to identify specific causes of variability relevant to the context of the problem.

Controlling causes of variability (CCV)

Three approaches to controlling causes of variability, some not feasible or desirable, were identified in responses:

- Informally recognising the need to identify possible causes of variability and include them in the model. For example, in response to a question that required consideration of possible causes of variability in weight gain in rabbits allocated to two different diets, one student mentioned the need for blocking to account for other sources of variability apart from diet.
- Removing the causes of variability by eliminating every factor apart from the factor(s) of interest. This was the most common approach. For example, in response to the rabbit diet question one student proposed using only female rabbits of a particular breed under one set of environmental conditions. When asked how findings could be generalised, the student talked about “replication” but was uncertain how this would help.
- Increasing the sample size. For one item a student stated that he would want a larger sample size as this would allow you “to get a trend of what you are really testing for, so those other minor outside factors will have less of an impact”.

Modeling and quantifying explained and unexplained variability (MQV)

The analysis highlighted a gap in the framework. This prompted the proposal of the new MQV component for the framework. Some responses went further than simply acknowledging variability around the signal but did not necessarily attempt to make links for inference. Some students attempted to quantify the variability around the signal, although sometimes in an idiosyncratic way. For example, one student, when describing the variability in the number of student disruptions before and after a remedial classroom intervention, made a calculation using the minimum number of disruptions before the intervention and the maximum number after.

DISCUSSION

The framework described in Table 1 represents important aspects of reasoning about explained and unexplained variation. The analysis of the responses reinforced the pre-existing five components of the framework from Reid et al. (2008) but also indicated that a new sixth component was needed. What was also apparent from the responses was that the various components may occur concurrently during reasoning and are not necessarily hierarchical. For example, students looked for causes of variability without acknowledging variability around a signal and vice-versa. Nonetheless, there are five important links (represented by arrows in Table 1) between various components within the framework. Students must be able to notice and acknowledge variability before they are capable of demonstrating achievement of any other component. From there, the framework has two paths: (i) *consideration of explained and unexplained variability* (AVS, MQV, and LVI on the left of Table 1) which should be considered hierarchical in terms of the level of reasoning about variability, and (ii) *consideration of causes of variability* (LCV, CCV on the right). The *causes of variability* should be considered as a necessary path to help reason about explained and unexplained variation with the view to reducing the amount of unexplained variation in the data. This leads to an increase in the power of the analysis, with inference as its objective. This is highlighted in Table 1 by the link from the right path (CCV) to the left path (MQV) of the framework.

IMPLICATIONS

The proposed framework provides researchers and educators with a structure for assessing reasoning about explained and unexplained variability. Used in combination with specifically designed assessment items, the framework draws attention to the various components of the reasoning. It should be noted that written responses do not always provide enough detail to determine whether particular components of reasoning have occurred. Further information may be obtained by providing other opportunities to develop their reasoning such as interviews, which can facilitate the expansion of responses to expose deeper reasoning. This is valuable for allowing researchers and educators to identify gaps in reasoning. Future research should seek to refine this framework by using it to assess the reasoning of a larger cohort of students.

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