

A SEMIOTIC ANALYSIS OF “MÔNICA’S RANDOM WALK”: ACTIVITY TO TEACH BASIC CONCEPTS OF PROBABILITY

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We analyzed the activity “Mônica’s random walk”¹, in the learning environment paper-and-pencil, which presents the basic concepts of probability. We use the onto-semiotic approach of cognition and mathematics instruction to study the types of mathematical objects (language, situations, concepts, procedures, propositions, properties and arguments), and the semiotic potential conflicts that may compromise the understanding and meaning of these concepts. The activity was implemented with 29 mathematics teachers in an in-service training program. The method adopted for this study was action research. The results showed the viability of the activity to contextualize such concepts. However, they indicate different semiotic conflicts, due to the teachers’ poor prior knowledge of probability, since many of them were seeing these concepts for the first time. These results imply rethinking of some of the assumptions, especially if we want to insert them into virtual learning environments, so that teachers can implement it with autonomy.

INTRODUCTION

This article is a result a number of activities aiming to give a theoretical and methodological support to the teaching sequences of the research project “Virtual Environment for Statistical Literacy (AVALE)” financed by FAPESB² for the years 2008-2010 in Bahia, Brazil. The objective is to analyze and evaluate the teaching - sequence “Monica’s random walk” in a paper and pencil environment introducing basic concepts of Probability for Basic Education.

For this aim we utilized the onto-semiotic approach of cognition and mathematics instruction to study the types of mathematical objects (language, situations, concepts, procedures, properties and arguments). Also were studied the possible semiotic conflicts which may compromise the understanding and meaning of basic concepts in Probability as we described below.

AN ONTO-SEMIOTIC APPROACH TO COGNITION AND THE MATHEMATIC INSTRUCTION – (OSA)

The OSA is an ontological and semiotic analysis that uses both theoretical constructs of semiotics as the ontology of mathematical objects, to try to answer the problem of understanding of mathematics, developed by Godino and collaborators (Godino & Batanero, 1994; Godino, 2002; Gusmão, 2006; Godino, Batanero & Font, 2007; Font, Godino & D’amore, 2007). The OSA is considered a practical approach (favors the quality of action), a semiotic approach (central role given to the use of symbols utilized in mathematics) and anthropological approach (studies the subject learning in an institutional framework and mathematics as a human activity).

The OSA deals with the meaning of mathematical objects in a personal and in an institutional level. It studies phenomena derived from the didactic transposition school dealing with the integration of ontological, syntactic, semantic, pragmatic and socio-cultural aspects (Godino, 2002; Gusmão & Cajaraville 2007). OSA define as primary entities: Language (terms , expressions, notations, graphics), Situations(problems, tasks, extra/intra mathematical modeling), Procedures (operations, algorithms, calculus techniques), Concepts (number, point, function ...), Propositions (proprieties, theorems,...), Arguments (discourses to validate and explain propositions) (Godino,2002). Using the OSA we focus on these entities, using the “onto semiotic analysis” , allowing us to characterize the institutional and personal meanings that are manifested during the mathematical activity and so to analyze in detail semiotic conflicts³ which arise from the resolution of concrete mathematical problems.

In order to carry out a mathematical activity/task in a situation like, for example, the instructions of the teaching sequence “Mônica’s random walk” the task solution agent (person or institution) needs several items of knowledge to fulfill it. First teachers needs to know and utilize a

certain verbal language (tossing a coin, randomness so on), and a symbolic language (H for heads, X for tails, HHHX, so on). This language is the apparent part of a series of concepts (sample space, events, probability, so on), propositions (equally likely events so on) and procedures (repeatedly tossing a coin and writing down the results, so on) that will be used to put together arguments in order to decide if the actions that make the practice are appropriate.

In this work we apply the technique of OSA to analyze how interact the two meanings (institutional and personal) of probability during the activity “Monica’s random walk”, and if this helps to teach probability, in an attempt to evaluate the outline of the teaching sequence aiming to adapt it in the future to the virtual environment of AVALE.

MÔNICA’S RANDOM WALK

This activity was adapted by Cazorla and Santana (2006) to be used in elementary school, from Fernandez and Fernandez (1999), whom proposed it to teach Binomial distribution to major students. The full activity has four different sections and 23 questions. In each section, questions must be answered based on a previous action. In the first section, teachers have to read this story:

“Mônica (the girl on the left corner down) and their friends live in the same quarter (four blocks). Mônica’s house is four blocks away to the houses of Horácio (dinosaur), Cebolinha (three-haired kid), Magali (girl at center), Cascão (the kid with braces) and Bidu (dog), according to Figure 1. Mônica used to visit her friends along weekdays on a pre-set order: Monday, Horácio; Tuesday, Cebolinha; Wednesday, Magali; Thursday, Cascão; and Friday, Bidu. For turning those visits more thrilling, the group decided that randomness should pick the friend to be visited by Mônica. To do so, when leaving home and on each crossing, Mônica has to toss a coin; when the head (H) she will walk a block North, and when the tail (T) she will walk a block East. Each tossing represents a block of her route. Mônica has to toss the coin four times in order to reach some friend’s house”.

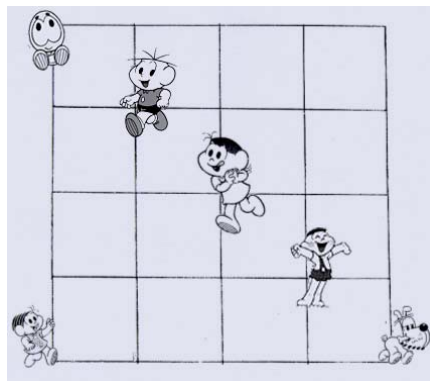


Figure 1. Map of Mônica’s random walk

After reading the story, teachers have to answer the some questions yet without tossing the coin. At section II, teachers carried out the experiment. At section III they have to construct the tree diagram; and at section IV they have to compare theoretical and frequentist probabilities, and between random and deterministic experiment.

Thus, this teaching sequence allows to work with probability concepts such as events, sample space, single event probability, to explore the difference between random and deterministic experiments, to approach probability from relative frequencies, to calculate theoretical probability from a tree diagram and to compare observed and expected patterns.

METHODOLOGY

The teaching sequence was applied by one of the teachers- researchers, who teaches the discipline, to a class of 29 mathematics basic education teachers who were studying the subject Probability and Statistics, in the course of teaching of mathematics, from the formation of teachers in service program of a public university, from the interior of Bahia.

The research was divided into two stages. The first stage was the presentation and development of the teaching sequence. The second stage the teachers doing the course applied the sequence with their students in school elaborating a document containing a report of the

application, thoughts and indications of limits and potentialities of the application, having in sight its improvement. At the present work we are reporting on the first stage of the research.

The activities started with a series of questions in order to expose the conceptions of the teachers about Probability. Afterwards each teacher performed the trials individually. Then the teacher-researcher mediated the systematization of the results building, on the black board, the Frequency Distribution Table (FDT), composed by the results of the experimentation done by all students. Also drawn on the board, with the help of the teachers doing the course, were the tree diagram and the bar graph side by side. The bar graph was constructed to compare for theoretical and frequentist probability. The teachers were asked to report verbally and in writing all their doubts, impressions difficulties and how their students would respond to the issues worked during the activity. The teacher-researcher also noted these observations.

TECHNICAL APPLICATION OF THE ONTO-SEMIOTIC ANALYSIS TO MÔNICA'S RANDOM WALK

Analysis of the aspects relating to language

The sequence includes terms like: order, luck, blocks, walking through, sequence. It also includes expression like: pre-established order (deterministic trials), northern route, western route over the squares, opposite corner, change in the rules. These terms and expressions do not create semiotic conflicts of expression/content (one who broadcasts the broadcaster and one who interprets the receiver of information). The use of notations associated with the trial like $\Omega = \{HHHH, HHHX, HHXC, \dots\}$, sample space associated with the trial "tossing of the coin four times consecutively", or $P(HH) = P(H)P(H)$ if the events are independent or even $P(H_1|H_2)$ to symbolize the conditional probability of an outcome of heads at the second toss when heads had been shown in the first toss, has created difficulties to the teachers doing the course.

Analysis of concepts, procedures, and proprieties underlying in the sequence

Among the probabilistic concepts which were worked on we found: deterministic and random trials, sample space, probability of events, equally likely, tree diagram, probability calculation of simple events (Heads of Tails), the routes (compound events), of the friends to be visited utilizing the relative frequency (frequentist), FDT, bar graph, difference between the expected frequency by Binomial distribution (Laplace) and the observed value (relative experimental frequency) and the independence of events.

We observed that when asked if all of Mônica's friends will have the same probability of being visited the majority of the teachers answered yes. Among the justifications given the teachers argued that her friends lived at the same distance from her house and because the probability of obtaining Heads or Tails is the same. This kind of answer demonstrates that the teachers failed to perceive that some of Mônica's friends had more chances of being visited due to the larger number of routes to their houses in spite of the probability of a Heads outcome being the same as a Tails outcome.

Afterwards, they were asked to do the trial (tossing the coin four times for Mônica). The teachers began to realize that some friends were more visited than others. It became more evident when the data was systematized in FDT. The procedures used were: to picture the map (Fig. 1), toss the coin four times for Mônica, write down the results, draw the route followed by Mônica, systematize the trial results in a FDT, constructing the tree diagram, the bar graph, so on.

Facing this procedure the teachers could not understand the reason of the unequal probability of being visited until they built the tree diagram. The systematization of the results in this diagram allowed the teachers doing the course to see rule of formation for each friend to be visited. The routes were equally probable, but the friends were not equally probable, because to visit Magali there are six routes, therefore her probability of being visited is 6/16. Yet to visit Cascão (or Cebolinha) there are four routes so the probability was 4/16. To visit Horácio (or Bidu) there is only one route so this probability is 1/16.

Among the proprieties emerging from the activities we have: a) If two events are mutually excluded and complementary their union comprise the sample space and their probabilities are added 1; b) If two events, A and B on a single performance of an experiment this is called the intersection or joint probability de A and B , denoted as $P(A \cap B)$ or $P(AB)$. If two events, A and B are independent then the joint probability is equal $P(A)$ times $P(B)$; c) The meaning of the logical

implication is conflictive when the understanding of this logic rule goes through the correct. Interpretation of the meaning of the counter reciprocity law: for example “if I do not walk North it is because it did not come out Heads”, among others.

We observed that many teachers were using intuitively the proprieties and axioms of probability to justify their arguments. They attribute $\frac{1}{2}$ to the probability of a heads outcome and immediately $\frac{1}{2}$ to a Tail outcome. The verbal justification given was: “It can only be $\frac{1}{2}$ because there are only two possible results and their sum of both must be 1”. The mathematical explanation of this way of thinking and the explicit use of the three axioms of probability surprised the teachers, who became aware of a series of concepts they never knew they were unaware.

FINAL CONSIDERATIONS

The use of the technique give us a glimpse of positive results because it allow us to have a closer look on the objects involved in the mathematical activity making room for detailed evaluation of the sequence aiming at a better planning, outlining and efficiency of its use. The results show that even with teachers, who are teaching mathematics for several years, it was neither possible to advance in terms of the generalization of Binomial distribution nor work with conditional probability and independence of events. For this reason we believe it is necessary replicate the trial with students of the teaching of mathematics course, of the regular course and also students from the degree in mathematics course in order to investigate if this limitations due to the peculiarities of the course formation of working teachers or they are part of the proposed sequence.

The results point to the sequence viability to teach basic concepts of probability, but they also point out to the presence of several semiotic conflicts due mainly to the difficulties of the teachers with some of the knowledge involved , because they were seeing for the first time some of the concepts.

These results imply rethinking of some of the assumptions, especially if we want to insert them into virtual learning environments, so that teachers can implement it with autonomy.

NOTES

¹ Mônica is a character's comic book, very popular in Brazil

² FAPESB: Foundation for Research of the State of Bahia.

³ [...] refer to any disparity or discrepancy between the meanings assigned to the same expression in two subjects (people or institutions) in communicative interaction and may explain the difficulties and limitations of teaching and learning implemented (GODINO, 2002, p. 246).

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