DEVELOPING ROBUST UNDERSTANDINGS OF VARIATION

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This retrospective phenomenological study investigates activities and actions identified by secondary statistics teachers who exhibit robust understandings of variation as deepening their understandings of statistical variation. Framed by Mezirow's transformation theory, analysis revealed learning factors that include interest in the field of statistics, motivation to encounter and resolve dilemmas, reflection on content, and opportunities to engage in learning and rational discourse. The extent to which these teachers seek and embrace learning opportunities may distinguish them from other teachers. This work contributes to understanding circumstances that may be conducive to developing deep understandings of statistical content. It also advances the use of retrospective methods within a theoretical frame for adult learning to investigate teacher learning.

BACKGROUND

The need for statistical literacy is clearly imperative in our increasingly data-filled, technological world. As suggested by work that connects teacher knowledge with student achievement (e.g., Hill, Rowan, & Ball, 2005), graduating statistically literate students requires statistically literate teachers. Expository literature documents views that many teachers lack appropriate knowledge and experiences to facilitate students' development of statistical literacy (e.g., Ben-Zvi & Garfield, 2004). Current research suggests that while researchers are uncovering characteristics of experiences that result in teachers' learning (e.g., Liu & Thompson, 2009), they are only beginning to reveal characteristics that lead to robust constructions of formal concepts. Current efforts can take years to produce insights to effect change in statistics teacher education.

This retrospective study with teachers who have robust understandings offers a viable and timely means to uncover characteristics of experiences associated with learning. Given the centrality of variation to statistics and the role of variation in statistical thinking (Wild & Pfannkuch, 1999), this study focuses on characteristics of experiences that contributed to teachers' construction of robust understandings of variation, which arguably provides insights into statistical learning in general. This study answers the question: For secondary AP Statistics teacher-leaders who exhibit robust understandings of variation, what are the activities and actions that contributed to their current understandings of variation as reflected in their perceptions and recollections?

THEORETICAL FRAMEWORK

Transformation theory is a theory of adult learning (Mezirow, 1991) that provides explanatory power for teacher learning. Consider an individual's initial belief that variation consists merely of dry techniques for calculating summary values to describe data variation. Learning about sampling and experimental design introduces additional types of variability, such as sampling variability, to the individual. Upon reflection, the individual may reject her belief and adopt a more encompassing view of variation, transforming her *meaning scheme*, or particular knowledge and beliefs (Mezirow, 1991), for variation. A transformed meaning scheme for variation may trigger examination and questioning of assumptions for other concepts, resulting in additional transformed meaning schemes. Although the individual may initially believe that statistics more generally is a subject of data manipulation, display, and calculation, the accrual of transformed meaning schemes may precipitate questioning and transforming her *meaning perspective* for statistics—her larger web of interwoven assumptions and predispositions about statistical concepts and relationships formed from culture, personality, and past experiences (Mezirow, 1991)—into statistics as a problem-solving process that allows decisions to be made from data.

The main phases of transformative learning that collectively result in perspective transformation are *critical reflection*, *rational discourse*, and *action* (Mezirow, 1991). Perspective transformation typically begins with one or more events that trigger cognitive conflict or

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examination of assumptions. An individual may become aware of previously implicit assumptions and then assess those assumptions through critical reflection (Mezirow, 2000). Upon critical reflection, the individual may explore options for new roles and actions by engaging in rational discourse with others in preparation for developing and acting on a plan to resolve the conflict.

METHODS AND DATA SOURCES

This study uses phenomenological methods (Moustakas, 1994) to explore the phenomenon of secondary teachers' development of robust understandings of statistical variation with a purposeful sample of 16 secondary statistics teacher-leaders representing 14 states in the United States. To determine the robustness of teachers' understandings, the researcher conducted a 90- to 120-minute semi-structured, task-based interview with each teacher using tasks developed from consideration of research about students' learning related to variation (e.g., delMas & Liu, 2005), expositions on what it means to understand variation (e.g., Garfield & Ben-Zvi, 2005), and analysis of pilot study interviews. Data from five teachers evidenced their perceptions and recollections of actions that contributed to their development of robust understanding. See Peters (2009) for details of the identification process. Data sources include questionnaires (Q), two learning-experience interviews (Int 1 and Int II), event history calendars (EHC), and critical incident descriptions (CI).

RESULTS

Evidence from the five teachers suggests that their meaning perspectives for statistics were transformed during the time they taught statistics. Description of their transformative processes begins with experiences that triggered dilemmas and continues with personal and environmental influences associated with learning and characteristics of their learning experiences.

Through events related to teaching, each of the five teachers experienced triggers that prompted self-awareness of limitations in their knowledge. They all encountered dilemmas triggered from active engagement in statistics workshop or conference activities. Consider Everett's reaction to a conference presentation on "Linear Models and the connections between Multiple Regression and ANOVA" (CI). Prior to teaching statistics, Everett's experiences with data analysis were minimal; content beyond AP Statistics triggered dilemmas. Everett notes, "I realized that I needed to fill in lots of missing steps if I was to ever understand the material" (CI). Everett left the conference with a dilemma triggered from awareness of limitations in his regression meaning scheme. This session, and others like it, stimulated action to fill in the "missing steps." Blake describes how interactions with colleagues and statisticians also triggered dilemmas, particularly when considering AP scoring rubrics. Discussion of responses for which students did or did not receive credit surfaced new issues. Each teacher also describes interactions with students in which students' perspectives and questions provided the impetus for learning. Additional triggers arose during lesson planning, particularly planning to teach AP Statistics for the first time. Dustin reports dilemmas related to design. "I think that's why the first couple years I struggled, because I really had not had any formal design. So I was… trying to figure out what I should be doing in AP stat, in the design area" (Int I).

Common to all five teachers are personal factors that may influence their reactions to dilemmas and their learning. Each has interests that spur him to learn statistics. Blake, for example, cites interests from a college statistics course. "I did see I was interested in probability and statistics…I did see in [Professor] the passion…it kind of sparked an interest" (Int I). Blake's instructor described analyses of sports data, and the combination of passion apparent in the storytelling, the context, and the clear practicality served as "the triggering mechanism[s] [for interest]" (Int I). Each teacher also exhibits interests in and actions towards learning the intricacies of statistics. Blake's motivation in part stems from a desire to resolve what he calls "conflicts." He seeks to understand why statistical formulas and procedures work and why particular procedures are useful in particular situations. Like Blake, each is motivated to resolve dilemmas. Consider Isaac's reaction to a dilemma triggered from activities with statisticians. "I was only dimly aware of what they were talking about" (Int II). Statisticians' discussions made him "realize…pretty much how shallow my knowledge was" (Int II). He describes taking notes during discussions, consulting reference sources, and spending considerable time to make sense of the discussions.

Teachers' interests, motivations, and confidence may contribute to their reflection on statistical ideas at the core of their dilemmas. Each teacher provides evidence of critical reflection in his accounts of learning. Blake, for example, talks about "resynthesizing" knowledge. When asked how, he indicates, "I think it's just a reflective piece" (Int I). Blake is reflective in describing progressions of thought—series of *how* and *why* questions that lead him to consider additional concepts. He describes the process as one in which resolving one "conflict" triggers another and creates a series of trigger-dilemma-resolution cycles. Other personal factors that may contribute to teachers' transformations are their commitments to their own learning and to their students, as evidenced by the time and thought they allot for statistical activities; insights gained from adequate prior knowledge but perhaps not formed earlier in their careers; and their expressed desire to see "the big picture" of statistics. Each credits either the organization of content in the AP Statistics course description or his efforts in planning to teach AP Statistics with his development of a "big picture" view.

Environmental factors that may be conducive to learning include a safe and "comfortable" learning environment in which teachers feel free to ask questions about content. Blake, for example, describes the AP Statistics electronic discussion group as a "very comfortable place" where he and others can direct questions to knowledgeable statisticians and secondary teachers (Int I). He notes that when people disagree, they do so "in a very, very nice way" (Int I) so that he and others feel comfortable in posting questions. The five teachers attribute a sense of community to secondary teachers and statisticians who are active in the AP program and describe the benefits they attribute to membership in this community, which seemingly fosters trusting relationships that are essential for rational discourse (Mezirow, 2000). All five teachers attribute much of their statistics learning to discourse-related activities that include collaboration and interactions with more knowledgeable others, including other statistics teachers and practicing statisticians. All five teachers describe opportunities to interact one-to-one with statisticians as wonderful learning experiences. Everett indicates that when he displays faulty reasoning, statisticians are generally "not shy about saying well that's not really how it is, but they'll also be happy to explain it" (Int I). He describes interactions as "more of a teammate kind of feeling" (Int I) than one would have in typical student-teacher classroom interactions. Isaac seemingly concurs, noting, "she [the statistician] would say I don't think this, this isn't really quite right… And she'd have a little explanation" (Int I). The comfortable learning environment coupled with practical insights from knowledgeable others creates an ideal learning environment for the teachers.

Listening to and considering alternative perspectives may motivate teachers to critically question previously unexamined assumptions and beliefs about statistics. Reading different textbook presentations of concepts similarly triggers critical assessment of assumptions. Consider Dustin's steps to resolve a dilemma triggered from textbooks attributing seemingly different meanings to confounding, lurking, and extraneous variables. "I'd sit there with three or four books open, and I'd read…and then try to figure out, okay, where the points of commonality were, where they were different… after a while I realized…they're really talking about the same thing" (Int II). Dustin suggests that reading and rereading different textbook explanations transformed his divergent images of lurking, confounding, and extraneous variables into a unified conception of variables different from the independent variable(s) of interest. Dustin assumed the authors' views and engaged in rational discourse with himself to resolve his dilemma.

The five teachers who exhibited robust understanding of variation attribute their learning to a variety of factors. None identify early courses as valuable learning experiences, but they suggest that those courses allowed them to construct a foundational base from which they could build more robust understandings. They speculate that a first course similar to AP Statistics should provide the familiarity with concepts and applications needed to make sense of statistical theory.

Each teacher identifies group engagement in statistics activities as significant. Dustin, for example, describes cooperatively designing and doing an experiment. "You got to consider…what kinds of things affect the variability…How can we, um, are there things we can control...and eliminate some of the variation that exists…, just reading a book, you don't necessarily get that kind of interaction" (Int II). Dustin suggests that by "playing" with the experimental treatments and groups and collaborating with others, more ideas about variables that contribute variability and ways to control variability are generated than by reading about a similar experiment in a book. The

teachers also describe learning benefits from problems that draw attention to fundamental statistical concepts and principles and experiencing presentations of key examples. Hudson describes an AP question that focuses on selecting experimental units in a way that removes a potential source of variation to make it easier to isolate treatment effects. The statistician who designed the problem discussed issues pertaining to this problem, and Hudson describes how her use of a key example—one focused in a context that "grabbed me better" (Int II)—enabled Hudson to see the variation effects of a particular variable. The teachers' comments suggest the nature of activities and problems that could trigger dilemmas in a first course or subsequent statistical experiences.

CONCLUSION

In describing their learning experiences, the five teachers with robust understandings of variation articulate a variety of characteristics indicative of transformative learning (Mezirow, 1991) for their meaning perspectives of statistics. While preparing to teach and teaching statistics, teachers experienced a number of dilemmas that prompted reflection, including critical reflection, of their assumptions related to statistics. During the course of resolving dilemmas, they often sought input from colleagues and more knowledgeable others and engaged in rational discourse. The consultations typically occurred as part of a plan of action to construct the knowledge and skills needed to resolve their dilemmas. As part of their meaning perspective transformations for statistics, some transformed their meaning schemes for variation, and others broadened their meaning schemes. Some possibilities for why these five teachers recognized and embraced opportunities to resolve dilemmas include their interests in the field of statistics, motivation to encounter and resolve dilemmas, reflection on content, and embracing of opportunities to engage in rational discourse and potential learning experiences. Perhaps the most striking feature of teachers' transformations is that a majority of their experiences are accessible to most teachers. The extent to which they embrace learning opportunities may distinguish them from other teachers, yet their collective stories offer insights for designing statistics education programs for teachers. Suggested features include a larger role for design, particularly in service of developing a "big picture" view of statistics, opportunities to engage with knowledgeable others and in communities of learners with real statistical investigation, and well-chosen problems that focus on fundamental statistical ideas in ways that are designed to trigger dilemmas and subsequent reflection on assumptions.

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