THE CONCEPT OF MEAN BY PRIMARY SCHOOL STUDENTS

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This research, which was undertaken in Brazil, focuses on the different invariants of the concept of mean. Activities from 16 mathematics textbooks were analyzed, and the invariants explored were identified. Afterwards, a test was designed and later undertaken by 179 elementary school students (3rd and 5th grades) and 31 teachers. The results show that the textbooks emphasize "the usage of mean as a number different from any value of the group" and "the usage of mean with no correspondence on real life". A lacking on the textbooks' approach of mean was identified, as regarded by the following invariants: "the relation between mean and standard deviation"; "the mean is in between extreme values"; "the inclusion of null and negative values". Regarding the students´ capacity, the majority of 3rd grade students failed on solving problems regarding mean, and some of the 5th grade students were able to solve problems by compensating values.

COMPREENSION OF MEAN: INVARIANTS AND REPRESENTATION

In general, when inferring both in the academic field and in everyday life, we use mean or compare means (Pollatsek, Lima & Well, 1981). For non-gathered data, the simple average is calculated as a quotient between the sum of all variable values and the number of observations involved in the sum. For pondered or gathered data, the values of variables must be pondered by their respective weights or frequencies, in which case, the mean is said to be pondered. Such mathematical notations are shown as follow:

$$\bar{x} = \frac{\sum_{i=1}^{n} k_i x_i}{\sum_{i=1}^{n} k_i} = \frac{k_1 x_1 + k_2 x_2 + \ldots + k_n x_n}{k_1 + k_2 + \ldots + k_n}$$

Batanero et al. (1994) say that the knowledge of how to calculate mean does not imply its comprehension. One of the difficulties is to interpret mean when the values involved are all integers and the mean is a decimal number, such as "*the mean of kids by couple equals to 2.3*". Strauss and Bichler (1988) argue that the concept of mean is intimately related to the comprehension of the following mathematics properties:

A. the average is located in between the extremes values (minimum value ≤ average ≤ maximum value);
B. the sum of the deviation from the average is zero (Σ(Xi – average)=0);
C. the average is influenced by each and by all the values (average = ΣXi/n);
D. the average does not necessarily coincide with one of the values which composes it;
E. the average may be a number that does not have a correspondent in the physical reality (for example, the average number of kids per couple can be 2.3);
F. the calculation of the average takes into consideration all the values, including negatives and zero;
G. the average is a representative value of the data from which it has been calculated.

In this research, we are assuming the Conceptual Field Framework (Vergnaud, 1988), which considers that a development of mathematics knowledge is based on a set of problems and situations to approach concepts, procedure and representations. Thus, it is based on three aspects: situations that give meaning to a concept, invariants mobilized by the students and representations used to organize, treat and communicate concepts or procedures.

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This paper starts with the acknowledgement of these properties, in order to analyze how the Brazilian mathematics textbooks approach the concept of mean, and as a starting point to investigate the invariants students and teachers mobilize while solving means problems, using different representations.

Stella (2003) investigated Brazilian high school students' abilities on the usage of mean, revealing difficulties to: use the algorithm for simple and pondered means, calculate the mean when the number of values were not explicit and interpret graphs. Cazorla (2003), in investigating Brazilian undergraduate students, also shows that they were able to calculate mean by using the algorithm, to calculate a new mean when a new value is joined to the data and to calculate an unknown value when given both the mean and the other values. Nonetheless, they did not understand mean as a representative value of a group of values, being only able to define it through the use of its algorithm.

BRAZILIAN TEXTBOOKS' ACTIVITIES FOR MEAN

Based both on the recent research findings and on the mathematics properties of mean (Strauss & Bichler, 1988), 16 Collections of Brazilian mathematics textbooks from the 6th to the 9th grade of elementary schools were analyzed. The textbooks were the ones used in Brazilian public schools. Two hundred seventy three (273) activities and explanatory sessions were analyzed by identifying the properties involved. In order to simplify this text, from now on, we will call all of them activities.

The results (Figure 1) show that the Brazilian mathematics textbooks emphasized, from all the mean activities, only two of the seven properties.

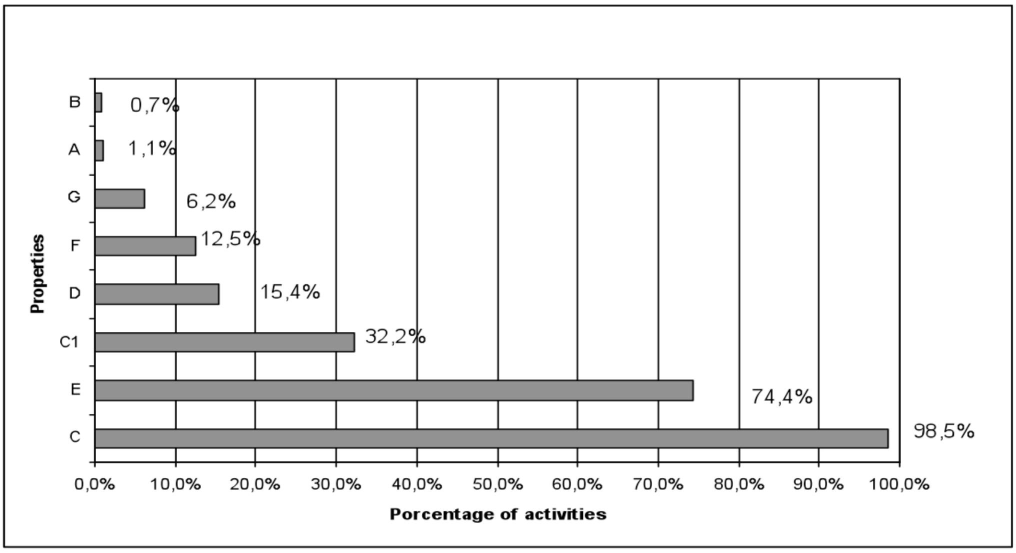

Figure 1. Percentage of activities that explores each of the properties

The properties A and B are very badly exploited. The fact that the *average is located in between the extreme values,* property A, have been observed only in three collections, representing 1.1% of the activities that explore average. Property B, *the sum of the deviation from the average is zero,* was observed only twice in a collection, representing 0.7% of the total of activities. In fact, the approach to standard deviation in Brazilian elementary mathematics textbooks is rare.

As regards to property C, the average is influenced by each and by all the values, it is explored in almost all the activities. Nonetheless, when examining those that require greater knowledge in order to calculate, this percentage decreases to 32.2%. This new percentage is coded as C1. Those findings are in accordance to Stella (2003) and Cazorla (2003), that point that students know the algorithm to calculate the mean, but have difficulties in using and interpreting mean, such as in this example: *The average height of a group of 5 basketball players is 1.85m. If another player with 1.97m became a new member of the group, the average height of the group* *will be ____m.* Moreover, these activities related to C1 are concentrated on five textbooks collections.

The percentage of the activities in which the mean does not coincide to one of the values of the group (property E) is quite high: 74.4%. In spite of that, a small percentage of activities explores a mean that is a number with no correspondence in the physical reality (property D): 15.4%.

Only 12.5% of the activities used zero or negative value on calculating mean (property F). Only two activities of different textbooks collections include negative values.

Very few activities emphasize mean in its representative aspect of a group. It was found in only 6.2% (property G). In general, it was observed that this property was emphasized when the group was not well represented by the mean.

Almost 14% of the activities give, in some sense, a meaning to average. Nonetheless, situations that involve different measures of central tendency, which force students to decide which measure is better to each point of view, were rarely found in the textbooks. Finally, we could observe that our results are in concordance with those by Stella (2003) and Cazorla (2003). Brazilian mathematics textbooks for 6th to 9th grade of elementary schools explore mainly the process of calculating mean. The students' difficulty in calculating the mean when one of the values, or the number of values, were not explicit, as reported by the authors, may be a reflection of the textbooks' emphasis on approaching mean through activities that calculate it by the use of its algorithm, while there are few activities exploiting other possible situations.

STUDENTS' AND TEACHERS' ABILITIES WITH MEAN

An investigation of students' and teachers' understanding of mean was undertaken with the usage of a test, which was taken by 179 students of the 2nd and 5th grades of elementary schools, and by 31 teachers. The subjects were asked to answer a test comprised of 7 questions related to the concept of mean, involving 4 of the seven properties pointed by Strauss and Bichler (1998): (C) *the average is influenced by each and by all the values (average =* $\Sigma Xi/n$); (D) *the average does not necessarily coincide with one of the values which are composed by it*; (E) *the average may be a number that does not have a correspondent in the physical reality*; (G) *the average is a representative value of the data from which it has been calculated*. Both the most exploited properties and the one that highlights its meaning were chosen from the analyzed textbooks. The following properties were considered for the test, with the usage of problems in which the meaning of mean varies: mean as equitable distribution; mean as a representative value of a group of values with a distribution that is approximately symmetric; to estimate an unknown quantity when there have been mistakes in the measures; to know what will be more probably obtained, when considering the existence of missing data on the distribution. The data on the activities were given in a natural language and with the usage of graphs.

Our data shows that students from both grades have had many difficulties in answering correctly. None of them show to understand *the mean as a representative value of a group in which the distribution was almost symmetric* that *it can be a number with no correspondence to physical reality* and that we also obtained very small percentage on understanding of the properties: (C) the average is influenced by each and by all the values (6.65% for 2nd and 9.6% for 5th grades students); (A) the average is located in between the extremes values (8% and 6.7%); (D) the average does not necessarily coincide with one of the values which composes it (5.3% e 3.8%); and (F) the calculation of the average takes into consideration all the values, including negatives and zero (1.3% and 4.8%).

The teachers also presented difficulties, since the percentage of right answers was 56.45% when dealing with activities that involve mean as a number that does not coincide to a value of the group, and 54.8% with those that present the mean in order to ask them to find a value that could compose the group.

As regard the properties, the results shows the following percentage of correct answer: *mean as a number which does not coincides to a value of the group* (61.3%); *the mean can be a value with no correspondence to physical reality* (19.4%); *the mean can be located between extremes values* (16.1%). The activities which present the mean in order as request them to find a value that could compose the group was correctly solved by 45.2% of the teachers.

According to the analysis of variance, it was found significant difference (p<.000) to all the questions between teachers and students, but it was not found difference among students from different grades.

There was no significant difference among the right answers to questions presented by natural languages or by graphic representation between none of the groups according to the analysis of variance. However, it does seem to influence the strategies used in answering, as well as the type of error. When the data were presented by graphs, the extreme values (maximum and minimum) were frequently used in the answer, approximately 20% of all questions. When the questions were presented in natural language, adding all the values was the most used answer (25%).

Thirty eight (38) students answered correctly one question, 12 students 2 questions and 3 students 3 questions. Thus, we can affirm that some students, even from the 2nd grade, were able to answer correctly some questions, thus revealing an understanding of the fact that the mean is influenced by each and by all the values of the group, including the nulls, as well as the knowledge that it is a representative value of the data from which it was computed. Thus, the starting of a systematic approach to mean even to early grade students shows promise.

CONCLUSIONS

This research points out that the Brazilian textbooks' approach to average is efficient on some aspects, excluding its concept. Contextualized situations are promoted so the students learn how to calculate the mean, and for them to understand that the mean can be a value different from those of the group.

Nonetheless, many properties of mean need more attention, such as the usage of mean represented by a number that does not have a correspondence in real life, the usage of nulls and negative values on calculating mean, the interpreting of the mean and the understanding of its representative aspect. Our findings point the same direction as Stella's (2003) and Cazorla's (2003).

Those findings may also be verified through the investigation of the students' and the teachers' comprehension. Nonetheless, the results show that even young children on the 2nd grade elementary school level can solve problems dealing with average.

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