A STATISTICAL GAME: THE SILENT COOPERATION PROBLEM

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In this paper an activity based on the following problem (Mosteller, 1965) is presented: Two strangers are separately asked to choose one of the positive integers and advised that if they both choose the same number, they both get a prize. If you were one of these people, what number would you choose? Some taste of statistical thinking and informal inference reasoning might be experienced by the students/teachers with this problem. Fifty-four teachers from different school levels participated in a two sessions experiment. In the first session, pairs of teachers were formed and each thought of a number in such way that they could not predict what number would be thought by his/her partner. The numbers thought were gathered, graphed in a distribution and analyzed. In the second, without advised in advance, the same experience was repeated. Distributions, patterns of variation, and making decisions are some themes that emerged.

INTRODUCTION

Statistics is used extensively in society to make decisions and to understand the world around us. However, statistics that is taught in schools fails to prepare students for such roles; students learn concepts and procedures but are not taught how to think with data. They are not given the opportunity to play "the game of statistics in an environment of exploratory data analysis" (Pfannkuch, 2008). Playing the game of statistics involves, at least, the conduct of research that follows the 'investigative cycle' model of statistical thinking of Wild and Pfannkuch (1999). The components of the model are: the problem, plan, data, analysis and conclusions (PPDAC).

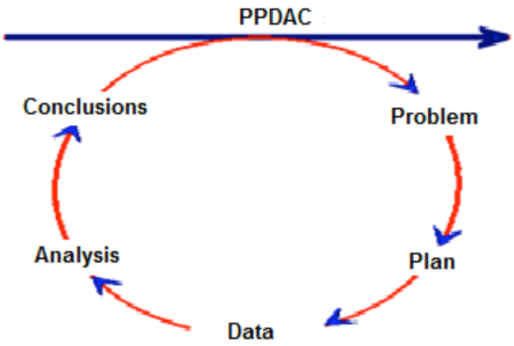

Figure 1. The research cycle

The challenge to design lessons where students take part in research is to find or produce statistical problems that are viable to work with in restrictive classroom conditions. It is not easy to overcome the challenge because many of the activities from which students can obtain data in the classroom may not be interesting (for example, measuring the heights of students in the group) or show deep-rooted concepts of statistics. The interesting activities are often difficult to design experimentally or to collect data.

The activity being proposed in this article is of interest to students and it can be done in the classroom. The activity also has some typical features of statistical research. First we will describe the problem and how to present it as an activity to students. Second, we will report on our findings when we presented the activity to teachers.

THE SILENT COOPERATION PROBLEM

The task, Silent Cooperation, proposed by Mosteller (1965) has features that allow lessons to be designed towards playing a statistics game. Problem number 11 of his fifty challenging problems in probability is:

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11. Silent Cooperation

> *Two strangers are separately asked to choose one of the positive whole numbers and advised that if they both choose the same number, they both get a prize. If you were one of these people, what number would you choose?*

In Mosteller's discussion of the Silent Cooperation problem he said:

> *I have not met anyone yet who would choose more than a one-digit number; and of these only 1, 3, and 7 have been chosen. Most of my informants choose 1, which seems on the face of it to be the natural choice. But 3 and 7 are popular choices* (p. 27).

Unfortunately Mosteller neither mentions anything about the environment under which his survey was carried out nor states the population he studied. He, however, suggests a variation pattern with the highest frequency being 1 then 3 and 7. We wonder if some other populations behave along the same pattern. This could be a good research problem.

THE POPULATION

Fifty four In-service teachers participated in a course of statistics (4 hours each fifteen days) as part of a three year teacher development program. Their ages oscillated between 24 to 45 years old. Seven of them are kinder garden teachers (4-5 years old), 8 are primary teachers (6-12 years old) and 32 are junior secondary teachers (13-15 years old), the other 7 assist other teachers without teach to students. Most of teachers had not taken a formal course of statistics; however, in this development program they took a probability course. In this, they studied the notion of probability and elementary distributions of probability. It could be convenient to clarify that the general level of teachers' mathematics knowledge was rather low.

THE ACTIVITY

The activity is carried out in two sessions. In the first session, held in the classroom, there was a contest to reward pairs of people who guessed the same number.

Teachers must ensure no foul play, namely; no pair converses before proposing their numbers. One way to avoid this is to form the pairs randomly and write them on a list that students do not know until after they have suggested their numbers.

A card is earlier given to them to write the number proposed. It is very important that students understand that the purpose is to think of a number which can also be proposed by their mate. However the teacher should not give any clues or suggestions, like saying "if you think of a very large number it is unlikely that your partner guess the same number". Students should be left to think freely a number.

Once the data are collected and the winning pairs chosen, the data are presented in a table of frequencies and then graphed. Computer software can also be used to get the graph of the distribution. The patterns of variation are analyzed, for example; did majority of the participants suggest numbers between 1 and 10? Are the odd numbers more frequent than even or vice versa? Which number has the highest frequencies? And so on. Here it is pertinent to explore the opportunities to win or not proposing large numbers.

In the second session, the participants, without previous notice, are asked to repeat the first activity. The teacher can argue that the activity is being repeated because there are still prizes left to be won (it may be just one prize, if it is attractive).

The activity is repeated, the data collected and analyzed just as was previously done but in addition the distribution obtained from the two activities are compared. This is to determine the change that has occurred in the frequencies of the second distribution as a result of knowledge or awareness of the first distribution.

This enables a discussion to be held on how to apply knowledge of the variation patterns observed in the first session to increase the likelihood of success in the second session.

THE ADVANTAGES OF THE ACTIVITY

The activity has the following statistical characteristics:

- Real data are collected in the classroom environment.

- There is uncertainty; a participant can neither predict the number that a pair thinks, nor the type of distribution that will be formed with the responses of the entire group.
- A frequency distribution is generated. For younger students, the activity may be a platform or motivation to understand and elaborate graphs of frequencies.
- The activity produces new statistical knowledge, which is the pattern of variation in the distribution of the numbers proposed.
- Knowledge of the pattern of variation can be utilized to make a better decision in the second session.

REPORT OF AN APPLICATION

Since we wondered whether Mosteller's findings on people's choice of number could be generalized to other populations we decided to conduct an experiment. We aimed to:

1. Investigate the variation pattern of the distribution of the predicted numbers
2. Determine whether people could learn from data by identifying reasons for their choice of number.

The procedure followed was similar to that given above: The task was explained to them, pairs are formed and small cards where they could write the number they think are given to them. The teachers were told that a pair who predicts the same number would get a prize. Once collected, the data were processed in Fathom and the corresponding frequency distribution drawn (Fig. 2). The teachers discussed the graph.

In the second session which was 15 days later, they were asked to write a number on a card given to them. This time was written the couples out on a list which they knew existed, but kept hidden; the purposed was that they did not know who their partner was. The cards were collected from them. They were then asked to respond in writing, the numbers they thought of in the first and second applications of the activity and to explain the reasons for choosing each number.

In the second session, just two pair won a prize. The frequency distribution of numbers predicted in this second session is shown in Figure 3.

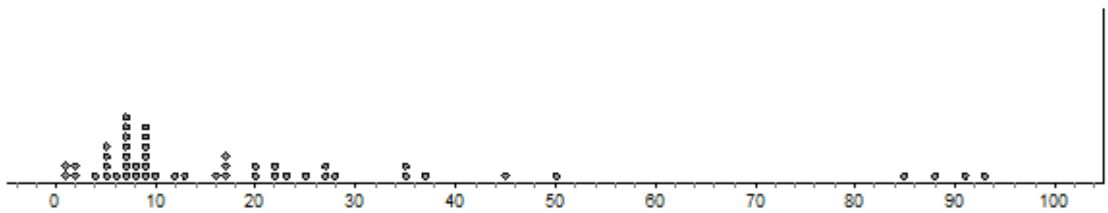

Figure 2. Distribution of numbers predicted in the first session

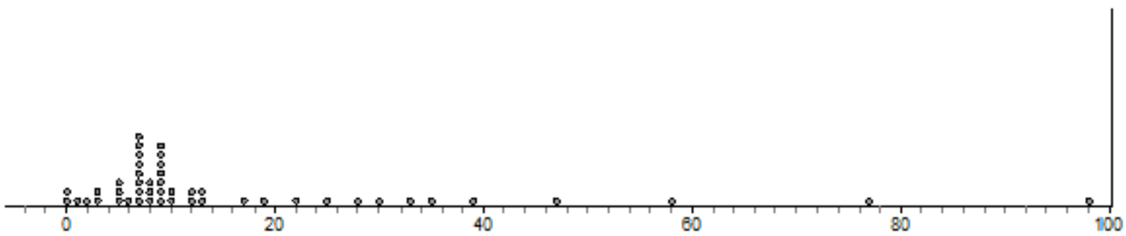

Figure 3. Distribution of numbers predicted in the second session

It can be observed that the changes were very few. In the distribution of numbers predicted in the first session, there are 26 answers between 1 and 10 (inclusive), in the distribution of the second session there are 28; an increase of just two units in the frequency of numbers between 1 and 10 predicted. Between 11 and 20, there were 8 in the distribution of the first session and 6 in the second session. Between 21 and 30, there were 7 and 4 respectively. The frequencies of the numbers 5, 7 and 9, increased by just one unit in the second session.

Five categories can be identified analyzing the possible reasons for the number first predicted:

1. *Personal Significance [PS]*. The choice was based only on personal experiences with a number; within these associations the most common is the birthday of the person.
2. *Plausibility that the number will be Predicted by the other* [PP]. Those who argue that they thought a number is likely to be predicted fall in this category. Such considerations are subjective and sometimes it is not clear the relationship between the number and nature of the alleged high possibility that the other will predict it.
3. *First Occurrence* [FO]. Classified in this group are those reasons which did not express a clear criterion for choosing the number, they say, for example: "it was the first number that come to my mind"
4. *Other*. The answers that do not fit into any of the above classifications. In this category were also classified reasons in which teachers argued that they had not understood that the number they predict should tally or match that of their partner.
5. *Based on the distributions* [BD]. The choice in the second session was based on the distribution of previous session [figure 1].

In the second session, 10 teachers proposed the same number that they had proposed in the first session, and 4 of them gave reasons based on the distribution in the second choice. Forty three teachers changed; but just 7 teachers could justify the change based on the distribution that was obtained in the first session. Can be seen in figure 4 that only 12 teachers gave reasons considering the distribution of the first session.

First choice \ Second choice	PS	PP	FO	Other	BD	
PS	12	2	0	4	5	23
PP	1	3	0	3	6	13
FO	5	1	0	1	0	7
Other	5	1	0	3	1	10
BD	0	0	0	0	0	0
	23	7	0	11	12	53

Figure 4. Frequency of type of reasons given in both sessions

CONCLUSIONS

The majority of the teachers were expected to choose a number based on their exposure to the distribution in the first session in order to raise their chances of matching that of their partners and get a prize. However, just 23% did, while the others continued to base their second choice on idiosyncratic criteria. This coincides with the observation of Burrill (2008) who notes that "... pre- and in-service teachers did not intuitively think of a graph as a tool to explore and understand data." We agree, but we would like to add to her comment "and as a tool to make decisions".

The activity we have presented can be used to argue that a distribution is not just an illustration of the data but also a means to know the reality of a phenomenon and how this knowledge should help in making decisions. If the teachers had understood how these ideas occurred in the activity that they performed, then they would have learned something about what it means to play a statistical game.

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