

## COMPARING THE EFFICIENCY OF MATHEMATICAL V. INTUITIVE EXPLANATIONS IN CONDITIONAL PROBABILITY.

Sigal Levy and Yelena Stukalin

Statistics Education Unit

The Academic College of Tel Aviv Yaffo, Tel Aviv, Israel

levy@mta.ac.il

*Some non-intuitive problems pose a challenge to students of probability. Such an example is the Monty Hall paradox, in which information provided by the "show host" is ignored. In our experiment, we provided three groups of students with different explanations of the solution: none, intuitive or technical/mathematical. Following that, we asked the students to solve another non-intuitive problem in conditional probability: what is the probability that twins are both male given (1) the older is male, or (2) one is male. Results show that students that had met the intuitive explanation performed better when solving the second problem. We therefore believe that presenting students with the intuition behind a solution may help them develop flexibility and better understanding of the underlying mechanism of conditional probability.*

### INTRODUCTION

Undergraduate level probability and statistics are considered by students to be intimidating topics, especially when their field of studies is not related to mathematics. In light of that, educators could benefit from a toolkit of strategies to improve the deep understanding of basic probabilistic and statistical concepts.

One such concept is that of conditional probability, that serves as the foundation for understanding statistical concepts such as hypothesis testing (Díaz and de la Fuente, 2006). The Monty Hall problem is an example of a commonly misunderstood problem in conditional probability. This problem describes a TV show in which the contestant is presented with 3 doors, one of which hides a valuable car while the other two hide useless goats. The contestant selects a door to open. Just before the door is opened, the hosts, who knows which door has a car behind it, opens one of the two doors that the contestant did not select. The door that the show hosts opens is guaranteed to have a goat behind it. At this point the contestant is given the option of changing his initial selection in favour of the third door, the one that is still closed. Should he indeed change his choice? Surprisingly, even though it seems that the additional information that is provided by the show host is irrelevant, the third door actually has a higher probability of hiding the car behind it than the door that was originally selected. This problem can be analysed both mathematically and intuitively (Freidman, 1998)

### METHOD

One hundred and fifty-nine first year, undergraduate biology students, composing 3 different groups, took part in the experiment. Even though the students had no statistical background, as this was the first course in statistics in their curriculum, they had above average mathematics background. The students were presented with the Monty Hall problem and then received one of 3 explanations to the counter-intuitive solution to the paradox: Group 1 served as a control group and did not receive any explanation. Group 2 students were shown a mathematical solution using a tree diagram. Group 3 students were presented with the following intuitive solution: "Imagine that after selecting a door at random you are given the choice of either holding on to your initial choice or opening the two remaining doors. Obviously the second option is better".

Following that example, the students were asked to solve a test question by themselves: "The Smith family just had non-identical twins. What is the probability that both twins are male, knowing that (1) the older is male, or (2) one of them is male?". We were interested in their answers to condition (2), as this is the counter-intuitive case: people automatically assume that the information regarding the child's birth-order is irrelevant, and that considering the gender of the other child can be done independently of that information. The question was presented in the form of a multiple-choice question, with the following options:

- 1/3 (correct answer),
- 1/2 (wrong, popular intuitive answer), and
- 1/4 (probability of the intersection – a common mistake in solving questions in conditional probability).

For ethical reasons, all groups were presented with both mathematical and intuitive solutions after they answered the test question.

## RESULTS

The distribution of the student's answers is shown in Table 1.

Table 1: *Distribution of responses by the type of explanation given. Numbers are N (%).*

	1/3 - Correct	1/2 - Intuitive	1/4
Control Group	8 (10%)	59 (77%)	10 (13%)
Mathematical Explanation	5 (10%)	26 (52%)	19 (38%)
Intuitive Explanation	8 (25%)	22 (69%)	2 (6%)

Results show that the group that was shown an intuitive solution to the Monty Hall problem performed better than the other groups in the test question ( $\chi^2(4)=20.3, p<.001$ ).

## CONCLUSION

These initial findings stress the importance of exposing statistics students to counter intuitive problems, and specifically to the underlying intuition behind the solutions of such problems. This exposure may help them develop flexibility and better understanding of the underlying mechanism of conditional probability. Further research is required in order to test these findings among students from different disciplines, whose mathematical background is weaker. Longitudinal study presenting more such problems, testing effect after students have seen more than one example.

This study is not without limitations. The setting of the experiment was such that we could not guarantee that all of the student who attended class indeed answered the test question. Additionally, even though the students were asked to solve the test question without consulting their fellow students, we had no way of preventing them from cooperating with each other.

## REFERENCES

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